UCLA

DEPARTMENT OF ELECTRICAL ENGINEERING

Winter 2007

Your Name (LAST, Middle, First):———————————————-

(Fill in the following lines — if applicable)

NAME OF PERSON ON YOUR LEFT:———————————————

NAME OF PERSON ON YOUR RIGHT:———————————————

EE102: SYSTEMS & SIGNALS

February 14

MIDTERM EXAMINATION

"SILENT" SOLUTIONS

Instructions:

(i) Closed Book, Calculators Are NOT Allowed. A Two-Sided Sheet (8.5 x 11.0) of "Goodies" Is Allowed

(ii) Please Write On ONE Side Only — otherwise, -5 pts/(second-sided) page

(iii) Questions Are Equally Weighted, Unless Otherwise Indicated

(iv) Please Put The First Letter of Your LAST NAME In The Upper Right Hand Corner of This Page

(v) Staple These Examination Papers With Your Papers — using your own stapler

Part 1: Time-Domain Analysis

(<u>Do NOT use Laplace Transforms. BT is your Good Friend!</u>)

1.1. (15%) The IPOP relation of a SISO system S is:

$$\mathbf{x(t)} \longrightarrow [\mathbf{S}] \longrightarrow \mathbf{y(t)}$$

$$y(t) = x(t) - 2\int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Write down the IRF $h(t,\tau)$ of S. Then compute its output $y(t)$ given that its input $x(t)$ is

$$x(t) = U(t) + U(-t), \quad t \in (-\infty, \infty)$$

SOLS.

$$y(t) = \int_{-\infty}^{\infty} [\delta(t-\tau) - 2e^{-(t-\tau)}] U(t-\tau)\, x(\tau)\, d\tau$$

$$\Rightarrow \quad \mathbf{h(t,\tau) = \delta(t) - 2e^{-(t-\tau)}\, U(t-\tau)} \ (= \mathbf{h(t-\tau)}).$$

$$x_1(t) := U(t), \quad x_2(t) := U(-t) \ \Rightarrow \ y(t) = y_1(t) + y_2(t).$$

$$\begin{aligned} y_1(t) &:= U(t) - 2\int_{-\infty}^{t} e^{-(t-\tau)} U(\tau) d\tau, \\ &= U(t) - 2\int_{0}^{t} e^{-(t-\tau)} 1\, d\tau = U(t) - 2(1-e^{-t})U(t), \\ \underline{y_1(t)} &= -U(t) + 2e^{-t}U(t). \end{aligned} \tag{1}$$

$$\begin{aligned} y_2(t) &:= U(-t) - 2\int_{-\infty}^{t} e^{-(t-\tau)} U(-\tau) d\tau, \\ y_2(t) &= U(-t) - 2\int_{-\infty}^{t} e^{-(t-\tau)} 1\ d\tau, \text{ for } t < 0, \\ \underline{y_2(t)} &= U(-t) - 2 = 1 - 2 = -1, \text{ for } t < 0, \qquad (2) \\ y_2(t) &:= U(-t) - 2\int_{-\infty}^{0} e^{-(t-\tau)}\ 1\ d\tau, \text{ for } t \geq 0, \\ \underline{y_2(t)} &= 0 - 2e^{-t}, \text{ for } t \geq 0. \qquad (3) \end{aligned}$$

(1), (2), (3) $\Rightarrow$

$$\begin{aligned} t \geq 0: \quad y(t) = y_1(t) + y_2(t) &= -U(t) + 2e^{-t} + [0 - 2e^{-t}] = -U(t), \quad (4) \\ t < 0: \quad y(t) = y_1(t) + y_2(t) &= -1. \quad (5) \end{aligned}$$

$$\Rightarrow \quad \mathbf{y(t) = -1}, \quad \forall\, \mathbf{t} \in (-\infty, \infty).$$

