UCLA — Electrical Engineering Dept.
ECE102: Systems and Signals — Final Exam
Monday, June 11, 2018

This exam has 5 questions, for a total of 75 points.

Closed book. One two-sided cheat-sheet allowed.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please, write your name and ID on the top of each loose sheet!

Name and ID: ______________________________

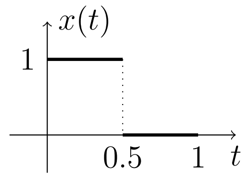
Name of person on your left: ______________________________

Name of person on your right: ______________________________

Question	Points	Score
1	10	
2	26	
3	15	
4	8	
5	16	
Total:	75	

Multiple-choice questions – Check all the answers that apply.

1. (a) (1 point) Consider the periodic signal $x(t)$, whose period is drawn below:



Are the Fourier coefficients, X_n, $|n| > 0$

- ◯ purely real?
- ◯ purely imaginary?
- ◯ neither purely real nor purely imaginary?

(b) (1 point) What is the region of convergence of $F(s) = \mathcal{L}[f](s)$, where

$$f(t) = \mathrm{e}^{-t^2}\,\mathrm{rect}(t-1).$$

- ◯ $\mathrm{Re}(s) > 0$
- ◯ $\mathrm{Re}(s) > -1$
- ◯ $\mathbb{C}$

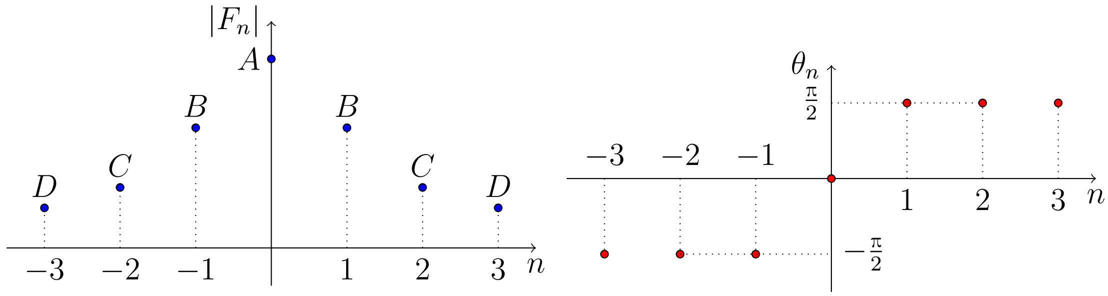
(c) (4 points) Consider the LTI system with impulse response

$$h(t) = \mathrm{e}^{-2t}\,\sin(t + \pi/3)\,\mathrm{u}(t).$$

Check the statements that are true:

- ◯ $\mathcal{F}[x](\omega) = \mathcal{L}[x](\mathrm{j}\omega)$
- ◯ The system is BIBO stable
- ◯ The ROC is given by $\mathrm{Re}(s) > -1$
- ◯ The Fourier transform of $h(t)$ does not exist

(d) (3 points) Consider the amplitude and phase spectra of a periodic signal below:



What information can you deduce about the time-domain signal $f(t)$, simply by looking at the spectra?

- ◯ The signal is real
- ◯ The signal is odd-symmetric

○ $\int_{-T/2}^{T/2} f(t)\,\mathrm{d}t = 0$, where T is the signal's period

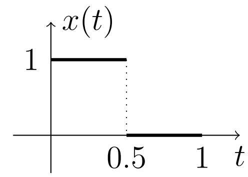
(e) (1 point) What is the Nyquist rate for signal $x(t) = \mathrm{sinc}(t) + \cos(2t)$?

○ $\Omega_\mathrm{s} = 4\,\mathrm{rad/s}$

○ $\Omega_\mathrm{s} = 2\,\mathrm{rad/s}$

○ $\Omega_\mathrm{s} = 1\,\mathrm{rad/s}$

2. Consider the periodic signal $x(t)$, whose period is drawn below:



(a) (2 points) What is the period, T, and what is the fundamental frequency, ω_0 in radians per second?

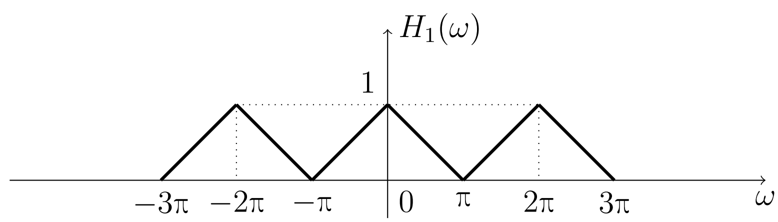
(b) (5 points) Compute the Fourier series coefficients, X_n, n integer. Make sure you compute them correctly, as the following questions will depend on them.

(c) (4 points) Plot the amplitude and phase spectra of X_n.

(d) (5 points) Write the expression of $\hat{x}_1(t)$, the approximation of $x(t)$ up to the first harmonic.

(e) (5 points) What is the mean squared error of the approximation up to the first harmonic, ϵ_1^2? Compute ϵ_1^2 by using the approximation $\pi \approx 3$.

