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**EE 102**
**SPRING 1995**
**TEST # 1**
April 27

**Your Name:** [redacted]
**(First, Middle, LAST)**

*CLOSED BOOK, ALL QUESTIONS TO BE ATTEMPTED*
*THE QUESTIONS ARE NOT EQUALLY WEIGHTED*
***Time allowed: 75 Minutes***
**Attach this sheet to your Test**

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**#1.**

A **Linear** system S is described by:

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$$2\frac{dy(t)}{dt} + y(t) = x(t), \quad t > 0, \quad y(0) = 0,$$

where $x(t) = 0$ for $t < 0$.

(i) Solve for $y(t)$ in terms of $x(t)$, then write down $h(t)$.
(ii) Find the unit step response $g(t)$ of S, i.e., the output due to the input $U(t)$.

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**#2.**

(i) Sketch the signal:

$$f(t) = \delta(t - \pi)\cos(t + \pi/2) + U(t-1)\,U(-t+2) + (t+1)\,U(1-t).$$

(ii) Compute:

$$\int_{-\infty}^{\infty} t e^{-t}\, U(t-a)\, U(b-t)\, dt,$$

where a and b are constants.

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**#3.**

Write down --*without proof*-- all the properties of the system having the following input-output relation:

$$y(t) = x(t)^2 + \int_{-\infty}^{\infty} (t - 2\tau)\, U(-t+\tau)\, x(\tau)\, d\tau, \quad \text{for } t \in (-\infty, \infty).$$

**(PTO)**

| | |
|---|---|
| 1 | 20/20 |
| 2 | 15/15 |
| 3 | 10/10 |
| 4 | 15/15 |
| 5 | 15/15 |
| 6 | 8/20 |

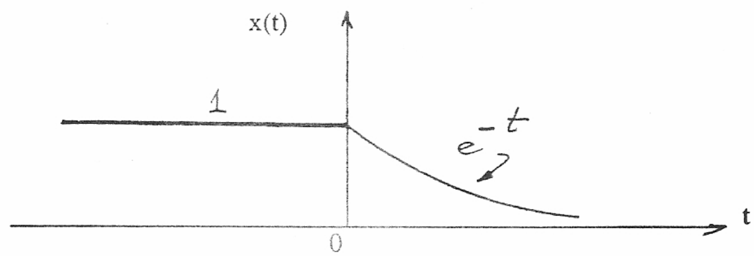
83/95

Class Avg ≈ 62/95

**#4**. Given:

$U(t) \longrightarrow \boxed{L, TI} \longrightarrow (1 - e^{-t})U(t)$

**Find the output y(t) when the input x(t) is:**

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**#5.** Given:

$U(t-2) \longrightarrow \boxed{L, TI, ①} \longrightarrow t\,U(t-2)$

and

$\delta(t) \longrightarrow \boxed{L, TI, ②} \longrightarrow \delta(t) + U(t)$

**Write down $h_{12}(t)$--the IRF of the cascaded system ①–②.**

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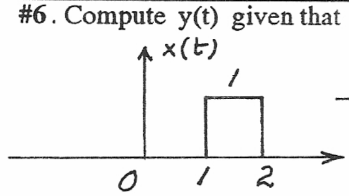
**#6**. Compute y(t) given that




$\longrightarrow \boxed{L, TI} \longrightarrow$ y(t) = ?

h(t) = t U(t)

**The End**

1. $2\frac{dy(t)}{dt} + y(t) = x(t)$, $t>0$, $y(0)=0$, $x(t)=0$ for $t<0$

$$\frac{dy(t)}{dt} + \frac{1}{2}y(t) = \frac{1}{2}x(t)$$

multiplication factor : $e^{\int_0^t \frac{1}{2} d\tau} = e^{\frac{1}{2}t}$

multiply $e^{\frac{1}{2}t}$ on both sides,

$$e^{\frac{1}{2}t}\frac{dy(t)}{dt} + \frac{1}{2}e^{\frac{1}{2}t}y(t) = \frac{1}{2}e^{\frac{1}{2}t}x(t)$$

$$\frac{d}{dy}\left[e^{\frac{1}{2}t}y(t)\right] = \frac{1}{2}e^{\frac{1}{2}t}x(t)$$

