ECE 102 Midterm

Andrew Tang

TOTAL POINTS

101 / 106

QUESTION 1

Signal/System/Convolution 35 pts

1.1 (a)i 5 / 5

✓ + **5 pts** **Correct**

+ **4 pts** Correctly computed periods of both signals and its ratio, but with insufficient justification

+ **2 pts** Attempted to justify the signal is not periodic, with insufficient or incorrect justification

+ **3.5 pts** Stated no common period, without computing or mentioning the ratio of periods

+ **0 pts** Incorrect answer or no Justification

1.2 (a)ii 5 / 5

✓ + **5 pts** **Correct**

+ **0 pts** Incorrect answer or no/incorrect justification

+ **2 pts** Insufficient Reasoning to justify the answer

1.3 (a)iii 5 / 5

✓ + **5 pts** **Correct answer**

+ **3 pts** One side is simplified correctly

+ **1.5 pts** Correctly used the sifting property on the right hand side

+ **1.5 pts** Correctly used the sampling property on the left hand side

+ **0 pts** Incorrect answer or reasoning

1.4 (b) 10 / 10

✓ + **10 pts** **Correct**

+ **5 pts** Correctly identify the linearity property

+ **0 pts** Incorrect

1.5 (c) 8 / 10

+ **10 pts** Correct h(t)

+ **8 pts** Correct h(t) for t<2

✓ + **8 pts** **Correct shape of h(t) but incorrect amplitude**

+ **2 pts** correct h(0)

+ **2 pts** correct h(1/4)

+ **2 pts** correct h(5/8)

+ **2 pts** Correct h(t) for t>2

+ **0 pts** Incorrect

QUESTION 2

LTI Systems 20 pts

2.1 (a) 4 / 4

✓ - **0 pts** **Correct**

- **1 pts** minor mistakes

- **2 pts** half correct

- **4 pts** incorrect

2.2 (b) 10 / 10

✓ + **3 pts** **use LTI properties.**

✓ + **3 pts** **correct convolution equation/multiplication in frequency domain.**

✓ + **2 pts** **correct convolution results/inverse FT results.**

✓ + **2 pts** **correct time shift for the last step.**

+ **0 pts** incorrect

2.3 (c) 6 / 6

✓ - **0 pts** **Correct.**

- **1 pts** constant term incorrect.

- **2 pts** time shift incorrect.

- **2 pts** inverse FT incorrect.

- **3 pts** missing/incorrect exponential/unit step function.

- **4 pts** calculate in frequency domain with wrong FT.

- **5 pts** incorrect calculation with a correct convolution equation.

- **6 pts** incorrect.

QUESTION 3

Fourier Series 20 pts

3.1 (a) 10 / 10

✓ **- 0 pts** **Correct**

- 0.5 pts Did not explicitly use the angular frequency ωg = aωo, or Tg = To/a in the Fourier Series.

- 5 pts Partially correct

- 7.5 pts Incorrect

- 10 pts See comment

- 10 pts No substantive answer

3.2 (b) 7 / 10

- 0 pts Correct

✓ **- 1 pts** **Did not mention divisibility of k by m1 and m2**

- 6.5 pts Incorrect

- 10 pts See comment

- 2 pts Missing alphas or different scale factor (that does not contain t, since it shouldn't)

✓ **- 2 pts** **k or mk instead of k/m**

- 10 pts No substantive answer

QUESTION 4

Fourier Transform 25 pts

4.1 (a) 10 / 10

✓ **+ 10 pts** **Correct**

+ 5 pts Attempted to use inverse FT formula and did many correct algebraic steps.

+ 3 pts Wrote equation of Fourier Transform but did not substitute omega = 0

+ 2 pts Attempted integral incorrectly, or other attempted math with Fourier transform (either of rect/sincs, or inverse FT).

+ 2 pts Explanation of property with no proof, or applying it to sinc(2t) incorrectly.

+ 0 pts Incorrect or no answer

4.2 (b) 5 / 5

✓ **+ 5 pts** **Correct, with Fourier Transform taken correctly.**

+ 4 pts Took the Fourier Transform correctly, did not calculate an area or did so incorrectly.