(f) (5 points) If the signal $x(t)$ is the input of an LTIC system with impulse response $h_1(t)$, whose Fourier transform is given in the figure below, what is the corresponding output? You should be able to answer without performing any computations. (Hint: note that $x(t)$ has a line spectrum.)



3. The output of a linear, time-invariant system is $y(t) = e^{-2t}\sin(t)\mathrm{u}(t)$ when the input is $x(t) = \delta(t) + \mathrm{u}(t)$.

   (a) (5 points) Find the transfer function $H(s) = \mathcal{L}[h](s)$ of the system and its region of convergence.

   (b) (5 points) Find the frequency response $\mathcal{F}[h](\omega)$ of the system (you do not need to simplify). Is $\mathcal{F}[h](\omega) = H(s)|_{s=\mathrm{j}\omega}$? Explain.

   (c) (5 points) The input $x_1(t) = 5 + \cos(t)$ is now applied to the system. Compute the corresponding output, $y_1(t)$.

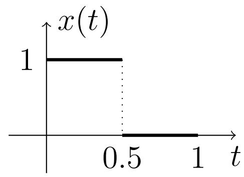
4. Consider an LTIC system with transfer function (Laplace transform of the impulse response) $H(s)$ given by:

$$H(s) = \frac{\mathrm{e}^{-s}}{(s+1)(s^2+2s+2)}.$$

(a) (3 points) Is the system BIBO stable? Explain.

(b) (5 points) Compute the impulse response, $h(t)$.

5. Let $y(t)$ be the output when $x(t)$ of problem 2 (a periodic signal whose main period is shown below) is applied to a system with transfer function $H(s)$ as in problem 4.



(a) (2 points) Is the output $y(t)$ periodic? If so, what is its period?

(b) (5 points) Compute the Fourier series coefficients of the output, Y_n. You do not need to simplify.

(c) (5 points) Compute the magnitude $|Y_n|$ and the angle $\measuredangle Y_n$ of the output signal's Fourier series coefficients.

(d) (4 points) From $|Y_n|$ and $\measuredangle Y_n$ can you conclude that the output is real? If so, explain.

$f(t)$	$F(s) = \mathcal{L}[f](s) = \int_{0-}^{\infty} \mathrm{e}^{-st} f(t)\ \mathrm{d}t$
$\delta(t-a)$	e^{-as}, ROC $= \mathbb{C}$
$\mathrm{u}(t)$	$\frac{1}{s}$, $\mathrm{Re}\{s\} > 0$
$t^n \mathrm{u}(t)$, $n \geq 0$ integer	$\frac{n!}{s^{n+1}}$, $\mathrm{Re}\{s\} > 0$
$\mathrm{e}^{at}\mathrm{u}(t)$	$\frac{1}{s-a}$, $\mathrm{Re}\{s\} > \mathrm{Re}\{a\}$
$t\,\mathrm{e}^{at}\mathrm{u}(t)$	$\frac{1}{(s-a)^2}$, $\mathrm{Re}\{s\} > \mathrm{Re}\{a\}$
$\cos(\omega_0 t)\mathrm{u}(t)$	$\frac{s}{s^2+\omega_0^2}$, $\mathrm{Re}\{s\} > 0$
$\sin(\omega_0 t)\mathrm{u}(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$, $\mathrm{Re}\{s\} > 0$
$\cosh(kt)\mathrm{u}(t)$	$\frac{s}{s^2-k^2}$, $\mathrm{Re}\{s\} > \lvert k\rvert$
$\sinh(kt)\mathrm{u}(t)$	$\frac{k}{s^2-k^2}$, $\mathrm{Re}\{s\} > \lvert k\rvert$
$\mathrm{u}(t-a)$	$\frac{\mathrm{e}^{-as}}{s}$, $\mathrm{Re}\{s\} > 0$
$\frac{\mathrm{d}}{\mathrm{d}t} f(t)$	$sF(s) - f(0-)$
$\frac{\mathrm{d}^n}{\mathrm{d}t^n} f(t)$	$s^n F(s) - s^{n-1} f(0-) - s^{n-2} f'(0-) - \cdots$ $\cdots - s f^{(n-2)}(0-) - f^{(n-1)}(0-)$
$\int_0^t f(\tau)\ \mathrm{d}\tau$	$\frac{1}{s} F(s)$
$\mathrm{e}^{at} f(t)$	$F(s-a)$
$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\ \mathrm{d}\tau$	$F(s)G(s)$
$t^n f(t)$, $n = 1, 2, \ldots$	$(-1)^n \frac{\mathrm{d}^n}{\mathrm{d}s^n} F(s)$
$\frac{f(t)}{t}$	$\int_s^{\infty} F(\sigma)\ \mathrm{d}\sigma$
$\mathrm{u}(t-a) f(t-a)$, $a \geq 0$	$\mathrm{e}^{-as} F(s)$
$f(at)$, $a > 0$	$\frac{1}{a} F\left(\frac{s}{a}\right)$