NOTE: The above is SloMo. Here is SuperMo:

$$x(t) = U(-t) + U(t) = 1, \quad \forall\, t \in (-\infty, \infty).$$

$$\Rightarrow \quad y(t) = 1 - 2\int_{-\infty}^{t} e^{-(t-\tau)}\, 1\, d\tau = 1 - 2 = -1, \quad \forall\, t \in (-\infty, \infty)!$$

1.2. A system S_1 is described by the IPOP relation:

$$x(t) \longrightarrow [S_1] \longrightarrow y(t), \quad y(t) = \int_{-\infty}^{\infty} \sinh(t-\tau)\, U(t-\tau)\, x(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

and a second system S_2 is described by the IPOP relation:

$$v(t) \longrightarrow [S_2] \longrightarrow w(t)$$

$$w(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)}\, U(t-\tau)\, v(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2\, S_1$, i.e., the system

$$IP \longrightarrow [\,[S_2] \rightarrow [S_1]\,] \longrightarrow OP$$

SOLS.

$$h_1(t) = \sinh t\, U(t), \quad h_2(t) = e^{-t} U(t).$$

$$\begin{aligned} \Rightarrow \quad h_{21}(t) &= \int_{-\infty}^{\infty} \sinh(t-\tau) U(t-\tau)\, e^{-\tau}\, U(\tau)\, d\tau, \\ \mathbf{h_{12}(t)} &= \int_0^t \sinh(t-\tau)\, e^{-\tau}\, d\tau = \frac{\mathbf{1}}{\mathbf{2}}\, (\sinh \mathbf{t} - \mathbf{e^{-t}\, t})\, \mathbf{U(t)}. \end{aligned}$$

1.3. Given the following information regarding a system S:

$$U(t) \longrightarrow [\mathbf{L,TI,C;\ IRF{:}\ h(t)U(t)}] \longrightarrow g(t)$$

Can you conclude that

$$g(t) = \int_0^t h(\tau)\, d\tau, \quad t > 0\ ?$$

Please give details of your answer.

Find $h(t)$ given that $\mathbf{g(t) = e^{-t} \sin t\, U(t)}$.

SOLS.

$$U(t) \longrightarrow [S] \longrightarrow g(t),$$

$$\begin{aligned}
\text{BT} \quad \Rightarrow \quad g(t) \quad &= \quad \int_{-\infty}^{\infty} h(t-\tau)U(t-\tau)\; U(\tau)\; d\tau, \\
&= \quad \int_0^t h(t-\tau)\; d\tau, \\
&= \quad \int_0^t h(\sigma)\; d\sigma, \quad (\sigma = t - \tau).
\end{aligned}$$

$$\Rightarrow \quad h(t) = \dot{g}(t) = \frac{d}{dt}\, \{e^{-t} \sin t\, U(t)\}.$$

$$\Rightarrow \quad h(t) = -e^{-t} \sin t\, U(t) + e^{-t}\{\cos t\, U(t) + \sin t\, \delta(t)\}$$

$$\Rightarrow \quad h(t) = -e^{-t} \sin t\, U(t) + e^{-t} \cos t\, U(t)$$

$$\Rightarrow \quad \mathbf{h(t) = e^{-t}[-\sin t + \cos t]\, U(t)}$$

Part 2: s-Domain Analysis

(<u>This is where Laplace Transforms shine</u>)

2.1. REDO **Problem 1.2** using Laplace Transforms:

Let system S_1 be described by the IPOP relation:

$$x(t) \longrightarrow\ [S_1]\ \longrightarrow y(t),\ y(t) = \int_{-\infty}^{\infty} \sinh(t-\tau)\, U(t-\tau)\, x(\tau)\, d\tau,\ t \in (-\infty, \infty)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \longrightarrow\ [S_2]\ \longrightarrow w(t)$$

$$w(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)}\, U(t-\tau)\, v(\tau)\, d\tau, \quad t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2\, S_1$, i.e., the system

$$IP \longrightarrow\ [\,[S_2] \to\ [S_1]\,]\ \longrightarrow OP$$

SOLS.