+ 2 pts Did not compute Fourier transform of sinc(2t) correctly, or other incorrect algebra.

+ 0 pts No appropriate work for partial credit or answer.

4.3 (c) 5 / 5

✓ **+ 5 pts** **Correct, omega_0 > 2*pi, or correct based on their answer to part (b).**

+ 4.5 pts Mistake on the amount of shift (e.g., 4pi instead of 2pi; or did not plug in omega = 0; or did not specify the shift precisely).

+ 3.5 pts Recognize the FT is time-shifted, did not correctly deduce when rect is 0 or other incorrect algebra.

+ 3 pts Recognized the FT is time-shifted.

+ 2 pts Incorrect answer due to incorrect rationale, or work appropriate for partial credit (such as showing some conditions where it's zero).

+ 0 pts No appropriate work for partial credit, or no answer.

4.4 (d) 5 / 5

✓ **+ 5 pts** **Correct, alpha=-1/2, or correct based on their answer to parts b or c. We required you to simplify to a number since we asked for a value -- it wasn't sufficient to keep things in terms of rects and sincs.**

+ 4 pts Would have had the correct answer if recalled sinc(0) = 1 or other minor algebraic constant.

+ 3 pts Didn't simplify rect(0) or sinc(0), but had the correct (or reasonable) equation; or other related error.

+ 2 pts Incorrect answer due to not treating the rect <-> sinc term correctly, or other partial work.

+ 1 pts Attempt to simplify the integral with incorrect arguments; or other incorrect / incomplete arguments.

+ 0 pts No appropriate work for partial credit, or no answer.

QUESTION 5

5 Bonus **6 / 6**

✓ **- 0 pts** **Correct for intended interpretation**

- 6 pts No answer or incorrect justification or no justification

- 3 pts Partially correct

- 5 pts If "any" was interpreted as "some" rather than "every": correct example of some causal system S

- 6 pts See comment

ECE102, Fall 2019
Department of Electrical and Computer Engineering
University of California, Los Angeles

Midterm
Prof. J.C. Kao
TAs: W. Feng, J. Lee & S. Wu

UCLA True Bruin academic integrity principles apply.
Open: Two cheat sheets allowed.
Closed: Book, computer, internet.
2:00-3:50pm.
Wednesday, 13 Nov 2019.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.
There is an extra blank space on page 16 to show your work if you run out of space on any questions.

Name: Andrew Tang

Signature: Andrew Tang

ID#: 905 014 422

Problem 1	____	/ 35
Problem 2	____	/ 20
Problem 3	____	/ 20
Problem 4	____	/ 25
BONUS	____	/ 6 bonus points
Total	____	/ 100 points + 6 bonus points

1. **Signal and System Properties + Convolution** (35 points).

(a) (15 points) Determine if each of the following statements is true or false. Briefly explain your answer to receive full credit.

i. (5 points) $x(t) = \cos(\sqrt{3}t) + \sin(-3t)$ is a periodic signal.

The period of $\cos(\sqrt{3}t)$ is $\frac{2\pi}{\sqrt{3}}$

The period of $\sin(-3t)$ is $\frac{2\pi}{3}$

$\sqrt{3}$ is an irrational number so there is no rational number multiple

such that $K_1 \frac{2\pi}{\sqrt{3}} = K_2 \frac{2\pi}{3} \Rightarrow \frac{K_2}{K_1} = \sqrt{3}$

Therefore, $x(t)$ is not periodic so the statement is false

ii. (5 points) A signal can be neither energy signal nor power signal.

True if a signal is $x(t) = \infty$, then it is not a Energy signal since $\int_{-\infty}^{\infty} |x(t)|^2 dt$ is infinite, and it's not a power signal since the power is the average and that's just ∞ or the DC term which is not bounded, meaning that it's not a power signal either.

iii. (5 points) Let $f(t) * g(t)$ denote the convolution of two signals, $f(t)$ and $g(t)$. Then,

$$\underline{f(t)[\delta(t) * g(t)]} = [f(t)\delta(t)] * g(t)$$

$$f(t)\,[\delta(t) * g(t)] = f(t)\cdot g(t)$$

by the identity property of convolution

$$[f(t)\,\delta(t)] * g(t)$$

$$= \int_{-\infty}^{\infty} f(\tau)\,\delta(\tau)\,g(t-\tau)\,d\tau$$

$$= f(0)\,g(t)$$

$f(0)g(t) \neq f(t)g(t)$ for all

signals therefore this is

<u>false!</u>

(b) (10 points) Determine if the following system is an LTI system. Explain your answer.

