EE102 **Signals and Systems**
2010 Fall Midterm **Jin Hyung Lee**

MIDTERM

Name:	UID:

You have 2 hours for 4 problems.

1. Show neat work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
2. Show your final answers in the boxes at the end of each question.

Closed book. Closed notes. You can use one hand-written letter sheet for your reference.

Problem	Score
1	
2	
3	
4	
Total	

1. *Linear Time Invariant System*

   Determine the linearity and time invariance of the systems given below.

   (a) $y(t) = x^2(t) + x(t-2) + 3$

   i. Input $\alpha x(t)$ into the system, the output is $\hat{y}(t) = \alpha^2 x^2(t) + \alpha x(t-2) + 3$. $\hat{y}(t)$ does not equal the scaled output $y(t)$: $\alpha y(t) = \alpha x^2(t) + \alpha x(t-2) + 3\alpha$. So the system is *NOT* linear.

   ii. Input delayed signal $\hat{x}(t) = x(t-T)$. Then, the corresponding output is $\hat{y}(t) = \hat{x}^2(t) + \hat{x}(t-2) + 3 = x^2(t-T) + x(t-2-T) + 3$.

   iii. Delayed $y(t)$ gives: $y(t-T) = x^2(t-T) + x(t-T-2) + 3$, which is equals $\hat{y}(t)$. Therefore, the system is *Time Invariant*.

The system is Linear	True	[✓ False]
The system is Time Invariant	[✓ True]	False

(b) $y(t) = \int_0^t x(\tau + 2) d\tau$

i. Input $\alpha x(t)$ into the system, the output is $\hat{y}(t) = \int_0^t \alpha x(\tau + 2) d\tau = \alpha \int_0^t x(\tau + 2) d\tau = \alpha y(t)$.

ii. Input $\hat{x}(t) = x_1(t) + x_2(t)$ into the system, the output is $\hat{y}(t) = \int_0^t (x_1(\tau + 2) + x_2(\tau + 2)) d\tau = \int_0^t x_1(\tau + 2) d\tau + \int_0^t x_2(\tau + 2) d\tau = y_1(t) + y_2(t)$. Thus, the system is *Linear*.

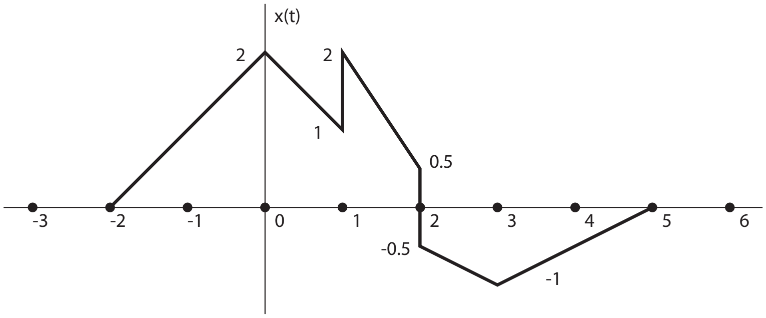
iii. Input delayed signal $\hat{x}(t) = x(t - T)$, then, the corresponding output is $\hat{y}(t) = \int_0^t x(\tau + 2 - T) d\tau = \int_{-T}^{t-T} x(\tau + 2) d\tau$.

iv. Delayed $y(t)$ gives: $y(t - T) = \int_0^{t-T} x(\tau + 2) d\tau$, which is *NOT* equal to $\hat{y}(t)$. Therefore, the system is *NOT Time Invariant*.

The system is Linear	[✓ True]	False
The system is Time Invariant	True	[✓ False]

2. *Signal Operations*

   A signal $x(t)$ is as given below:



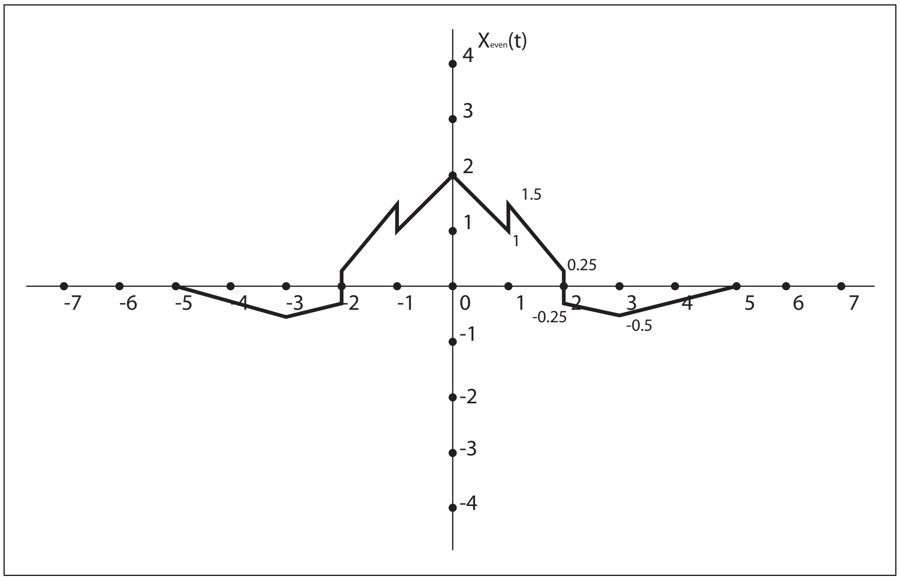
   (a) Try to decompose the signal into an combination of scaled *Trianglular* and *Rectangular* functions. Write the expression for x(t) in the box.

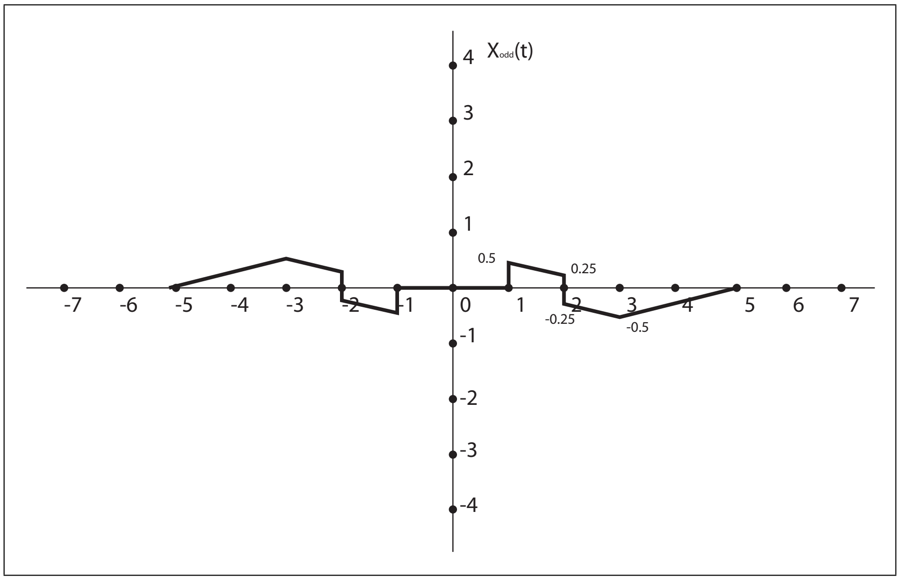
$$tri(t) = \begin{cases} 1-|t| & |t| \le 1 \\ 0 & otherwise \end{cases}$$

$$rect(t) = \begin{cases} 1 & |t| \le \frac{1}{2} \\ 0 & otherwise \end{cases}$$

$$\boxed{x(t) = 2tri(\tfrac{1}{2}t) + rect(t-\tfrac{3}{2}) - tri[\tfrac{1}{2}(t-3)]}$$

