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# EE 102 – Midterm I

10/24/07

Duration: 1 hour and 30 minutes

*The midterm is closed book and closed lecture notes.*
*You can use a single page of handwritten notes.*
*Please carefully justify all your answers.*

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**Problem 1:** Consider the systems $S_1 : [Reals \to Reals] \to [Reals \to Reals^+]$, $S_2 : [Reals \to Reals^+] \to [Reals \to Reals]$ and $S_3 : [Reals \to Reals] \to [Reals \to Reals]$ defined by:

$$\forall x \in [Reals \to Reals],\ \forall t \in Reals, \qquad S_1(x)(t) = x^2(t) + x(t) + 1$$
$$\forall x \in [Reals \to Reals^+],\ \forall t \in Reals, \qquad S_2(x)(t) = \sqrt{x(-2t)}$$
$$\forall x \in [Reals \to Reals],\ \forall t \in Reals, \qquad S_3(x)(t) = -x(t)$$

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1. Is the system $S_1$ memoryless? yes; depends only on input at certain time and not before or after

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2. Is the system defined by the diagram in Figure 1 memoryless?

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3. Can we construct a system according to the diagram in Figure 2?

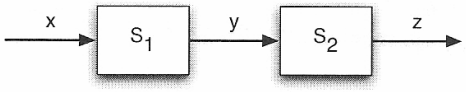


Figure 1: System from Problem 1.

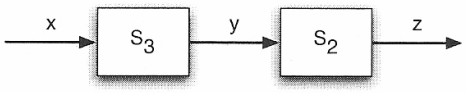


Figure 2: System from Problem 1.

no because y is negative and $S_2$ takes the root and cannot take root of negative # and make it into Real+

Figure 1 is a composition, therefore it is not memoryless because $S_2$ depends on the output of $S_1$ ?

**Problem 2:** Consider the finite state system $S$ described by the transition diagram in Figure 3.

1. What are the sets *States*, *Inputs* and *Outputs* for system $S$?
2. Construct the functions *update* and *output* for system $S$.
3. Consider the signal $x : \{1, 2, \ldots, 6\} \to \mathit{Inputs}$ defined by:

$$x(1) = 0,\ x(2) = 1,\ x(3) = 0,\ x(4) = 1,\ x(5) = 1,\ x(6) = 0$$

   What is the signal $y = S(x)$?

4. Show that $S(x)(n) = g(x(n), x(n-1))$ for $n > 1$ where $g : \{0,1\} \times \{0,1\} \to \{0,1\}$ is the function defined by the graph:

$$\{((0,0),0),((1,0),0),((0,1),0),((1,1),1)\}$$

5. Is system $S$ memoryless?

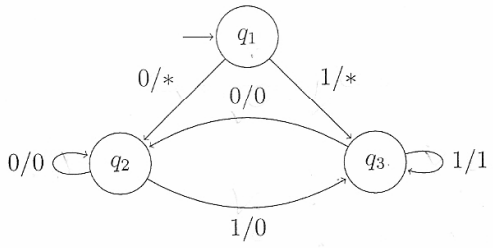


Figure 3: Transition diagram for Problem 2.

1) States = $\{q_1, q_2, q_3\}$
Inputs = $\{0, 1\}$
outputs = $\{0, 1, *\}$

2) update = $\begin{cases} q_1 & \text{if } n = 0 \text{ (or } s(0) \text{ - initial position)} \\ q_2 & \text{if } (s(n) = q_1 \vee s(n) = q_2 \vee s(n) = q_3) \wedge x(n) = 0 \\ q_3 & \text{if } (s(n) = q_1 \vee s(n) = q_2 \vee s(n) = q_3) \wedge x(n) = 1 \end{cases}$

not really needed or does not belong in update, just shows initial condition

output = $\begin{cases} 0 & \text{if } s(n) = q_2 \wedge (x(n) = 0 \vee x(n) = 1) \\ 0 & \text{if } s(n) = q_3 \wedge x(n) = 0 \\ 1 & \text{if } s(n) = q_3 \wedge x(n) = 1 \\ * & \text{if } s(n) = q_1 \wedge (x(n) = 0 \vee x(n) = 1) \end{cases}$

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**Problem 3:** Let $S$ be the system defined by:

$$\forall x \in [Integers \to Reals],\ \forall n \in Integers, \qquad S(x)(n) = 2x(n) + 3x(n-1) + 4x(n-2)$$

6.5 1. Construct the $[A, B, C, D]$ representation of this system.

6.5 2. Compute the impulse response.

0 3. Assume that the initial state is zero and consider the signal:

$$x(0) = 0,\ x(1) = 4,\ x(2) = 3,\ x(3) = 2,\ x(4) = 1,\ x(n) = 0 \text{ for all } n \notin \{0, 1, 2, 3, 4\}.$$

Compute $y = S(x)$ using the impulse response.

4 4. Compute $y = S(x)$ directly, that is, without using the impulse response. Do the results agree?

6.5 5. Pick an initial state of your choice, but different from zero, and compute $y = S(z)$ where the signal $z$ satisfies $z(n) = 0$ for all $n \in Integers$.

5.5 6. Combine your answer to the previous two questions to obtain $y = S(x)$ when the system starts at the initial state used in 5.

* $y = S(x) \to S(x)(n) = y(n) = 2x(n) + 3x(n-1) + 4x(n-2)$

1) choose $s(n) = \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix}$ ✓

$\begin{bmatrix} x(n-1) \\ 0 \end{bmatrix} + \begin{bmatrix} x(n-1) \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} x(n)$

$s(n+1) = A\,s(n) + b\,x(n) \to \begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = A \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + b\,x(n)$

$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

$A^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$y(n) = c^T s(n) + d\,x(n)$

$= \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x(n-2) \end{bmatrix} + d\,x(n)$ ✓

$c^T \qquad\qquad\qquad\qquad\quad d = 2$

$= 3x(n-1) + 4x(n-2) + 2x(n) = y(n)$

$A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\to A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad d = 2$ ✓

2) impulse response $\forall n \in Integers,\ h(n) = \begin{cases} 0 & \text{if } n < 0 \\ d & \text{if } n = 0 \\ c^T A^{n-1} b & \text{if } n \geq 1 \end{cases}$ ✓



$h(n) = \begin{cases} 0 & \text{if } n < 0 \\ 2 & \text{if } n = 0 \\ \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}^{n-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \text{if } n \geq 1 \end{cases}$ ✓