$$y(t) = \frac{x(t-1)}{t} + x(t-2) \tag{1}$$

No,

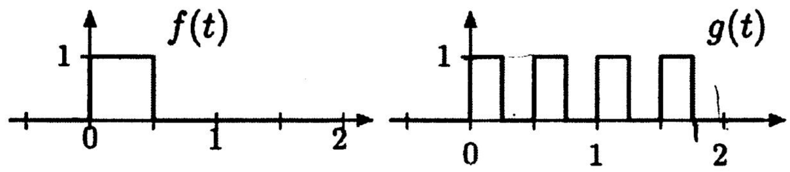
$$y(t-\tau) = \frac{x(t-1-\tau)}{t-\tau} + x(t-2-\tau)$$

however if we plug into $t' = t-\tau$ for $x(t')$ we have

$$\frac{x(t-1-\tau)}{t} + x(t-2-\tau).$$

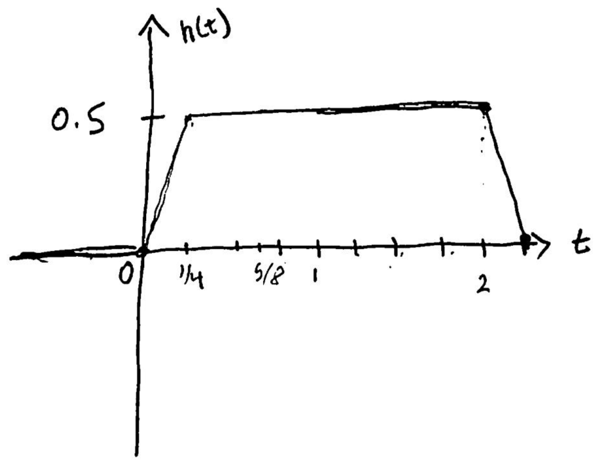
Therefore the system is not time invariant, since a time shift in the input and output are different.

(c) (10 points) For signals $f(t)$ and $g(t)$ plotted below, graphically compute the convolution signal $h(t) = f(t) * g(t)$. To receive partial credit, you may show $h(0)$, $h(1/4)$ and $h(5/8)$ in the graph when illustrating the convolution using the "flip and drag" technique.



$h(t) = f(t) * g(t) = g(t) * f(t)$ $\qquad$ $0.5 \cdot 1 = 0.5$

so we flip and drag $f(t)$.

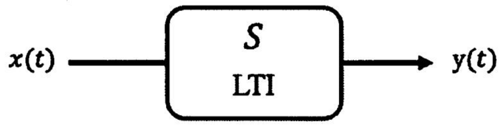


$h(t)$ stays constant from $1/4$ to 2 since there is always the same amount of overlap.

→ It first meets at $t=0$ then maxes out at $t = 1/4$, then it stagnates since whenever you interact w/ the next skinny rectangle from $g(t)$ you leave the previous one at the same rate. This holds until $t=2$ where you fully exit from the nonzero part of $g(t)$ afterwards.

2. **LTI Systems** (20 points).

Consider the following LTI system S:



Consider an input signal $x_1(t) = e^{-2t}u(t-2)$. It is given that

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$\frac{dx_1(t)}{dt} \xrightarrow{S} -2y_1(t) + e^{-2t}u(t)$$

(a) (4 points) Show that:

$$\frac{dx_1(t)}{dt} = -2x_1(t) + e^{-2t}\delta(t-2)$$

$$\frac{d(x_1(t))}{dt} = \frac{d}{dt}\left(e^{-2t}u(t-2)\right) =$$

by chain rule and that the derivative of a step is a delta function

$$-2e^{-2t}u(t-2) + e^{-2t}\delta(t-2) =$$

$$-2\left(e^{-2t}u(t-2)\right) + e^{-2t}\delta(t-2) =$$

$$\boxed{-2x_1(t) + e^{-2t}\delta(t-2)}$$

(b) (10 points) Find the impulse response $h(t)$ of S.

