ECE 102 Midterm

Kyle Cameron Osborn

TOTAL POINTS

80.5 / 106

QUESTION 1

Signal/System/Convolution 35 pts

1.1 (a)i 5 / 5

✓ **+ 5 pts Correct**

+ 4 pts Correctly computed periods of both signals and its ratio, but with insufficient justification

+ 2 pts Attempted to justify the signal is not periodic, with insufficient or incorrect justification

+ 3.5 pts Stated no common period, without computing or mentioning the ratio of periods

+ 0 pts Incorrect answer or no Justification

1.2 (a)ii 0 / 5

+ 5 pts Correct

✓ **+ 0 pts Incorrect answer or no/incorrect justification**

+ 2 pts Insufficient Reasoning to justify the answer

1.3 (a)iii 5 / 5

✓ **+ 5 pts Correct answer**

+ 3 pts One side is simplified correctly

+ 1.5 pts Correctly used the sifting property on the right hand side

+ 1.5 pts Correctly used the sampling property on the left hand side

+ 0 pts Incorrect answer or reasoning

1.4 (b) 10 / 10

✓ **+ 10 pts Correct**

+ 5 pts Correctly identify the linearity property

+ 0 pts Incorrect

1.5 (c) 8 / 10

+ 10 pts Correct h(t)

✓ **+ 8 pts Correct h(t) for t<2**

+ 8 pts Correct shape of h(t) but incorrect amplitude

+ 2 pts correct h(0)

+ 2 pts correct h(1/4)

+ 2 pts correct h(5/8)

+ 2 pts Correct h(t) for t>2

+ 0 pts Incorrect

QUESTION 2

LTI Systems 20 pts

2.1 (a) 4 / 4

✓ **- 0 pts Correct**

- 1 pts minor mistakes

- 2 pts half correct

- 4 pts incorrect

2.2 (b) 8 / 10

✓ **+ 3 pts use LTI properties.**

✓ **+ 3 pts correct convolution equation/multiplication in frequency domain.**

+ 2 pts correct convolution results/inverse FT results.

✓ **+ 2 pts correct time shift for the last step.**

+ 0 pts incorrect

2.3 (c) 3 / 6

- 0 pts Correct.

- 1 pts constant term incorrect.

- 2 pts time shift incorrect.

- 2 pts inverse FT incorrect.

✓ **- 3 pts missing/incorrect exponential/unit step function.**

- 4 pts calculate in frequency domain with wrong FT.

- 5 pts incorrect calculation with a correct convolution equation.

- **6 pts** incorrect.

QUESTION 3

Fourier Series 20 pts

3.1 (a) 7.5 / 10

- **0 pts** Correct

- **0.5 pts** Did not explicitly use the angular frequency ωg = aωo, or Tg = To/a in the Fourier Series.

- **5 pts** Partially correct

- **7.5 pts** Incorrect

✓ **- 10 pts See comment**

- **10 pts** No substantive answer

+ 7.5 Point adjustment

Why isn't this Tg? In the next line also, the change of variables is not correct

3.2 (b) 5 / 10

- **0 pts** Correct

✓ **- 1 pts Did not mention divisibility of k by m1 and m2**

- **6.5 pts** Incorrect

- **10 pts** See comment

✓ **- 2 pts Missing alphas or different scale factor (that does not contain t, since it shouldn't)**

✓ **- 2 pts k or mk instead of k/m**

- **10 pts** No substantive answer

QUESTION 4

Fourier Transform 25 pts

4.1 (a) 10 / 10

✓ **+ 10 pts Correct**

+ **5 pts** Attempted to use inverse FT formula and did many correct algebraic steps.

+ **3 pts** Wrote equation of Fourier Transform but did not substitute omega = 0

+ **2 pts** Attempted integral incorrectly, or other attempted math with Fourier transform (either of rect/sincs, or inverse FT).

+ **2 pts** Explanation of property with no proof, or applying it to sinc(2t) incorrectly.

+ **0 pts** Incorrect or no answer

4.2 (b) 5 / 5

✓ **+ 5 pts Correct, with Fourier Transform taken correctly.**

+ **4 pts** Took the Fourier Transform correctly, did not calculate an area or did so incorrectly.

+ **2 pts** Did not compute Fourier transform of sinc(2t) correctly, or other incorrect algebra.

+ **0 pts** No appropriate work for partial credit or answer.