$$H_{21}(s) = H_2(s).H_1(s) = \frac{1}{s+1}\,\frac{1}{s^2-1} = \frac{1}{4}\,\frac{1}{s-1} - \frac{1}{4}\,\frac{1}{s+1} - \frac{1}{2}\,\frac{1}{(s+1)^2}.$$

$$\Rightarrow \quad \mathbf{h_{12}(t) = \frac{1}{2}\,(\sinh t - e^{-t} t)\, U(t)}.$$

2.2. REDO **Problem 1.3** using Laplace Transforms:

Given the following information regarding a system S:

$$U(t) \longrightarrow\ [\mathbf{L,TI,C;\ IRF:\ h(t)U(t)}]\ \longrightarrow g(t)$$

Can you conclude that

$$g(t) = \int_0^t h(\tau)\, d\tau, \quad t > 0\ ?$$

Please give details of your answer.

Find $h(t)$ given that $\mathbf{g(t) = e^{-t} \sin t\, U(t)}$.

SOLS.

$$G(s) = H(s)\, U(s) = \frac{1}{s}\, H(s) = \mathcal{L}_s\{\int_0^t h(\tau)\, d\tau\} \ \Rightarrow\ g(t) = \int_0^t h(\tau)\, d\tau.$$

$$H(s) = s\,G(s) = s\;\mathcal{L}_s\{e^{-t}\sin t\,U(t)\} = s\;\{\frac{1}{(s+1)^2+1}\}$$

$$H(s) = \frac{s}{(s+1)^2+1} = \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}$$

$$\Rightarrow h(t) = e^{-t}\;[\cos t - \sin t]\,U(t)$$

2.3 Find $y(t)$ whose Laplace Transform $Y(s)$ is

$$Y(s) = \frac{s}{s^2+7s+10}.$$

Now take the signal $y(t)$ — you just found — as the OP of **a** L, TI, C system S when an input $x(t)$ is applied to S. Your problem is to find the IRF $h(t)U(t)$ of S as well as the signal $x(t)$:

$$\mathbf{x(t)}(?) \longrightarrow\; [\mathbf{S:L,TI,C:h(t)U(t)}\,(?)]\;\longrightarrow \mathbf{y(t)}(\leftarrow \text{known})$$

SOLS.

$$Y(s) = \frac{s}{(s+2)(s+5)} = -\frac{2}{3}\,\frac{1}{s+2} + \frac{5}{3}\,\frac{1}{s+5}$$

$$\Rightarrow \quad \mathbf{y(t)} = \{-\frac{\mathbf{2}}{\mathbf{3}}\,\mathbf{e^{-2t}} + \frac{\mathbf{5}}{\mathbf{3}}\,\mathbf{e^{-5t}}\}\,\mathbf{U(t)}.$$

$$Y(s) = \frac{s}{(s+2)(s+5)} = [\frac{s}{(s+2)}]\,[\frac{1}{s+5}]$$

$$\Rightarrow \quad H(s) = \frac{s}{s+2} = 1 - \frac{2}{s+2} \quad \Rightarrow \quad \mathbf{h(t)} = \delta(\mathbf{t}) - \mathbf{2e^{-t}}\,\mathbf{U(t)}.$$

$$\Rightarrow \quad X(s) = \frac{1}{s+5} \quad \Rightarrow \quad \mathbf{x(t)} = \mathbf{e^{-5t}}\,\mathbf{U(t)}.$$

Part 3: Time-Domain and/or s-Domain

(Use whatever method(s) which you are most comfortable with)

3.1. A system S :

$$x(t) \longrightarrow [S] \longrightarrow y(t)$$

is described by the differential equation:

$$\begin{aligned} \frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) &= \frac{dx(t)}{dt}, \quad t > 0, \\ y(0) &= 0 = \dot{y}(0), \quad x(0) = 0. \end{aligned}$$

Find the IRF $h(t)$ of S. Can this system be realized by cascading two L, TI, C, systems? If so write down the IRF's of the two systems.