Hint: Since we have not provided S, we cannot straightforwardly input an impulse into the system and measure the output. One approach is to solve for $h(t)$ by writing the output of S in terms of a convolution when the input is ${dx_1(t)}/{dt}$, i.e.,

$$\frac{dx_1(t)}{dt} * h(t)$$

$$\frac{dx_1(t)}{dt} * h(t) = -2y_1(t) + e^{-2t}u(t)$$

$$\left(-2x_1(t) + e^{-2t}\delta(t-2)\right) * h(t) = -2y_1(t) + e^{-2t}u(t)$$

By the distributivity of convolution

$$\underbrace{-2(x_1(t) * h(t))}_{\beta} + e^{-2t}\delta(t-2) * h(t) = -2y_1(t) + e^{-2t}u(t)$$

$$-2y_1(t) + e^{-2t}\delta(t-2) * h(t) = -2y_1(t) + e^{-2t}u(t)$$

$$e^{-2t}\delta(t-2) * h(t) = e^{-2t}u(t)$$

$$\int_{-\infty}^{\infty} h(\tau)e^{-2(t-\tau)}\delta(t-\tau-2)d\tau = e^{-2t}u(t)$$

$$\cancel{e^{-2t}}\int_{-\infty}^{\infty} h(\tau)e^{2\tau}\delta(t-\tau-2)d\tau = e^{-2t}u(t)$$

$$\int_{-\infty}^{\infty} h(\tau)e^{2\tau}\delta(t-\tau-2)d\tau = u(t)$$

$$h(t-2)e^{2(t-2)} = u(t)$$

$$h(t-2) = \frac{u(t)}{e^{2(t-2)}}$$

$$\boxed{h(t) = \frac{u(t+2)}{e^{2t}}}$$

(c) (6 points) Consider a new system, S_2, whose impulse response is $h_2(t) = e^{-3t}u(t+3)$. Find this system's output to the following input signal:

$$x_2(t) = \cos\left(\frac{\pi}{4}t\right)\delta(t-1)$$

$$\cos\left(\frac{\pi}{4}t\right) = \frac{1}{2}\left(e^{j\frac{\pi}{4}t} + e^{-j\frac{\pi}{4}t}\right)$$

$$x_2(t) = \frac{1}{2}\left(e^{j\frac{\pi}{4}t} + e^{-j\frac{\pi}{4}t}\right)\delta(t-1)$$

Let $y_2(t)$ be the output

$$y_2(t) = x_2(t) * h_2(t) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-3\tau}u(\tau+3)\cdot\left(e^{j\frac{\pi}{4}(t-\tau)} + e^{-j\frac{\pi}{4}(t-\tau)}\right)\delta(t-\tau-1)\,d\tau$$

$$= \frac{1}{2}\left(e^{j\frac{\pi}{4}t}\int_{-\infty}^{\infty} e^{(-3-j\frac{\pi}{4})\tau}u(\tau+3)\,\delta(t-\tau-1)\,d\tau + e^{-j\frac{\pi}{4}t}\int_{-\infty}^{\infty} e^{(-3+j\frac{\pi}{4})\tau}u(\tau+3)\,\delta(t-\tau-1)\,d\tau\right)$$

$t-1$

$$= \frac{1}{2}\left(e^{j\frac{\pi}{4}t}\cdot e^{(-3-j\frac{\pi}{4})(t-1)}u(t+2) + e^{-j\frac{\pi}{4}t}\cdot e^{(-3+j\frac{\pi}{4})(t-1)}u(t+2)\right)$$

$-3t - \frac{j\pi}{4} + 3 + j\frac{\pi}{4}$

$$= \frac{1}{2}\left(e^{-3t+3+\frac{j\pi}{4}}u(t+2) + e^{-3t+3-\frac{j\pi}{4}}u(t+2)\right)$$

$$= \boxed{\frac{u(t+2)}{2}e^{-3t+3}\left(e^{j\pi/4} + e^{-j\pi/4}\right)}$$

3. **Fourier Series** (20 points).