4.3 (c) 5 / 5

✓ **+ 5 pts Correct, omega_0 > 2*pi, or correct based on their answer to part (b).**

+ **4.5 pts** Mistake on the amount of shift (e.g., 4pi instead of 2pi; or did not plug in omega = 0; or did not specify the shift precisely).

+ **3.5 pts** Recognize the FT is time-shifted, did not correctly deduce when rect is 0 or other incorrect algebra.

+ **3 pts** Recognized the FT is time-shifted.

+ **2 pts** Incorrect answer due to incorrect rationale, or work appropriate for partial credit (such as showing some conditions where it's zero).

+ **0 pts** No appropriate work for partial credit, or no answer.

4.4 (d) 5 / 5

✓ **+ 5 pts Correct, alpha=-1/2, or correct based on their answer to parts b or c. We required you to simplify to a number since we asked for a value -- it wasn't sufficient to keep things in terms of rects and sincs.**

+ **4 pts** Would have had the correct answer if recalled sinc(0) = 1 or other minor algebraic constant.

+ **3 pts** Didn't simplify rect(0) or sinc(0), but had the correct (or reasonable) equation; or other related error.

+ **2 pts** Incorrect answer due to not treating the rect

<-> sinc term correctly, or other partial work.

+ 1 pts Attempt to simplify the integral with incorrect arguments; or other incorrect / incomplete arguments.

+ 0 pts No appropriate work for partial credit, or no answer.

QUESTION 5

5 Bonus **0 / 6**

- 0 pts Correct for intended interpretation

✓ **- 6 pts** **No answer or incorrect justification or no justification**

- 3 pts Partially correct

- 5 pts If "any" was interpreted as "some" rather than "every": correct example of some causal system S

- 6 pts See comment

ECE102, Fall 2019
Department of Electrical and Computer Engineering
University of California, Los Angeles

Midterm
Prof. J.C. Kao
TAs: W. Feng, J. Lee & S. Wu

UCLA True Bruin academic integrity principles apply.
Open: Two cheat sheets allowed.
Closed: Book, computer, internet.
2:00-3:50pm.
Wednesday, 13 Nov 2019.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.
There is an extra blank space on page 16 to show your work if you run out of space on any questions.

Name: Kyle Osborn

Signature: Kyle Osborn

ID#: 005-101-089

Problem 1	____	/ 35
Problem 2	____	/ 20
Problem 3	____	/ 20
Problem 4	____	/ 25
BONUS	____	/ 6 bonus points
Total	____	/ 100 points + 6 bonus points

1. **Signal and System Properties + Convolution** (35 points).

   (a) (15 points) Determine if each of the following statements is true or false. Briefly explain your answer to receive full credit.

      i. (5 points) $x(t) = \cos\left(\sqrt{3}t\right) + \sin(-3t)$ is a periodic signal.

$T_1 = \frac{2\pi}{\sqrt{3}}$ $T_2 = \frac{2\pi}{-3}$

False, there is no common period T_0 where $T_0 = k_1 T_1 = k_2 T_2$

      ii. (5 points) A signal can be neither energy signal nor power signal.

False, A signal can be either an energy signal or a power signal

where an energy signal $y(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

and power signal $z(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

iii. (5 points) Let $f(t) * g(t)$ denote the convolution of two signals, $f(t)$ and $g(t)$. Then,

$$f(t)[\delta(t) * g(t)] = [f(t)\delta(t)] * g(t)$$

False, $x(t) * \delta(t) = x(t)$

so $\delta(t) * g(t) = g(t)$

Thus, $f(t)[\delta(t) * g(t)] = f(t)[g(t)]$

On the RHS: $[f(t)\delta(t)] * g(t) = [f(0)\delta(t)] * g(t)$ (sampling property)

$= f(0)g(t)$

LHS $\neq$ RHS so false

(b) (10 points) Determine if the following system is an LTI system. Explain your answer.

$$y(t) = \frac{x(t-1)}{t} + x(t-2) \tag{1}$$

Checking if Linear:

$$S[ax_1 + bx_2(t)] = \frac{ax_1(t-1) + bx_2(t-1)}{t} + ax_1(t-2) + bx_2(t-2)$$

$$= \frac{ax_1(t-1)}{t} + ax_1(t-2) + \frac{bx_2(t-1)}{t} + bx_2(t-2)$$

$$= a\left(\frac{x_1(t-1)}{t} + x_1(t-2)\right) + b\left(\frac{x_2(t-1)}{t} + x_2(t)\right)$$

$$= aS[x_1(t)] + bS[x_2(t)]$$ Linear ✓

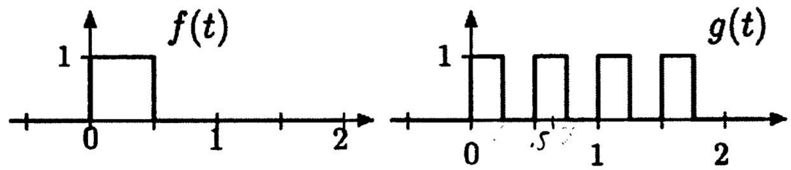
Testing T-I: Delaying input: $\frac{x(t-1-\tau)}{t} + x(t-2-\tau)$

