ECE102, Systems and Signals **Name:** ______________
UCLA Summer 2020
Final Exam
08/17/2020-08/18/2020
Time Limit: 24 hours

(a) This exam contains 9 pages (including this cover page) and 7 problems. Total of points is 100.

(b) You must scan and submit your solutions in a single file (preferably in PDF format) via the CCLE portal by **Tuesday 08/18/20, 6:00pm at the latest**. Please note that the portal will shut down at 6:00pm sharp.

(c) Please make sure to write your full name on your papers and also **include your name as part of your file name**.

(d) Please fully justify your answers and clearly show ALL the intermediate steps and calculations in all your solutions. And, when appropriate, box your final answer.

(e) Open books and notes.

(f) Feel free to use any calculators and computers, including MATLAB/Python, if you like. But, except when specifically asked for, using them is not advisable and will likely waste your time without helping much for the solutions. And, regardless, please remember that all your intermediate steps and calculations must be included in your answers.

(g) And finally, no collaboration please. You are expected to work on the solutions individually. Unreasonably similar write-ups and calculation steps would be heavily penalized on all parties suspected of collaboration.

(h) **Good Luck...**

Grade Table (for instructor use only)

Question	Points	Score
1	10	
2	14	
3	12	
4	18	
5	12	
6	12	
7	22	
Total:	100	

1. (10 points) **Review**
Carefully read each statement below and identify it as *True* or *False* and, for each answer, **explain your reasoning briefly in 1-2 sentences.**

    1. The phase response of a causal filter can never be identically zero.
    True or False? Why?
    2. For a filter to be distortion-less, its group delay must be constant over all frequencies.
    True or False? Why?
    3. A signal can be both time-limited in the time domain, and band-limited in the frequency domain.
    True or False? Why?
    4. The frequency spectrum of a discrete-time signal will always be periodic in frequency.
    True or False? Why?
    5. The higher the damping ratio of a second order system, the larger the peak in the magnitude of its frequency response.
    True or False? Why?
    6. The Region of Convergence for a non-causal infinite-duration signal will always be the entire s-plane.
    True or False? Why?
    7. If all poles of a system reside on the open left half plane, it will be BIBO stable, regardless of whether or not it is causal.
    True or False? Why?
    8. The natural response of a system can be affected by both its input as well as its initial conditions.
    True or False? Why?
    9. The step response of a system with a strictly proper transfer function will always be continuous at $t = 0$.
    True or False? Why?
    10. The Final Value Theorem can be used to obtain the steady-state response of a marginally stable system to a step input.
    True or False? Why?

2. (14 points) Consider the continuous-time signal $x(t)$ given as:

$$x(t) = 1000\,\text{sinc}^2(1000t)$$

(a) (5 points) Obtain its Fourier Transform $X(f)$ and plot its magnitude and phase.

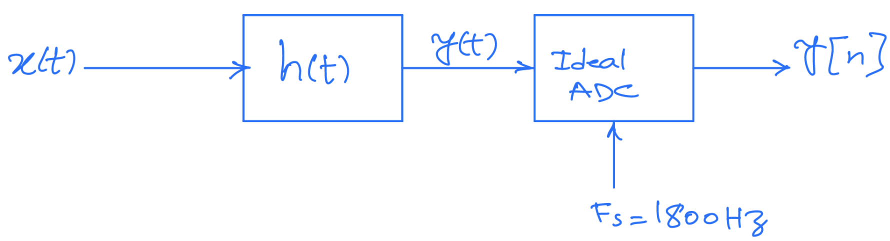
(b) (2 points) Assume that the signal $x(t)$ is sampled at the rate of $F_s = 1800$ Hz. Will we be able to reconstruct the signal $x(t)$ from its samples? Please explain.