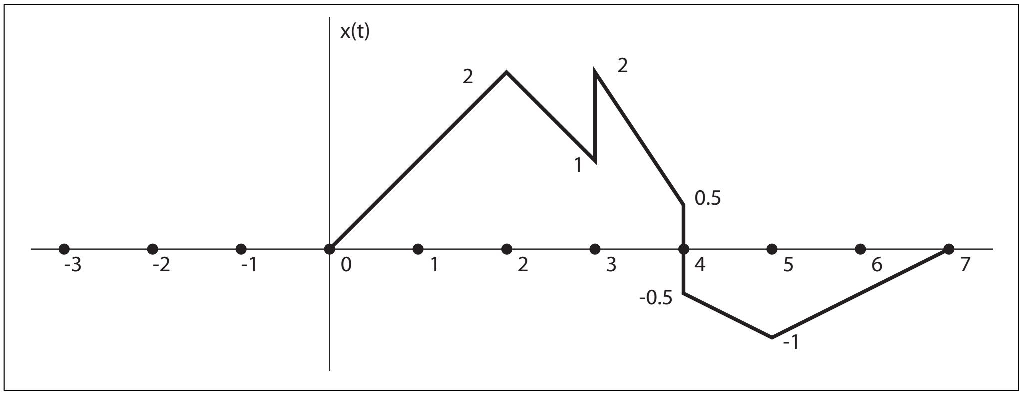
(b) Find and plot the *even* and *odd* part of the signal.



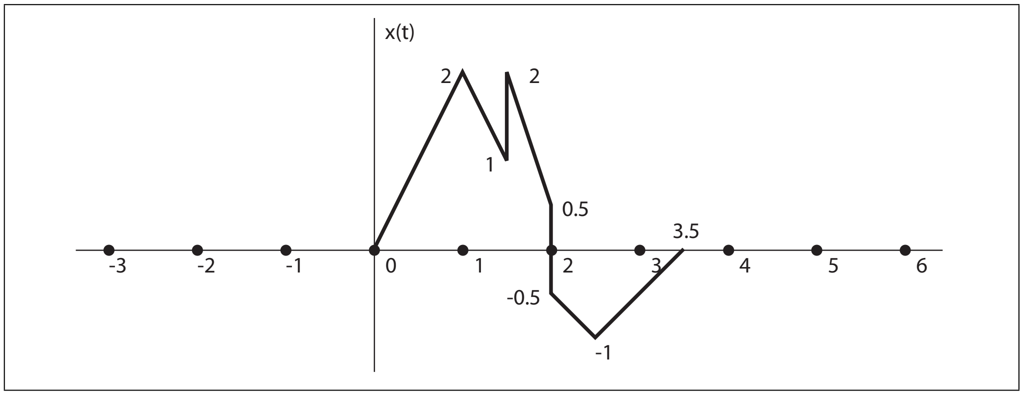


(c) Plot the following signals:

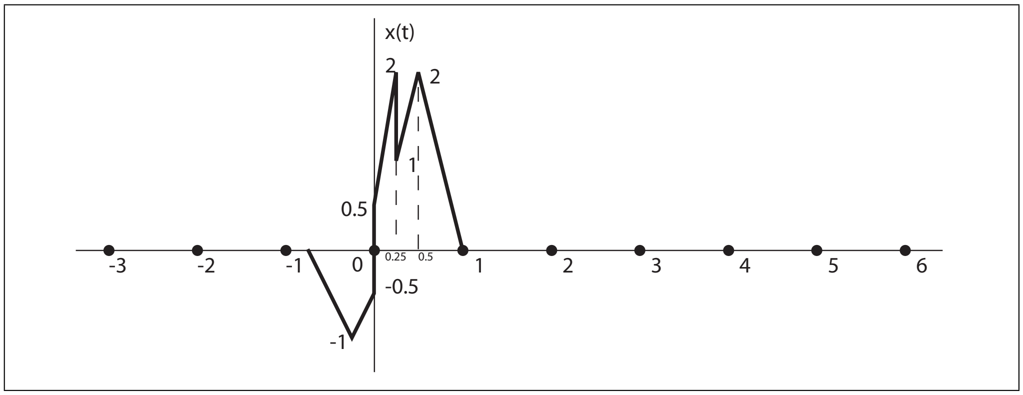
i. $x(t) * \delta(t-2)$.



ii. $x(2t-2)$.