$f(t)$	$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$
$e^{-at}u(t),\ a > 0$	$\dfrac{1}{a + j\omega}$
$e^{at}u(-t),\ a > 0$	$\dfrac{1}{a - j\omega}$
$e^{-a\|t\|},\ a > 0$	$\dfrac{2a}{a^2 + \omega^2}$
$te^{-at}u(t),\ a > 0$	$\dfrac{1}{(a + j\omega)^2}$
$t^n e^{-at}u(t),\ a > 0$	$\dfrac{n!}{(a + j\omega)^{n+1}}$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\sin(\omega_0 t)$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$	$\dfrac{2}{j\omega}$
$e^{-at}\cos(\omega_0 t)u(t),\ a > 0$	$\dfrac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
$e^{-at}\sin(\omega_0 t)u(t),\ a > 0$	$\dfrac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
$\cos(\omega_0 t)u(t)$	$\dfrac{\pi}{2}[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)] + \dfrac{j\omega}{\omega_0^2 - \omega^2}$
$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$	$T\mathrm{sinc}\left(\frac{T\omega}{2}\right)$
$\mathrm{sinc}\left(\dfrac{t}{T}\right) = \dfrac{\sin(t/T)}{t/T}$	$\pi T\mathrm{rect}\left(\frac{T\omega}{2}\right)$
$\mathrm{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - \|t\|/T, & -T < t < T \\ 0, & \text{otherwise} \end{cases}$	$T\left[\mathrm{sinc}\left(\frac{T\omega}{2}\right)\right]^2$
$\left[\mathrm{sinc}\left(\frac{t}{T}\right)\right]^2$	$\pi T\mathrm{tri}\left(\frac{T\omega}{2}\right)$

$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
$f(t - t_0)$	$\mathrm{e}^{-\mathrm{j}\omega t_0} F(\omega)$
$\mathrm{e}^{\mathrm{j}\omega_0 t} f(t)$	$F(\omega - \omega_0)$
$f(-t)$	$F(-\omega)$
$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
$(f_1 * f_2)(t)$	$F_1(\omega)F_2(\omega)$
$f_1(t)f_2(t)$	$\frac{1}{2\pi}(F_1 * F_2)(\omega)$
$\frac{\mathrm{d}}{\mathrm{d}t} f(t)$	$\mathrm{j}\omega F(\omega)$
$\int_{-\infty}^{t} f(t)\,\mathrm{d}t$	$\frac{1}{\mathrm{j}\omega} F(\omega) + \pi F(0)\delta(\omega)$
$tf(t)$	$\mathrm{j}\frac{\mathrm{d}}{\mathrm{d}\omega} F(\omega)$
$f(t)$ is real	$F(\omega) = F^{\star}(-\omega)$
$f(t)$ is real and even	$F(\omega)$ is real and even
$f(t)$ is real and odd	$F(\omega)$ is purely imaginary and odd
$F(t)$	$2\pi f(-\omega)$

Parseval's Theorem:

$$\int_{-\infty}^{\infty} f(t) g^*(t)\,\mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) G^*(\omega)\,\mathrm{d}\omega.$$

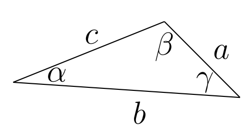
$$\int_{-\infty}^{\infty} \lvert f(t) \rvert^2\,\mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert F(\omega) \rvert^2\,\mathrm{d}\omega.$$

Product to Sum	Sum to Product
$2\sin(\alpha)\sin(\beta) = \cos(\alpha-\beta) - \cos(\alpha+\beta)$	$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta)$	$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta)$	$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta)$	$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Sum/Difference	Pythagorean Identity
$\sin(\alpha\pm\beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$	$\sin^2(\alpha) + \cos^2(\alpha) = 1$
$\cos(\alpha\pm\beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$	
$\tan(\alpha\pm\beta) = \dfrac{\tan(\alpha)\pm\tan(\beta)}{1\mp\tan(\alpha)\tan(\beta)}$	

Even/Odd	Periodic Identities
$\sin(-\alpha) = -\sin(\alpha)$	$\sin(\alpha + 2\pi n) = \sin(\alpha)$
$\cos(-\alpha) = \cos(\alpha)$	$\cos(\alpha + 2\pi n) = \cos(\alpha)$
$\tan(-\alpha) = -\tan(\alpha)$	$\tan(\alpha + \pi n) = \tan(\alpha)$

Double-Angle Identities	Half-Angle Identities
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$	$\sin\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\alpha)}{2}}$
$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$	$\cos\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1+\cos(\alpha)}{2}}$
$\tan(2\alpha) = \dfrac{2\tan(\alpha)}{1-\tan^2(\alpha)}$	$\tan\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\alpha)}{1+\cos(\alpha)}}$

Laws of Sines, Cosines, and Tangents	Mollweide's Formula
$\sin(\alpha)/a = \sin(\beta)/b = \sin(\gamma)/c$	$(a+b)/c = \cos\left((\alpha-\beta)/2\right)/\sin\left(\gamma/2\right)$
$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$	
$\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$	