(c) (3 points) Consider a filter with the following impulse response:

$$h(t) = 1600\,\text{sinc}(1600t)$$

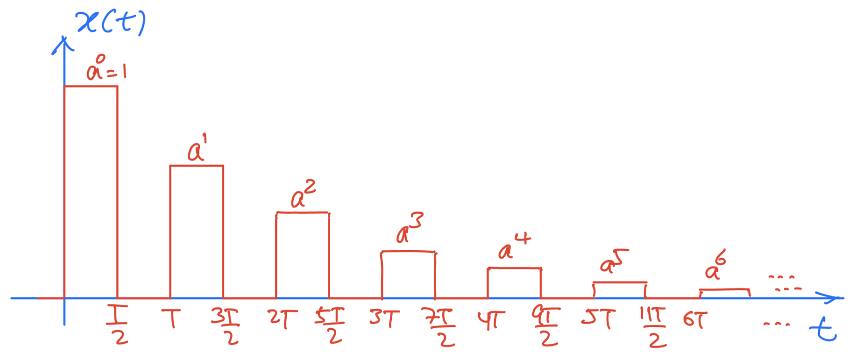
What kind of a filter is this? What is its cut-off frequency in Hz? Plot the magnitude and phase of its frequency response.

(d) (4 points) Imagine the same signal $x(t)$ in Part (a) first passes through the filter in Part (c) and then gets sampled, as shown below:



Plot the frequency spectrum of $y[n]$ with respect to frequency in Hz. Can we now reconstruct the signal $y(t)$ from its samples $y[n]$? Please explain.

3. (12 points) Consider the signal $x(t)$ as shown below, where $T > 0$ and $0 < a < 1$ are constant parameters. The amplitude of the k-th pulse is equal to a^k, $\quad (k = 0, \ldots, \infty)$ and its duration is $\frac{T}{2}$.

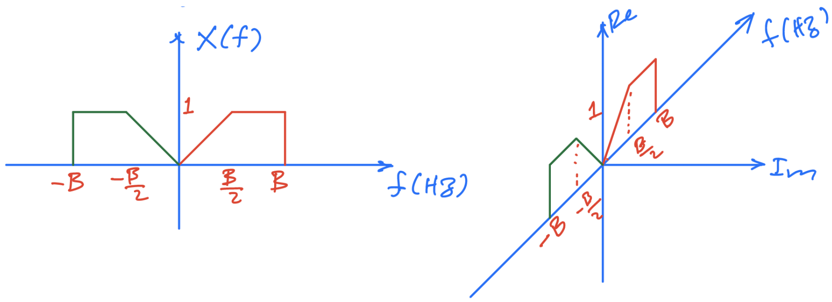


(a) (9 points) Obtain its Laplace Transform. (Bilateral and Unilateral Laplace Transforms would be the same, why?).

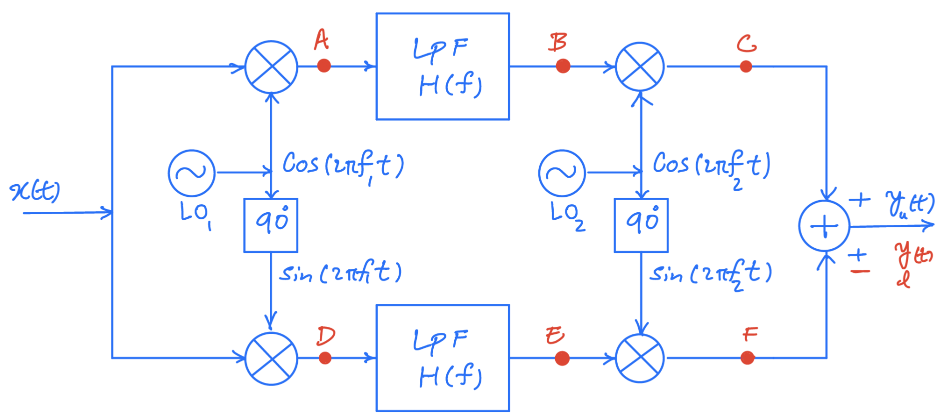
Hint: Write $x(t)$ as a summation of scaled and delayed unit step functions.

(b) (3 points) Determine the Region of Convergence (ROC) for the Laplace Transform you obtained in Part (a).

4. (18 points) Consider an information-carrying signal $x(t)$ with one-sided bandwidth B Hz and the frequency spectrum shown below:



This signal is then modulated using a modulator with the following architecture:



As shown, $x(t)$ first goes through a so-called quadrature mixer whereby it gets multiplied (mixed) with $\cos(2\pi f_1 t)$ and $\sin(2\pi f_1 t)$ outputs of the first Local Oscillator (LO), respectively on the upper and lower paths. The signals on the two paths then pass though two identical and ideal lowpass filters with frequency response $H(f)$, and subsequently pass through a second set of quadrature mixers, and get multiplied by $\cos(2\pi f_2 t)$ and $\sin(2\pi f_2 t)$. Finally, the two signal paths may be added or subtracted to produce the final modulator output.