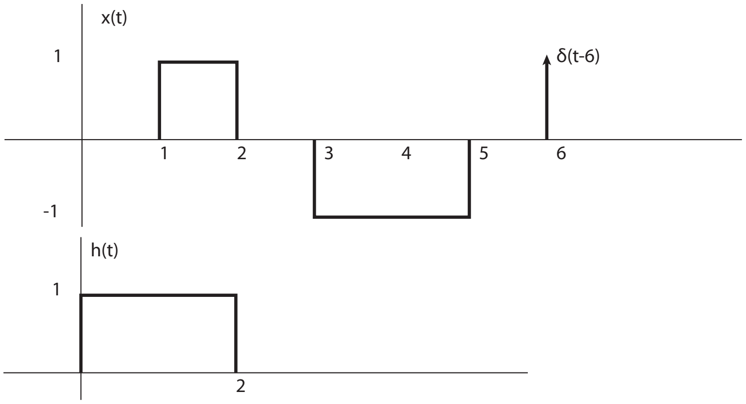


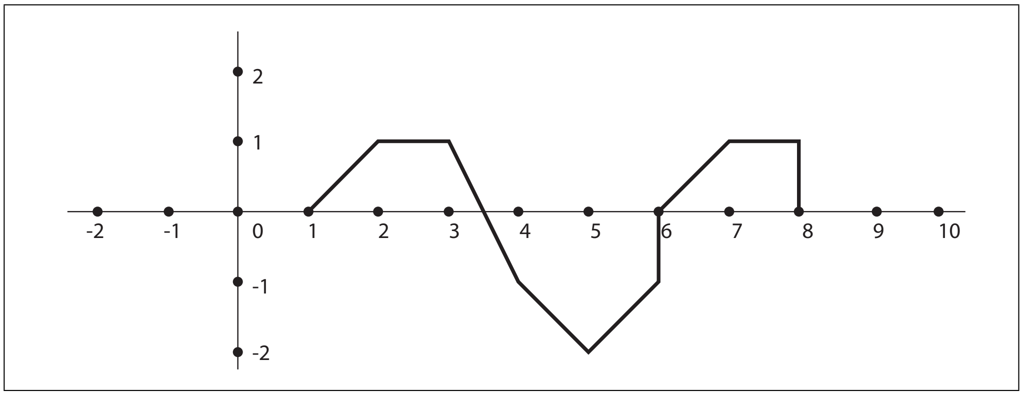
iii. $x(2-4t)$.



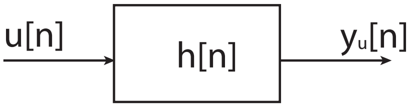
3. *Convolution*

(a) Convolve the following two signals and plot your result.
(hint: Utilizing the linearity of the system, consider decomposing $x(t)$ into simpler signals, and convolving them with $h(t)$ separately.)

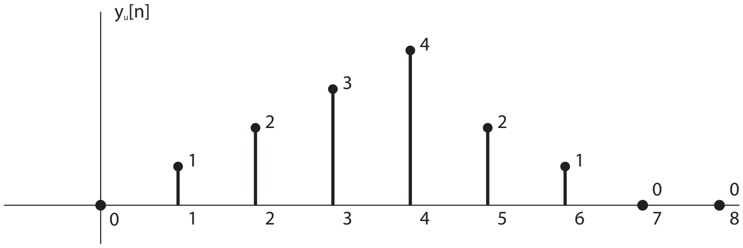


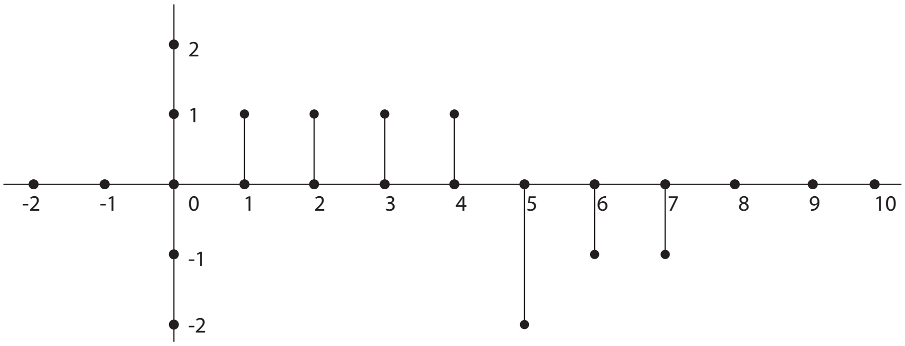


(b) An LTI system's *Step Response* is the output when the *Unit Step* signal $u[n]$ is igiven as input to the system:



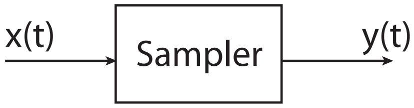
Suppose the following signal $y_u[n]$ is the step response of the system. Find its *Impulse Response* $h[n]$.
(hint: $\delta[n] = u[n] - u[n-1]$)



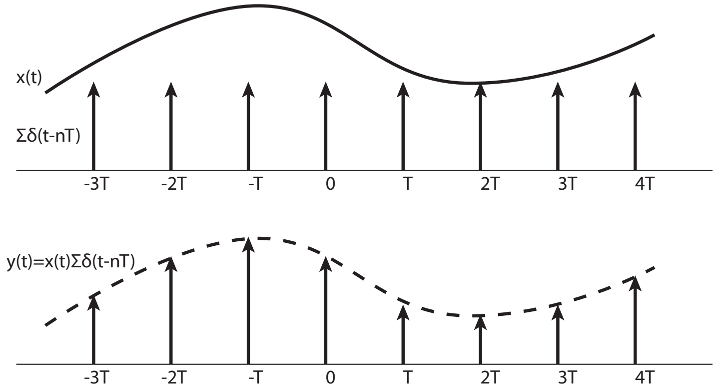


4. *Fourier Series*

A sampler is a system which is very often used in Analog to Digital Conversions.



An ideal sampling process can be represented as multiplying the signal $x(t)$ by a sampling impulse train $\Delta(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$. (See figure below.) The output of the sampling system can be written as $y(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT)$.



(a) Is this system Linear and Time Invariant?

   i. Input $\alpha x(t)$, output is $\alpha x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \alpha y(t)$.

   ii. Input $x_1(t) + x_2(t)$, output is $(x_1(t) + x_2(t)) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = y_1(t) + y_2(t)$, thus the system is linear.

   iii. If delay the input $x(t)$ by $0 \leq s \leq T$, then the sampled point will be totally different, and cannot be represented by delayed $y(t)$. Thus the system is *NOT Time Invariant*.

The system is *Linear*	[✓ True]	False
The system is *Time Invariant*	True	[✓ False]

(b) Find the Fourier Series of the sampling impulse train $\Delta(t) = \sum\limits_{n=-\infty}^{+\infty} \delta(t - nT)$.

$$a_k = \frac{1}{T} \int\limits_{t=-\frac{T}{2}}^{\frac{T}{2}} \delta(t) e^{-j\frac{2\pi k t}{T}} dt$$

$$a_k = \frac{1}{T} \int\limits_{t=-\frac{T}{2}}^{\frac{T}{2}} \delta(t) e^{-j\frac{2\pi k 0}{T}} dt$$

$$a_k = \frac{1}{T} \int\limits_{t=-\frac{T}{2}}^{\frac{T}{2}} \delta(t) dt$$

$$a_k = \frac{1}{T}$$

$$\boxed{a_k = \frac{1}{T}}$$

(c) Find the Fourier Series b_k of a shifted and scaled Δ signal:

$$\Delta_2(t) = \Delta(\frac{T}{4} - t)$$

To get $\Delta_2(t)$ there are two steps:
1. flip $\Delta(t)$ get $\Delta_1(t)$
then the Fourier Series of $\Delta_1(t)$: $c_k = a_{-k}$.
2. Shift $\Delta_1(t)$ to the right by $\frac{T}{4}$ get $\Delta_2(t)$
then the Fourier Series of $\Delta_2(t)$: $b_k = c_k e^{-j\frac{2\pi k(T/4)}{T}} = c_k e^{-j\frac{k\pi}{2}} = a_{-k} e^{-j\frac{k\pi}{2}} = \frac{1}{T} e^{-j\frac{k\pi}{2}}$

$$\boxed{b_k = \frac{1}{T} e^{-j\frac{k\pi}{2}}}$$