Integrate,

$$e^{\frac{1}{2}t}y(t) = \int \frac{1}{2}e^{\frac{1}{2}t}x(t)dt + k$$

$$= \int_0^t \frac{1}{2}e^{\frac{1}{2}\tau}x(\tau)d\tau + k$$

integrant=0 for $t<0$, so start at 0!

$$\Rightarrow y(t) = e^{-\frac{1}{2}t}\int_0^t \frac{1}{2}e^{\frac{1}{2}\tau}x(\tau)d\tau + k$$

Set $t=0$,

$$0 = e^0 \underbrace{\int_0^0 \frac{1}{2}e^{\frac{1}{2}\tau}x(\tau)d\tau}_{\to 0} + k$$

$\Rightarrow$ $k=0$,

So rewrite,

$$y(t) = e^{-\frac{1}{2}t}\int_0^t \frac{1}{2}e^{\frac{1}{2}\tau}x(\tau)d\tau$$

$$= \int_0^t \frac{1}{2}e^{-\frac{1}{2}(t-\tau)}x(\tau)d\tau \quad \checkmark$$

Since the system is given to be linear and it looks linear to me.

Then, by the

BIG Theorem (next page)

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, x(\tau)\, d\tau$$

$$= \int_{0}^{t} \frac{1}{2} e^{-\frac{1}{2}(t-\tau)} x(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \underbrace{\frac{1}{2} e^{-\frac{1}{2}(t-\tau)} U(t-\tau)}_{h(t,\tau)} x(\tau) U(\tau)\, d\tau$$

$$\Rightarrow h(t,\tau) = \frac{1}{2} e^{-\frac{1}{2}(t-\tau)} U(t-\tau)$$

We can see $h(t,\tau)$ is T[illegible] there exists a pattern of $t-\tau$ that can [illegible]

$$\Rightarrow h(\sigma) = \frac{1}{2} e^{-\frac{1}{2}\sigma} U(\sigma)$$

$$\therefore \boxed{h(t) = \frac{1}{2} e^{-\frac{1}{2}t} U(t)}$$

ii) when input is $U(t)$

$$g(t) = y(t) = \int_{0}^{t} \frac{1}{2} e^{-\frac{1}{2}(t-\tau)} [illegible]$$

$$= 2 \times \frac{1}{2} e^{-\frac{1}{2}(t-\tau)} \Big]_{0}^{t} \cdot U(t)$$

$$= \left(e^{0} - e^{-\frac{1}{2}t}\right) U(t)$$

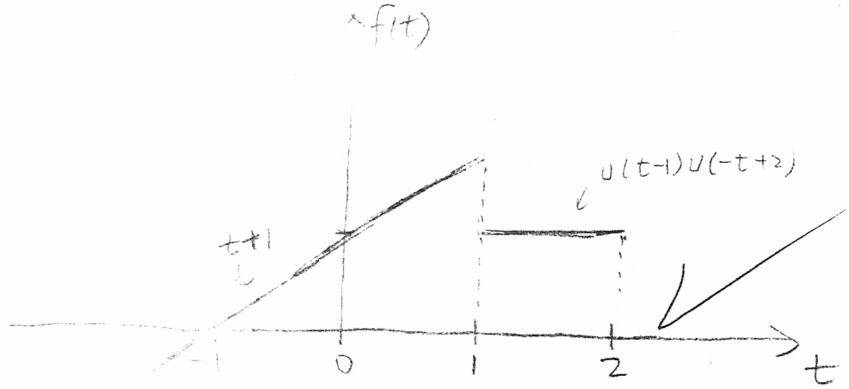
$$= \boxed{\left(1 - e^{-\frac{1}{2}t}\right) U(t)}$$