SOLS.

$$(s^2 + 3s + 2)Y(s) = sX(s) \quad \Rightarrow \quad H(s) = \frac{s}{s^2 + 3s + 2}$$

$$H(s) = \frac{s}{s^2 + 3s + 2} = \frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$\mathbf{h(t) = [-e^{-t} + e^{-2t}]\, U(t)}.$$

$$H(s) = \frac{s}{s^2 + 3s + 2} = \{\frac{s}{(s+1)}\}\, \{\frac{1}{(s+2)}\} := H_1(s)\,.\,H_2(s).$$

$$\Rightarrow \quad \mathbf{h_1(t) = \delta(t) - e^{-t}\, U(t)}, \quad \mathbf{h_2(t) = e^{-2t} U(t)}.$$

3.2.(15%) Consider the cascaded combination S_{12} of L, TI, C systems S_1 and S_2 as shown below:

$$x(t) \rightarrow [S_1] \rightarrow [S_2] \rightarrow z(t)$$

where

$$x(t) = U(t)$$

and

$$z(t) = (\sin t + \cos t - e^{-t})\, U(t)$$

Compute the IRF function $h_{12}(t)$ of the cascaded system S_{12}.
If system S_1 is described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} \cos(t - \sigma)\, x(\sigma) d\sigma, \quad t > -\infty$$

Your problem is to write down the IPOP description of system S_2.

SOLS.

$$X(s) = \frac{1}{s}, \quad Z(s) = \frac{1+s}{s^2+1} - \frac{1}{s+1} = \frac{2s}{(s+1)(s^2+1)}$$

$$\Rightarrow \quad H_{12}(s) = \frac{Z(s)}{X(s)} = \frac{2s^2}{(s+1)(s^2+1)} = \frac{1}{s+1} + \frac{(2/3)s-1}{s^2+1}$$

$$\Rightarrow \quad \mathbf{h_{12}(t) = [e^{-t} + \frac{2}{3}\cos t - \sin t]\, U(t).}$$

$$H_1(s) = \mathcal{L}_s\{\cos t\, U(t)\} = \frac{s}{s^2+1}, \quad H_{12}(s) = \frac{2s^2}{(s+1)(s^2+1)}$$

$$\Rightarrow \quad H_2(s) = \frac{2s}{s+1} \quad \Rightarrow \quad h_2(t) = 2[\delta(t) - e^{-t}U(t)].$$

$$\Rightarrow \quad \mathbf{y(t) = \int_{-\infty}^{\infty} 2[\delta(t-\tau) - e^{-(t-\tau)}\, U(t-\tau)]\, x(\tau)\, d\tau.}$$

3.3. Find the system function $H(s)$ of the system S described by the IPOP relation

$$x(t) \longrightarrow [S] \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau)\, d\tau + \int_{-\infty}^{\infty} U(t-\tau)\, x(\tau)\, d\tau.$$

Then write down the differential equation involving any input $x(t)$ and corresponding output $y(t)$ of S.

SOLS.

$$h(t) = e^{-t}\, U(t) + U(t) \quad \Rightarrow \quad \mathbf{H(s) = \frac{1}{s+1} + \frac{1}{s}.}$$

$$\Rightarrow \quad H(s) = \frac{2s+1}{s^2+s} = \frac{Y(s)}{X(s)}.$$

$$\Rightarrow \quad (s^2+s)Y(s) = (2s+1)X(s) \quad \Rightarrow$$

$$\mathbf{\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} = 2\frac{dx(t)}{dt} + x(t), \quad y(0) = 0 = \dot{y}(0),\; x(0) = 0.}$$