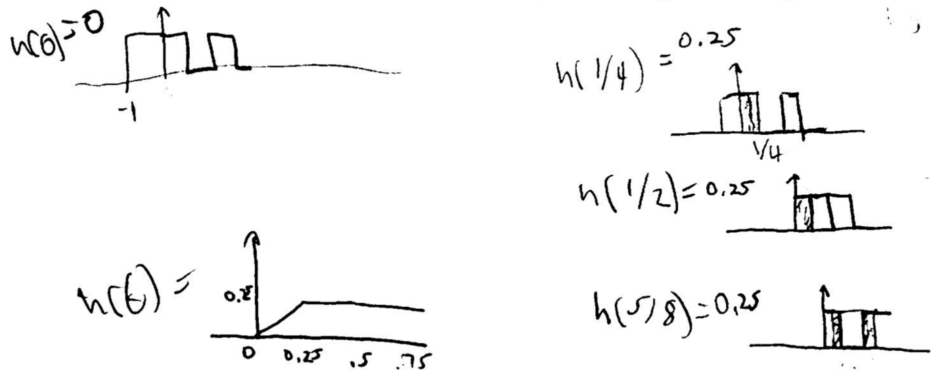
Delaying output: $\frac{x(t-1-\tau)}{t-\tau} + x(t-2-\tau)$

Not T-I, $y(t-\tau) \neq S[x(t-\tau)]$

Not an LTI system, although its linear, its not time-invariant

(c) (10 points) For signals $f(t)$ and $g(t)$ plotted below, graphically compute the convolution signal $h(t) = f(t) * g(t)$. To receive partial credit, you may show $h(0)$, $h(1/4)$ and $h(5/8)$ in the graph when illustrating the convolution using the "flip and drag" technique.

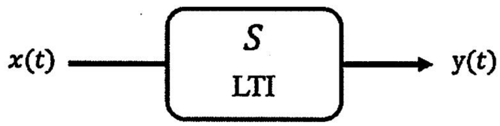




$$h(t) = \frac{1}{4}u(t-0.25) + r\left(\frac{t}{4}\right) - r\left(\frac{t}{4} - 0.25\right)$$

2. **LTI Systems** (20 points).

Consider the following LTI system S:



Consider an input signal $x_1(t) = e^{-2t}u(t-2)$. It is given that

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$\frac{dx_1(t)}{dt} \xrightarrow{S} -2y_1(t) + e^{-2t}u(t)$$

(a) (4 points) Show that:

$$\frac{dx_1(t)}{dt} = -2x_1(t) + e^{-2t}\delta(t-2)$$

$x_1(t) = e^{-2t}u(t-2)$

$\frac{d\,u(t)}{dt} = \delta(t)$

Chain Rule: $\frac{dx_1(t)}{dt} = -2e^{-2t}u(t-2) + e^{-2t}\delta(t-2)$

$\frac{dx_1(t)}{dt} = -2x_1(t) + e^{-2t}\delta(t-2)$ ✓

(b) (10 points) Find the impulse response $h(t)$ of S.

Hint: Since we have not provided S, we cannot straightforwardly input an impulse into the system and measure the output. One approach is to solve for $h(t)$ by writing the output of S in terms of a convolution when the input is $dx_1(t)/dt$, i.e.,

$$\frac{dx_1(t)}{dt} * h(t)$$

$x_1(t) = e^{-2t}u(t-2)$

$\frac{d\,x_1(t)}{dt} = -2x_1(t) + e^{-4t}\delta(t-2)$

$-2y_1(t) + e^{-2t}u(t) = \int_{-\infty}^{\infty}$

$x_1(t) * h(t) = y_1(t)$

$\frac{d_1 x(t)}{dt} * h(t) = -2y_1(t) + e^{-2t}u(t)$

$= -2x_1(t) * h(t) + e^{-2t}\delta(t-2) * h(t)$

$= -2y_1(t) + e^{-2t}(u(t))$

So $\delta(t-2) * h(t) = u(t)$

Delayed identity; $x(t) * \delta(t-t_0) = x(t-t_0)$

So $h(t-2) = u(t)$

Thus, $h(t) = u(t+2)$.