(a) (12 points) Assume the frequency of the first LO is set to $f_1 = \frac{B}{2}$ Hz and the second LO has the frequency $f_2 = f_c + \frac{B}{2}$ for some frequency $f_c \gg B$. Also assume $H(f)$ is an ideal lowpass filter with cut-off frequency at $\frac{B}{2}$ Hz. Finally, assume that the two signal paths get added at the final junction to produce $y_u(t) = x_C(t) + x_F(t)$, where the subscripts on x correspond to the indicated points on the block diagram. Carefully obtain and plot the frequency spectra of all signals, i.e., $X_A(f), X_B(f), X_C(f), X_D(f), X_E(f), X_F(f)$ and the final output $Y_u(f)$.

(b) (6 points) For this part, we only change the frequency of the second LO and we subtract the two signal paths. Specifically, assume $f_1 = \frac{B}{2}$ Hz and $f_2 = f_c - \frac{B}{2}$ for some frequency $f_c \gg B$. And the same ideal lowpass filter $H(f)$ with cut-off at

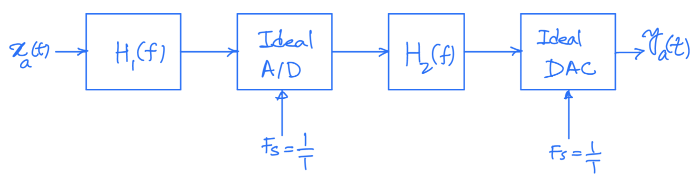
$\frac{B}{2}$ Hz. And assume the two signal paths are now subtracted, i.e., the modulator output is $y_l(t) = x_C(t) - x_F(t)$. Carefully obtain and plot the frequency spectra $X_C(f), X_F(f)$, and the output $Y_l(f)$.

Hint #1: It would be best if you use two pen colors to show the two sidebands as I have done in my spectrum plot above.

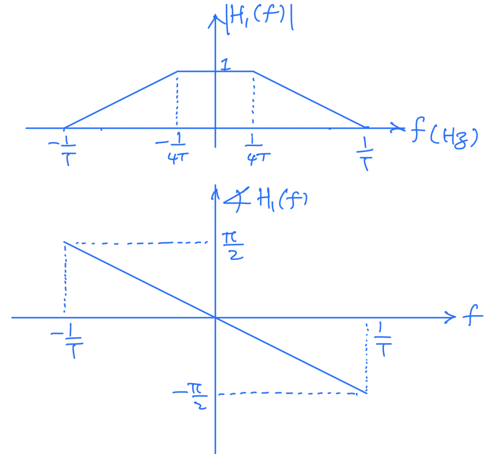
Hint #2: It would be best to plot in 3D with real and imaginary axes, as shown on the right in my spectrum plot above. This would help you properly track the phase rotations as the signals pass through the mixers.

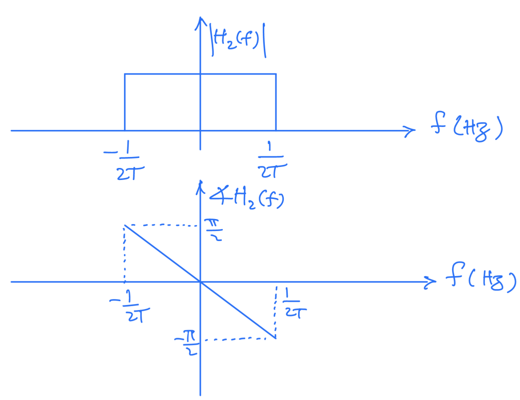
Hint #3: When parts of the spectrum overlap, show them as overlapping, i.e., in the intermediate points in the signal paths, you don't need to actually add the overlapping spectra to find the aggregate result.