(a) (10 points) Let the Fourier Series coefficients of $f(t)$ be denoted f_k, and the Fourier Series coefficients of $g(t)$ denoted g_k. Let T_o be the period of $f(t)$. If $g(t) = f(a(t-b))$, where $a > 0$, show that

$$g_k = e^{-j2\pi \frac{ab}{T_o}k} f_k.$$

$$\omega_{f(t)} = \frac{2\pi}{T_o}$$

$$f(t) = \sum_{k=-\infty}^{\infty} f_k e^{jk\omega_o t}$$

$$g(t) = f(a(t-b)) = \sum_{k=-\infty}^{\infty} f_k e^{jk\frac{2\pi}{T_o}\cdot a(t-b)}$$

$$= \sum_{k=-\infty}^{\infty} f_k e^{jk\frac{2\pi}{T_o}(at-ab)}$$

$$= \sum_{k=-\infty}^{\infty} f_k e^{jk\frac{2\pi}{T_o}at} \cdot e^{-jk\frac{2\pi}{T_o}ab}$$

Since $g(t) = f(a(t-b))$ w/ the t scaled by a, the period of $g(t)$ is equal to $\frac{T_o}{a}$ or

$\omega_{g(t)} = a\omega_{f(t)}$ so we have

$$g(t) = \left(\sum_{k=-\infty}^{\infty} f_k e^{-jk\frac{2\pi}{T_o}ab}\right) \cdot e^{jk\frac{2\pi}{T_o}at}$$

and that

$$g(t) = \sum_{-\infty}^{\infty} g_k \cdot e^{jk\frac{2\pi}{T_o}at}$$, so therefore

$$\boxed{g_k = f_k \cdot e^{-jk\frac{2\pi}{T_o}ab}}$$

(b) (10 points) Let the Fourier Series coefficients of $x(t)$ and $y(t)$ be x_k and y_k respectively, with respective periods T_1 and T_2. We define $f(t) = \alpha_1 x(t) + \alpha_2 y(t)$ with non-zero α_1, α_2, with period $T_o = m_1 T_1 = m_2 T_2$. What are the Fourier Series coefficients f_k in terms of x_k and y_k?

$$f(t) = \sum_{-\infty}^{\infty} f_k e^{j\frac{2\pi}{T_0}kt} = \alpha_1 \sum_{n=-\infty}^{\infty} x_n e^{j\frac{2\pi}{T_1}nt} + \alpha_2 \sum_{m=-\infty}^{\infty} y_m e^{j\frac{2\pi}{T_2}mt}$$

$m_1 T_1$

The $e^{j\frac{2\pi}{T_0}kt}$ muste equal the $e^{j\frac{2\pi}{T_1}nt}$ and $e^{j\frac{2\pi}{T_2}mt}$ for the fourier series coefficients of $x(t)$ and $y(t)$ to contribute to that of $f(t)$.

(1)

So we have

$$f_k = \alpha_1 X_{m_1 k} + \alpha_2 y_{m_2 k}$$

Since for the e terms to match ~~we~~ with T_1 and T_2 in the denominator n and m must be m_1 ~~and~~ and m_2 times k respectively, That means we are looking at $X_{m_1 k}$ and $y_{m_2 k}$.

4. **Fourier Transform** (25 points).
Consider the signal

$$x(t) = \text{sinc}(2t)$$

and let the Fourier transform of $x(t)$ be denoted $X(j\omega)$. We are interested in calculating the area under the curve of $x(t)$.

(a) (10 points) Prove that the following relationship holds.

$$\int_{-\infty}^{\infty} x(t)dt = X(j\omega)|_{\omega=0}$$

$$\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = X(j\omega)$$

by definition.

so if we evaluate both sides at $\omega = 0$ we have

$$\int_{-\infty}^{\infty} x(t)\, e^{-j\cdot 0\cdot t}\, dt = \boxed{\int_{-\infty}^{\infty} x(t)\, dt = X(j\omega)\big|_{\omega=0}}$$

Therefore the relationship holds.

