This exam has 6 questions, for a total of 90 points.

Closed book. Two two-sided cheat-sheets allowed.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please, write your name and ID on the top of each loose sheet!

Name and ID: ____________________

Name of person on your left: ____________________

Name of person on your right: ____________________

Question	Points	Score
1	10	
2	15	
3	20	
4	15	
5	15	
6	15	
Total:	90	

1. The output of a linear, time-invariant, causal system is $y(t) = \mathrm{e}^{-2t}\sin(t)\mathrm{u}(t)$ when the input is $x(t) = \delta(t) + \mathrm{u}(t)$.

   (a) (5 points) Find the transfer function $H(s)$ of the system and its region of convergence.

   (b) (5 points) Find the frequency response $\mathcal{F}[h](\omega)$ of the system. Is $\mathcal{F}[h](\omega) = H(s)|_{s=\mathrm{j}\omega}$? Why?

2. Consider a linear, time-invariant, causal system described by

$$H(s) = \frac{s}{(s+2)^2}.$$

(a) (5 points) Derive the impulse response function, $h(t)$.

(b) (10 points) Compute the output, $y(t)$, corresponding to the input $x(t) = \cos(t)$.

3. Consider the signal, $x(t)$, whose Fourier transform, $X(\omega) = |X(\omega)|e^{j\Theta(\omega)}$, is defined as follows:

$$|X(\omega)| = \begin{cases} 1, & -2 < \omega < -1 \text{ and } 1 < \omega < 2 \\ 0, & \text{elsewhere} \end{cases} \qquad \Theta(\omega) = \begin{cases} \frac{\pi}{2}, & \omega < 0 \\ -\frac{\pi}{2}, & \omega > 0 \end{cases}$$

(a) (5 points) Write down the mathematical expression for $X(\omega)$ by using the unit step function, $u(\omega)$.

(b) (5 points) Do you expect $x(t)$ to be a real signal or an imaginary signal? Why? Answer without computing $x(t)$ explicitly.

(c) (10 points) Derive the expression for $x(t)$. Simplify as much as you can, for example, write all complex exponentials in terms of their real and imaginary parts.

4. (a) (8 points) The signal $y(t)$ is obtained by multiplying the signal $x(t)$ of problem 3 with $c(t) = \cos(3t)$, i.e., $y(t) = x(t)\, c(t)$. Compute the Fourier transform of $y(t)$, $Y(\omega)$.

   (b) (7 points) The signal $y(t)$ is now passed through a linear, time-invariant system with impulse response equal to

$$h(t) = \frac{3}{\pi}\text{sinc}(3t).$$

   Compute the output, $z(t)$, of this system. [Hint: It may be easier to work in the frequency domain, i.e., to first compute $Z(\omega)$.]

5. The impulse response of a linear, time-invariant system is

$$h(t) = 2\,\frac{\sin(2t)}{\pi t}\,[\cos(6t)]^2 .$$

(a) (10 points) Find the frequency response of the system, $H(\omega)$.

(b) (5 points) Find the output, $y(t)$, when the input is $x(t) = 1 + \sin(2.5t) + \cos(5t)$.

6. Consider the signal $x(t)$ periodic of period $T = 2$, defined as $x(t) = t^2$, $-1 < t < 1$. The Fourier sine-cosine series expansion of $x(t)$ is given by

$$x(t) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi t).$$

(a) (10 points) Using Parseval's theorem, calculate $\sum_{n=1}^{\infty} n^{-4}$.

(b) (5 points) Write the expression for the mean square error ϵ_1^2 when $x(t)$ is approximated by terms up to the first harmonic.