(c) (6 points) Consider a new system, S_2, whose impulse response is $h_2(t) = e^{-3t}u(t+3)$. Find this system's output to the following input signal:

$$x_2(t) = \cos\left(\frac{\pi}{4}t\right)\delta(t-1)$$

output $= x_2(t) * h_2(t)$

sampling prop: $x_2(t) = \cos\left(\frac{\pi}{4}\right)\delta(t-1)$

output $= \cos\left(\frac{\pi}{4}\right)\int_{-\infty}^{\infty} e^{-3\tau}u(\tau+3)\,\delta(t-1-\tau)\,d\tau$

$= \cos\left(\frac{\pi}{4}\right)\int_{-3}^{\infty} e^{-3\tau}\delta(t-1-\tau)\,d\tau$ $\quad$ $\delta(0)$ at $\tau = t+1$ so we sampling prop

$= \cos\left(\frac{\pi}{4}\right)e^{-3(t+1)}\int_{-3}^{\infty}\delta(t-1-\tau)\,d\tau$

$y_2(t) = \boxed{\cos\left(\frac{\pi}{4}\right)e^{-3(t+1)}}$

3. **Fourier Series** (20 points).

(a) (10 points) Let the Fourier Series coefficients of $f(t)$ be denoted f_k, and the Fourier Series coefficients of $g(t)$ denoted g_k. Let T_o be the period of $f(t)$. If $g(t) = f(a(t-b))$, where $a > 0$, show that

$$g_k = e^{-j2\pi \frac{ab}{T_o} k} f_k.$$

$$T_g = \frac{T_0}{a}$$

fourier series coeff eq:

$$f_k = \frac{1}{T_0} \int_{\tau}^{\tau+T_0} f(t) e^{-jk\frac{2\pi}{T_0}t} \, dt$$

$$g_k = \frac{a}{T_0} \int_{\tau}^{\tau+T_0} f(a(t-b)) \, e^{-jk\frac{2\pi a}{T_0}t} \, dt$$

$$g_k = \frac{1}{T_0} \int_{\tau}^{\tau+T_0} f(t) e^{-jk\frac{2\pi a}{T_0}(\frac{t}{a}+b)} \, dt$$

let $t = at - ab$ so $t = \frac{t+ab}{a} = \frac{t}{a}+b$

$dt = a\,dt$

$dt = \frac{dt}{a}$

$$g_k = \frac{1}{T_0} \int_{\tau}^{\tau+T_0} f(t) e^{-\frac{jk2\pi}{T_0}t} e^{-\frac{jk2\pi ab}{T_0}k} \, dt$$

$$g_k = e^{-\frac{jk2\pi ab}{T_0}} \left(\frac{1}{T_0} \int_{\tau}^{\tau+T_0} f(t) e^{-\frac{jk2\pi}{T_0}t} \, dt \right)$$

$$g_k = e^{-\frac{jk2\pi ab}{T_0}} f_k \checkmark$$

(b) (10 points) Let the Fourier Series coefficients of $x(t)$ and $y(t)$ be x_k and y_k respectively, with respective periods T_1 and T_2. We define $f(t) = \alpha_1 x(t) + \alpha_2 y(t)$ with non-zero α_1, α_2, with period $T_o = m_1 T_1 = m_2 T_2$. What are the Fourier Series coefficients f_k in terms of x_k and y_k?

$$f_k = \frac{1}{T_0}\int_{\tau}^{\tau+T_0} \left(\alpha_1 x(t) + \alpha_2 y(t)\right) e^{-jk\frac{2\pi}{T_0}t}\, dt$$

$$f_k = \frac{1}{m_1 T_1}\int_{\tau}^{\tau+m_1T_1} \alpha_1 x(t) e^{\frac{-jk2\pi t}{m_1 T_1}}\, dt + \frac{1}{m_2 T_2}\int_{\tau}^{\tau+m_2T_2} \alpha_2 y(t) e^{\frac{-jk2\pi}{m_2 T_2}}\, dt$$

X_k of form $\frac{1}{T_1}\int_{\tau}^{\tau+T_1} x(t)\, e^{\frac{-jk2\pi t}{T_1}}\, dt$

follow similar form but multiplied by α_1 and divided by m_1

$$f_k = \frac{\alpha_1 X_k}{m_1} + \frac{\alpha_2 Y_k}{m_2}$$

4. **Fourier Transform** (25 points).
Consider the signal

$$x(t) = \text{sinc}(2t)$$

and let the Fourier transform of $x(t)$ be denoted $X(j\omega)$. We are interested in calculating the area under the curve of $x(t)$.

