UCLA — Electrical Engineering Dept.
ECE102: Systems and Signals — Midterm Exam
Wednesday, May 2, 2018

This exam has 4 questions, for a total of 50 points.

Closed book. No calculator. No cheat-sheet.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please, write your name and ID on the top of each loose sheet!

Name and ID: ______________________________

Name of person on your left: ______________________________

Name of person on your right: ______________________________

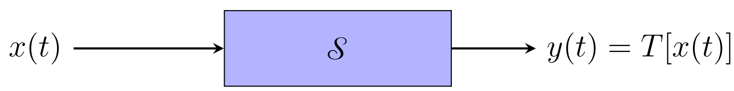
Question	Points	Score
1	12	
2	10	
3	11	
4	17	
Total:	50	

Please go to next page...

Multiple-choice questions – Check all the answers that apply.

1. (a) (3 points) Consider the system with input-output relation:

$$y(t) = T[x(t)] = x\left(\sqrt{t}\right).$$



Is it

☐ linear?
☐ time-invariant?
☐ causal?

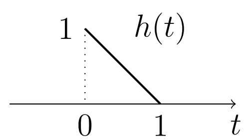
(b) (3 points) Consider the system with input-output relation;

$$y(t) = \int_{-\infty}^{t} \mathrm{e}^{x(\tau)}\,\mathrm{d}\tau.$$

Is it

☐ linear?
☐ time-invariant?
☐ causal?

(c) (3 points) Consider the linear, time-invariant, causal system with impulse response given by:



What is the step response of this system?

☐ t^2, $0 < t < 1$; 1, $t > 1$?
☐ $1 - t^2$, $0 < t < 1$; 0, $t > 1$?
☐ $t - \frac{1}{2}t^2$, $0 < t < 1$; $\frac{1}{2}$, $t > 1$?

(d) (3 points) Consider the system described by the following differential equation:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + \frac{2t}{1+t^2}y(t) = \frac{1}{1+t^2}x(t), \quad t > 0, y(0) = 0.$$

Is this system

☐ linear?
☐ time-invariant?
☐ causal?

2. Consider a linear, time-invariant, causal system $\mathcal{S}$. When the input is $x_1(t) = \mathrm{u}(t) - \mathrm{u}(t-1)$, the output is

$$y_1(t) = \begin{cases} 1 - \mathrm{e}^{-t}, & 0 < t < 1, \\ (\mathrm{e}-1)\mathrm{e}^{-t}, & t > 1. \end{cases}$$

(a) (3 points) What is the system's response to

$$x_2(t) = \mathrm{u}(t) - \mathrm{u}(t-2)?$$

(b) (4 points) From the answer to the previous part, argue what the response is to the input $x_3(t) = \mathrm{u}(t)$.

(c) (3 points) What is the impulse response of $\mathcal{S}$?

Please go to next page...

3. Consider the series of two linear, time-invariant systems, with impulse responses $h_1(t) = \mathrm{e}^{-t}\mathrm{u}(t)$ and $h_2(t) = \mathrm{e}^{t}\mathrm{u}(-t)$.
   (a) (2 points) Are the systems causal?
   (b) (3 points) What is the impulse response of the overall system?
   (c) (3 points) Is the overall system linear? Time-invariant? Causal?
   (d) (3 points) What is the system's step response?

4. (a) (3 points) Using the table or the definition, compute the Laplace transform of

$$x_1(t) = \mathrm{u}(t) - \mathrm{u}(t-1),$$

and indicate the region of convergence. (Hint: make sure you consider what happens for $s = 0$. You may want to use L'Hôpital's rule, i.e., if you have a zero-over-zero indeterminate case, you compute the ratio of the derivative of the numerator and the derivative of the denominator.)

(b) (2 points) Compute the Laplace transform of

$$x_2(t) = \delta(t) - \delta(t-1)$$

and provide the region of convergence.

(c) (2 points) Observing the relationship between answers to (a) and (b), establish the Laplace transform of $x_1(t)$.

(d) (5 points) Using your answer to part (b), provide the Laplace transform of

$$x_3(t) = \sum_{k=0}^{\infty} (-1)^k \delta(t-k).$$

(Hint: you may need the series $\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$ for $|a| < 1$.)