2. i) Sketch

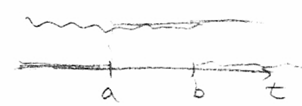
$$f(t) = \delta(t-\pi)\cos(t+\pi/2) + U(t-1)U(-t+2) + (t+1)U(1-t).$$

$$= \delta(t-\pi)\cos(\tfrac{3}{2}\pi) + U(t-1)U(-t+2) + (t+1)U(1-t)$$

$$= U(t-1)U(-t+2) + (t+1)U(1-t)$$




ii) $$\int_{a}^{b} t e^{-t} U(t-a) U(b-t)\, dt$$




$$= \int_a^b t e^{-t}\, dt \cdot U(t-a) \quad \text{if } a \leq b$$

$u = t,\ du = dt$
$dv = e^{-t} dt,\ v = -e^{-t}$

$$= \left( -t e^{-t} \Big|_a^b + \int_a^b e^{-t}\, dt \right) U(t-a) \cdot U(b-a)$$

$$= \left( -b e^{-b} + a e^{-a} - e^{-b} + e^{-a} \right) \cancel{U(t-a)} U(b-a)$$

3. write down all the properties

$$y(t) = x(t)^2 + \int_{-\infty}^{\infty} (t-2\tau) u(-t+\tau) x(\tau) d\tau, \text{ for } t \in (-\infty, \infty)$$

first of all, there is a $x(t)^2$, so

system is non-linear

check if it's T.I.

$$z(t) = T[x(t-T)] = x(t-T)^2 + \int_{-\infty}^{\infty} (t-2\tau) u(-t+\tau) x(t-T) \dots$$

$$y(t-T) = x(t-T)^2 + \int_{-\infty}^{\infty} (t-T-2\tau) u(-t+\dots) x(\tau) d\tau$$

check if $z(t) \stackrel{?}{=} y(t-T)$

sub $t - T - 2\tau = \sigma$

Integral of $y(t-T)$ $\int_{-\infty}^{\infty} \sigma u(-T-\sigma) x\left(\frac{t-\sigma}{3}\right) d\sigma$

which has no way to be equal to $z(t)$

$\Rightarrow$ System is T.V.

$$y(t) = x(t)^2 + \int_{-\infty}^{\infty} (t-2\tau) u(-t+\tau) x(\tau) d\tau$$

$$= x(t)^2 + \int_{t}^{\infty} (t-2\tau) x(\tau) d\tau$$

it depends on the future values

$\Rightarrow$ System is not Causal

$\therefore$ System is
- non-linear
- T.V.
- not causal

$$\int_{t}^{\infty} x(T) dT$$

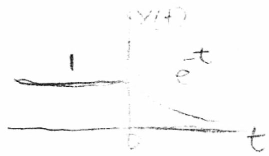
4. Given L, TI system $y(t) = (1-e^{-t})u(t)$ when
$x(t) = u(t)$

So, $y(t) = (1-e^{-t})u(t) = g(t)$ unit-step response fun.

$$h(t) = \frac{d}{dt} g(t) = \frac{d}{dt}\left[(1-e^{-t})\, u(t)\right]$$
$$= (1-e^{-t})\,\delta(t) + u(t)\, e^{-t}$$
$$= e^{-t}u(t) + (1-e^{0})\,\delta(t)$$
$$= e^{-t}u(t)$$

By BIG Theorem,
$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)}\, u(t-\tau)\, x(\tau)\, d\tau$$

Now, $x(t) = u(-t) + e^{-t}u(t)$




if $t<0$
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, u(t-\tau)\, u(-\tau)\, d\tau \cdot u(-t)$$
$$= \left(e^{-t+\tau}\Big|_{-\infty}^{t} = e^{0} - e^{-\infty}\right) = 1 \cdot u(-t)$$
$$= u(-t)$$

if $t \geq 0$
$$y(t) = \int_{-\infty}^{0} e^{-(t-\tau)}\, u(t-\tau)\, u(-\tau)\, d\tau + \int_{0}^{t} e^{-(t-\tau)}\, u(t-\tau)\, e^{-\tau}\, d\tau$$
$$= \left(e^{-(t-\tau)}\Big|_{-\infty}^{0} + \int_{0}^{t} e^{-t}\, d\tau\right) u(t)$$
$$= (e^{-t} + te^{-t})\, u(t)$$

