EE 102
SPRING 1995
TEST # 1
April 27

Your Name: [redacted]
(First, Middle, LAST)

CLOSED BOOK, ALL QUESTIONS TO BE ATTEMPTED
THE QUESTIONS ARE NOT EQUALLY WEIGHTED
Time allowed: 75 Minutes
Attach this sheet to your Test

#1.

A **Linear** system S is described by:

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$$2\frac{dy(t)}{dt} + y(t) = x(t), \quad t > 0, \quad y(0) = 0,$$

where $x(t) = 0$ for $t < 0$.

(i) Solve for $y(t)$ in terms of $x(t)$, then write down $h(t)$.
(ii) Find the unit step response $g(t)$ of S, i.e., the output due to the input $U(t)$.

#2.

(i) Sketch the signal:

$$f(t) = \delta(t - \pi)\cos(t + \pi/2) + U(t - 1)\,U(-t + 2) + (t + 1)\,U(1 - t).$$

(ii) Compute:

$$\int_{-\infty}^{\infty} t\,e^{-t}\,U(t - a)\,U(b - t)\,dt,$$

where a and b are constants.

#3.

Write down --*without proof*-- all the properties of the system having the following input-output relation:

$$y(t) = x(t)^2 + \int_{-\infty}^{\infty} (t - 2\tau)\,U(-t + \tau)\,x(\tau)\,d\tau, \quad \text{for } t \in (-\infty, \infty).$$

(PTO)

1	2/20
2	15/15
3	10/10
4	10/15
5	5/15
6	20/20

14/15

9/15

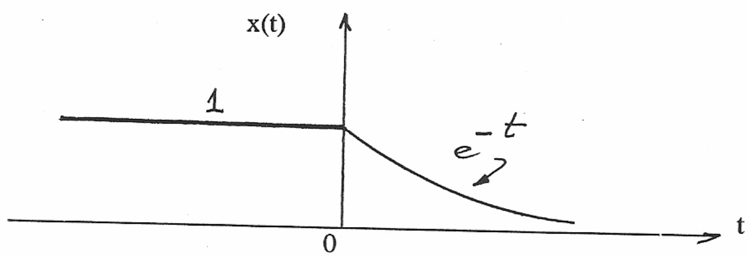
70/95

~~62~~/95

#4. Given:

$U(t) \longrightarrow$ [L, TI] $\longrightarrow (1 - e^{-t})U(t)$

Find the output $y(t)$ when the input $x(t)$ is:



#5. Given:

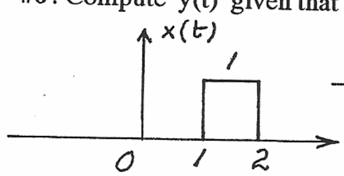
$U(t-2) \longrightarrow$ [L, TI, (1)] $\longrightarrow t\,U(t-2)$

and

$\delta(t) \longrightarrow$ [L, TI, (2)] $\longrightarrow \delta(t) + U(t)$

Write down $h_{12}(t)$--the IRF of the cascaded system (1)—(2) .

#6. Compute $y(t)$ given that



$\longrightarrow$ [L, TI] $\longrightarrow$ $y(t) = ?$

$h(t) = t\,U(t)$

The End

[redacted]

1. $2y' + y = x$ $\qquad u = e^{\int \frac{1}{2} dx} = e^{x/2}$

$y' + \frac{y}{2} = \frac{x}{2}$ $\qquad \int uv' = uv - \int u'v$

$y' e^{x/2} + \frac{y}{2} e^{x/2} = \frac{x}{2} e^{x/2}$

$(y e^{x/2})' = \frac{x}{2} e^{x/2}$

$y e^{x/2} = \frac{1}{2} \int x e^{x/2} dx$

$y e^{x/2} = \frac{1}{2} \left[2e^{x/2} x - \int 2e^{x/2} \right] + C$

$y(t) = e^{x/2} e^{-x/2} x - 2e^{x/2} e^{-x/2} + Ce^{-x/2}$

$y(t) = x - 2 + Ce^{-x/2}$

$y(0) = 0 - 2 + C = 0$

$C = 2$

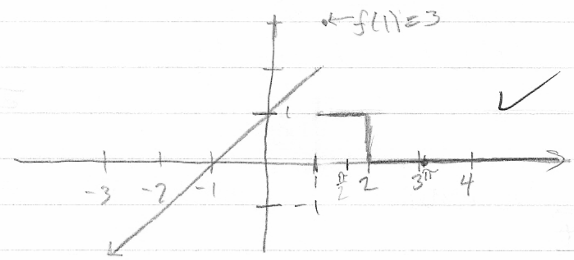
i) $\boxed{y(t) = x - 2 + 2e^{-x/2}}$

$\boxed{h(t, \tau) = \delta(t - \tau) - 2 + 2e^{-\delta(\frac{t-\tau}{2})}}$

Please Review your P.E!

ii) $\boxed{g(t) = U(t) - 2 + 2e^{-U(t)/2}}$

2, i)



ii) $\int_{-\infty}^{\infty} te^{-t} U(t-a) U(b-t) dt = \int_a^b te^{-t} dt \quad b \geq a$, 0 else

$= -te^{-t} + \int e^{-t}$

$= -te^{-t} - e^{-t} \Big|_a^b$

$= -(t+1)e^{-t}\Big|_a^b = \begin{cases} -(b+1)e^{-b} + (a+1)e^{-a} & b \geq a \\ 0 & \text{else} \end{cases}$

6) $x(t) = U(t-1)\,U(2-t)$

$$y(t) = \int_{-\infty}^{\infty} (t-\tau)\,U(t-\tau)\,U(\tau-1)\,U(2-\tau)\,d\tau$$

$$= \int_1^2 (t-\tau)\,U(t-\tau)\,d\tau$$

$$= \begin{cases} \int_1^2 (t-\tau)\,d\tau & t > 2 \\ \int_1^t (t-\tau)\,d\tau & 1 \le t \le 2 \\ 0 & t < 1 \end{cases}$$

$$= \begin{array}{ll} \tau t - \frac{1}{2}\tau^2 \Big|_1^2 & t > 2 \\ \tau t - \frac{1}{2}\tau^2 \Big|_1^t & 1 \le t \le 2 \\ 0 & t < 1 \end{array}$$

$$= \begin{array}{ll} 2t - t - 2 + \frac{1}{2} & t > 2 \\ t^2 - t - \frac{1}{2}t^2 + \frac{1}{2} & 1 \le t \le 2 \\ 0 & t < 1 \end{array}$$

$$= t - \tfrac{3}{2} \qquad t > 2$$

$$= \tfrac{1}{2}t^2 - t + \tfrac{1}{2} \qquad 1 \le t \le 2$$

$$= 0 \qquad t < 1$$

BRAVO

$$\left(t - \tfrac{3}{2}\right)U(t-1) + \left(\tfrac{1}{2}t^2 - t + \tfrac{1}{2} - t + \tfrac{3}{2}\right)U(t-1)\,U(2-t)$$

$$T[x(t)] = \left(t - \tfrac{3}{2}\right)U(t-1) + \left(\tfrac{1}{2}t^2 - 2t + 2\right)U(t-1)\,U(2-t)$$