(a) (10 points) Prove that the following relationship holds.

$$\int_{-\infty}^{\infty} x(t)dt = X(j\omega)|_{\omega=0}$$

Fourier Transformation Def.

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

So $X(jw)|_{w=0} = \int_{-\infty}^{\infty} x(t)e^{-j(0)t}dt$ 1

Thus, $X(jw)|_{w=0} = \int_{-\infty}^{\infty} x(t)\,dt$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \Rightarrow \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

$t \to \infty$

(b) (5 points) Use the result of part (a) to calculate:

$$\int_{-\infty}^{\infty} x(t)dt$$

for $x(t) = \text{sinc}(2t)$.

$$\int_{-\infty}^{\infty} \text{sinc}(2t)\, dt$$

Since $\int_{-\infty}^{\infty} x(t) = X(j\omega)\big|_{\omega=0}$

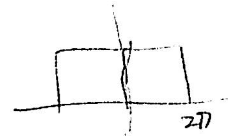
and $\text{sinc}(t) \Rightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$

$\text{sinc}(2t) \Rightarrow \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$

$X(0) = \frac{1}{2}\text{rect}(0) = \frac{1}{2}$

Thus, $\boxed{\int_{-\infty}^{\infty} \text{sinc}(2t)\, dt = \frac{1}{2}}$

(c) (5 points) Consider the following system:



$$y(t) = e^{-j\omega_0 t}x(t)$$

Let $x(t) = \text{sinc}(2t)$ and consider only $\omega_0 > 0$. Are there any values of ω_0 for which

$$\int_{-\infty}^{\infty} y(t)dt = 0$$

and if so, what value(s) of ω_0 does this hold for?

$x(t) \xrightarrow{\mathcal{F}} \frac{1}{2}\,\text{rect}\left(\frac{\omega}{4\pi}\right)$

$e^{-j\omega_0 t}x(t) \xrightarrow{\mathcal{F}} \frac{1}{2}\,\text{rect}\left(\frac{\omega+\omega_0}{4\pi}\right)$

for $\int_{-\infty}^{\infty} y(t)dt = 0$, $\quad Y(0)$: $\frac{1}{2}\,\text{rect}\left(\frac{\omega_0}{4\pi}\right) = 0$

Thus, for $\omega_0 > 2\pi$ $\int_{-\infty}^{\infty} y(t)\,dt = 0$

(d) (5 points) Consider the following system:

$$y(t) = x(t) + \alpha \text{rect}(t)$$

Let $x(t) = \text{sinc}(2t)$. Are there any values of α for which

let $z(t) = \alpha \, \text{rect}(t)$

$$\int_{-\infty}^{\infty} y(t)dt = 0$$

and if so, what value(s) of α does this hold for?

$Y(j\omega) = X(j\omega) + Z(j\omega)$

from b) $X(j\omega) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$

$X(0) = \frac{1}{2}\text{rect}(0) = \frac{1}{2}$

So for $\int_{-\infty}^{\infty} y(t)\,dt = 0$ $\quad Z(0) = -\frac{1}{2}$

$Z(j\omega) = \alpha \, \text{sinc}\left(\frac{\omega}{2\pi}\right)$

$Z(0) = \alpha \, \text{sinc}(0) = \alpha = -\frac{1}{2}$

So for $\boxed{\alpha = -\frac{1}{2}}$, $\int_{-\infty}^{\infty} y(t)\,dt = 0$.

Bonus (6 points) Suppose $x(t) = \cos(\omega_o t)$ is an eigenfunction of an LTI system S for any ω_o, and S cannot be defined as $S[x(t)] = ax(t)$ for some constant a. Is the system S causal? Justify your answer.

$\cos(\omega_0 t) = \frac{1}{2}e^{j\omega t} + \frac{1}{2}e^{-j\omega t}$

False

Yes, exponentials are the eigenfunctions of LTI systems and cos(wot) can be written as above, for system where $h(t)=0$ when $t \leq 0$, the system is causal.

cos >0 at $t<0$

This is an extra piece of paper to show your work. If you use this space for a question, for that question, please write "Refer to page 16."