(e) (5 points) Using the tables, provide the Laplace transform of

$$x_4(t) = \int_{t_0}^{t} \sin(\omega_0(\tau - t_0))\mathrm{e}^{-\tau}\,\mathrm{d}\tau.$$

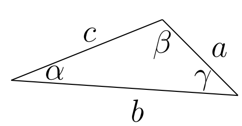
$f(t)$	$F(s) = \mathcal{L}[f](s) = \int_{0-}^{\infty} \mathrm{e}^{-st} f(t)\ \mathrm{d}t$
$\delta(t-a)$	e^{-as}, ROC $= \mathbb{C}$
$\mathrm{u}(t)$	$\frac{1}{s}$, $\mathrm{Re}\{s\} > 0$
$t^n \mathrm{u}(t)$, $n \geq 0$ integer	$\frac{n!}{s^{n+1}}$, $\mathrm{Re}\{s\} > 0$
$\mathrm{e}^{at}\mathrm{u}(t)$	$\frac{1}{s-a}$, $\mathrm{Re}\{s\} > \mathrm{Re}\{a\}$
$t\,\mathrm{e}^{at}\mathrm{u}(t)$	$\frac{1}{(s-a)^2}$, $\mathrm{Re}\{s\} > \mathrm{Re}\{a\}$
$\cos(\omega_0 t)\mathrm{u}(t)$	$\frac{s}{s^2+\omega_0^2}$, $\mathrm{Re}\{s\} > 0$
$\sin(\omega_0 t)\mathrm{u}(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$, $\mathrm{Re}\{s\} > 0$
$\cosh(kt)\mathrm{u}(t)$	$\frac{s}{s^2-k^2}$, $\mathrm{Re}\{s\} > \lvert k \rvert$
$\sinh(kt)\mathrm{u}(t)$	$\frac{k}{s^2-k^2}$, $\mathrm{Re}\{s\} > \lvert k \rvert$
$\mathrm{u}(t-a)$	$\frac{\mathrm{e}^{-as}}{s}$, $\mathrm{Re}\{s\} > 0$
$\frac{\mathrm{d}}{\mathrm{d}t} f(t)$	$sF(s) - f(0-)$
$\frac{\mathrm{d}^n}{\mathrm{d}t^n} f(t)$	$s^n F(s) - s^{n-1} f(0-) - s^{n-2} f'(0-) - \cdots$ $\cdots - s f^{(n-2)}(0-) - f^{(n-1)}(0-)$
$\int_0^t f(\tau)\ \mathrm{d}\tau$	$\frac{1}{s} F(s)$
$\mathrm{e}^{at} f(t)$	$F(s-a)$
$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\ \mathrm{d}\tau$	$F(s)G(s)$
$t^n f(t)$, $n = 1, 2, \ldots$	$(-1)^n \frac{\mathrm{d}^n}{\mathrm{d}s^n} F(s)$
$\frac{f(t)}{t}$	$\int_s^{\infty} F(\sigma)\ \mathrm{d}\sigma$
$\mathrm{u}(t-a) f(t-a)$, $a \geq 0$	$\mathrm{e}^{-as} F(s)$
$f(at)$, $a > 0$	$\frac{1}{a} F\left(\frac{s}{a}\right)$

Product to Sum	Sum to Product
$2\sin(\alpha)\sin(\beta) = \cos(\alpha-\beta) - \cos(\alpha+\beta)$	$\sin(\alpha)+\sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta)$	$\sin(\alpha)-\sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta)$	$\cos(\alpha)+\cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta)$	$\cos(\alpha)-\cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Sum/Difference	Pythagorean Identity
$\sin(\alpha\pm\beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$	$\sin^2(\alpha)+\cos^2(\alpha) = 1$
$\cos(\alpha\pm\beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$	
$\tan(\alpha\pm\beta) = \dfrac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$	

Even/Odd	Periodic Identities
$\sin(-\alpha) = -\sin(\alpha)$	$\sin(\alpha+2\pi n) = \sin(\alpha)$
$\cos(-\alpha) = \cos(\alpha)$	$\cos(\alpha+2\pi n) = \cos(\alpha)$
$\tan(-\alpha) = -\tan(\alpha)$	$\tan(\alpha+\pi n) = \tan(\alpha)$

Double-Angle Identities	Half-Angle Identities
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$	$\sin\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\alpha)}{2}}$
$\cos(2\alpha) = \cos^2(\alpha)-\sin^2(\alpha)$	$\cos\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1+\cos(\alpha)}{2}}$
$\tan(2\alpha) = \dfrac{2\tan(\alpha)}{1-\tan^2(\alpha)}$	$\tan\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\alpha)}{1+\cos(\alpha)}}$

Laws of Sines, Cosines, and Tangents	Mollweide's Formula
$\sin(\alpha)/a = \sin(\beta)/b = \sin(\gamma)/c$	$(a+b)/c = \cos\left((\alpha-\beta)/2\right)/\sin\left(\gamma/2\right)$
$a^2 = b^2+c^2-2bc\cos(\alpha)$	
$\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$	