5. (12 points) Consider the following filtering and sampling system where $F_s = \frac{1}{T}$ is our sampling rate. As shown, the input signal first passes through a filter $H_1(f)$, then gets ideally sampled (i.e., multiplied by an impulse train), then passes through a second filter $H_2(f)$, and finally ideally reconstructed:



Assume the following frequency responses for the two filters:



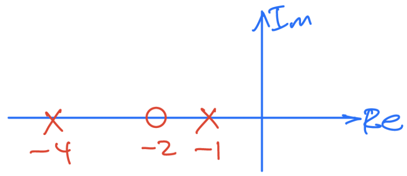


Assume the input signal is given as follows:

$$x_a(t) = 10 + 2\cos\left(\frac{\pi t}{2T} + \frac{\pi}{8}\right) + 6\sin\left(\frac{3\pi t}{2T} + \frac{3\pi}{8}\right) + 5\cos\left(\frac{5\pi t}{2T} + \frac{\pi}{4}\right)$$

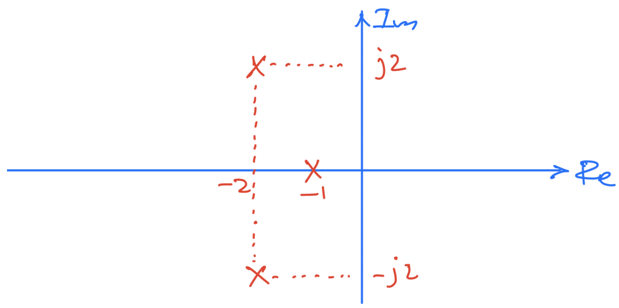
Find $y_a(t)$ and carefully plot its frequency spectrum $Y_a(f)$.

6. (12 points) (a) (6 points) We have a causal continuous-time LTI system whose transfer function has the following pole-zero diagram:



Obtain its *Step Response* $g(t)$, assuming that, in the steady-state, $g(t) \to 1$ as $t \to \infty$.

(b) (3 points) Consider another causal LTI system with the following pole-zero diagram:

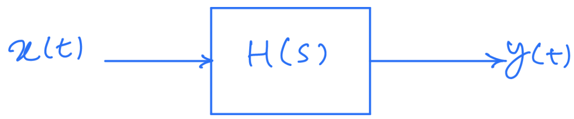


We applied a periodic signal at the input to this system, and measured the *DC term* at its output, and observed that the output DC term is half the DC term at the input, i.e., $b_0 = a_0/2$ where b_k and a_k are Fourier Series coefficients for the output signal and the input signal respectively. Find the complete Transfer Function for this system.

(c) (3 points) Consider the same system with the transfer function you obtained in Part (b), and assume we apply a periodic input signal with the fundamental period $T = \pi$ seconds. What would be the ratio of the second harmonic of the output signal to the second harmonic of the input signal, i.e., $\frac{b_2}{a_2}$?

7. (22 points) We have a causal continuous-time LTI system described by the following Linear Constant Coefficient Differential Equation (LCCDE):

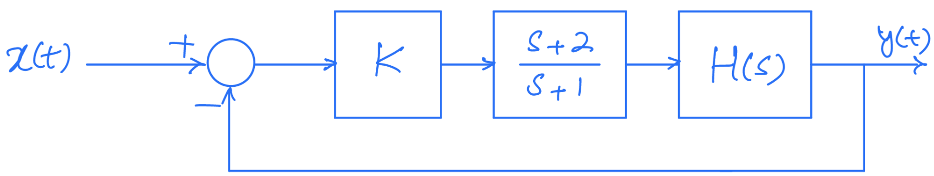
$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + x(t)$$



(a) (2 points) Find the transfer function for this system $H(s)$.

(b) (3 points) Realize this system in Direct Form, i.e, present a block diagram implementation using only gains, integrators, and summing junctions.

(c) (3 points) We now use the above system in a feedback loop as shown below:



Obtain the closed-loop transfer function from input $x(t)$ to the output $y(t)$, and find an acceptable range for the gain K that would ensure the full closed-loop system would remain BIBO stable.

(d) (8 points) Consider the original open-loop system in Part (a), with the given LCCDE. Assume the input signal $x(t) = (1+e^{-3t})u(t)$ along with the initial conditions $y(0^-) = -1$ and $y^{(1)}(0^-) = 3$. Obtain the full system response $y(t)$, and identify the *zero-state* and *zero-input* components in the system response.

(e) (3 points) Identify the *transient* and *steady-state* responses of the system for the same input and initial conditions.

(f) (3 points) Identify the *forced* and *natural* responses of the system for the same input and initial conditions.