(b) (5 points) Use the result of part (a) to calculate:

$$\int_{-\infty}^{\infty} x(t)dt$$

for $x(t) = \text{sinc}(2t)$.

$$\int_{-\infty}^{\infty} x(t)\,dt = X(j\omega)\Big|_{\omega=0}$$

Using the transform table

$$x(2t) = \frac{1}{2} X\left(\frac{j\omega}{2}\right)$$

so

$$x(t) = \text{sinc}(2t) \xrightarrow{\text{transforms to}} \frac{1}{2}\,\text{rect}\left(\frac{\omega}{4\pi}\right)$$

$$\frac{1}{2}\,\text{rect}\left(\frac{\omega}{4\pi}\right)\Bigg|_{\omega=0} = \frac{1}{2}\cdot 1 = \boxed{\frac{1}{2}}$$

(c) (5 points) Consider the following system:

$$y(t) = e^{-j\omega_0 t}x(t)$$

Let $x(t) = \text{sinc}(2t)$ and consider only $\omega_0 > 0$. Are there any values of ω_0 for which

$$\int_{-\infty}^{\infty} y(t)dt = 0$$

and if so, what value(s) of ω_0 does this hold for?

$$\int_{-\infty}^{\infty} y(t)\, e^{0}\, dt = Y(j\omega)\Big|_{\omega=0}$$

$$y(t) \xrightarrow{\text{transformed}} \frac{1}{2}\,\text{rect}\left(\frac{(\omega-\omega_0)}{4\pi}\right)$$

so $\frac{1}{2}\,\text{rect}\left(\frac{0-\omega_0}{4\pi}\right)$ $\quad \omega_0 > 0$

rect is 1 from $-\frac{1}{2}$ to $\frac{1}{2}$ and 0 everywhere else so when

$$-\frac{\omega_0}{4\pi} < -\frac{1}{2}$$

$$\frac{\omega_0}{4\pi} > \frac{1}{2}$$

$\boxed{\text{when } \omega_0 > 2\pi \text{ then } \int_{-\infty}^{\infty} y(t)dt = 0}$

(d) (5 points) Consider the following system:

$$y(t) = x(t) + \alpha \text{rect}(t)$$

Let $x(t) = \text{sinc}(2t)$. Are there any values of α for which

$$\int_{-\infty}^{\infty} y(t)dt = 0$$

and if so, what value(s) of α does this hold for?

$$\int_{-\infty}^{\infty} \text{sinc}(2t)\, dt + \alpha \int_{-\infty}^{\infty} \text{rect}(t)\, dt =$$

$$\frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)\Big|_{\omega=0} + \alpha\, \text{sinc}\left(\frac{\omega}{2\pi}\right)\Big|_{\omega=0} = 0$$

$$\frac{1}{2} + \alpha\, \text{sinc}(0) = 0$$

$$\frac{1}{2} + \alpha \cdot 1 = 0$$

$$\boxed{\alpha = -\frac{1}{2}}$$

Bonus (6 points) Suppose $x(t) = \cos(\omega_o t)$ is an eigenfunction of an LTI system S for any ω_o. and S cannot be defined as $S[\vec{x}(t)] = ax(t)$ for some constant a. Is the system S causal? Justify your answer.

$$\frac{1}{2}\left(e^{j\omega t} + e^{-j\omega_o t}\right)$$

No, b/c if $\cos(\omega_o t)$ is an eigenfunction then $y(t) \leftarrow$ output must be even, b/c it's a constant multiple of $\cos(\omega_o t)$ and when we convolute the $h(t)$ and $x(t)$, we know that $h(t)$ must be even too, since

$$y(-t) = \int_{-\infty}^{\infty} h(-\tau)\, x(-t-\tau)\, d\tau$$

$$x(-t) = x(t) \quad 10 \qquad \tau' = -\tau$$

$$y(-t) = y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t+\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(-\tau')\, x(t-\tau')\, d\tau'$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

For this to hold $h(-\tau) = h(\tau)$ meaning that $h(t)$ is even. If $h(t)$ is even then $h(t) \neq 0$ ~~when~~ for all $t<0$ ~~be~~ b/c otherwise it would be the case where $S[x(t)] = 0 \cdot x(t)$. If $h(t) \neq 0$ for ~~any~~ $t<0$ then it is not causal.

This is an extra piece of paper to show your work. If you use this space for a question, for that question, please write "Refer to page 16."