$$\therefore\ y(t) = \boxed{u(-t) + (e^{-t} + te^{-t})\, u(t)}$$

5 Given: $u(t-2) \rightarrow \boxed{LTI_{1}} \rightarrow t\,u(t-2)$

translate $t-2=\tau$ $\quad u(\tau) \rightarrow \boxed{LTI_{1}} \rightarrow (\tau+2)u(\tau)$

$$\Rightarrow g_1(t) = (t+2)u(t)$$

$$h_1(t) = \frac{d}{dt} g_1(t) = \frac{d}{dt}\left[(t+2)u(t)\right]$$

$$= u(t) + (t+2)\delta(t)$$

$$= u(t) + 2\delta(t)$$

$$\delta(t) \longrightarrow \boxed{LTI_{2}} \longrightarrow \delta(t) + u(t)$$

So $h_2(t) = \delta(t) + u(t)$

By definition,

$$h_{12}(t) = \int_{-\infty}^{\infty} h_2(t-\tau)\,h_1(\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} \left[\delta(t-\tau) + u(t-\tau)\right]\left[u(\tau) + 2\delta(\tau)\right] d\tau$$

$$= \int_{-\infty}^{\infty} \delta(t-\tau)u(\tau)\,d\tau + 2\int_{-\infty}^{\infty} \delta(t-\tau)\delta(\tau)\,d\tau + \int_{-\infty}^{\infty} u(t-\tau)u(\tau)\,d\tau$$

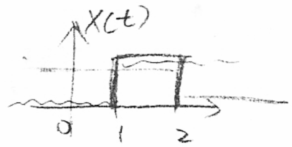
$$+ 2\int_{-\infty}^{\infty} \delta(\tau)u(t-\tau)\,d\tau$$

$$= u(t) + 2\delta(t) + \int_0^t d\tau \cdot u(t) + 2u(t)$$

$$= u(t) + 2\delta(t) + t\,u(t) + 2u(t)$$

$$= \boxed{(t+3)u(t) + 2\delta(t)}$$

✓

6. 


$$X(t) = U(2-t)\,U(t-1)$$

$\rightarrow \boxed{L,TI} \rightarrow$

$h(t) = t\,U(t)$

By B.T,

$$y(t) = \int_{-\infty}^{\infty} (t-\tau)\,U(t-\tau)\,U(2-\tau)\,U(\tau-1)\,d\tau$$

Since $X(t) = 0$ when $t$ is not bet...

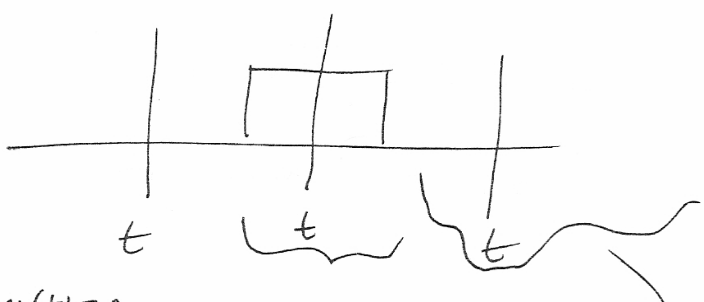
$\Rightarrow 1 \leq t \leq 2$

$$y(t) = \int_{1}^{t} (t-\tau)\,U(t-\tau)\,U(2-\tau)\,U(\tau-1)\,d\tau \cdot U(2-t)\,U(t-1)$$

$$= \int_{1}^{t} (t-\tau)\,d\tau \cdot U(2-t)\,U(t-1)$$

$$= t\tau - \tau^2 \Big]_{1}^{t}\, U(2-t)\,U(t-1)$$

$$= (-t+1)\,U(2-t)\,U(t-1)$$




B.D!

$y(t) = 0$

$y(t) = \frac{t^2}{2} - t + \frac{1}{2}$

$y(t) = t - \frac{3}{2}$

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