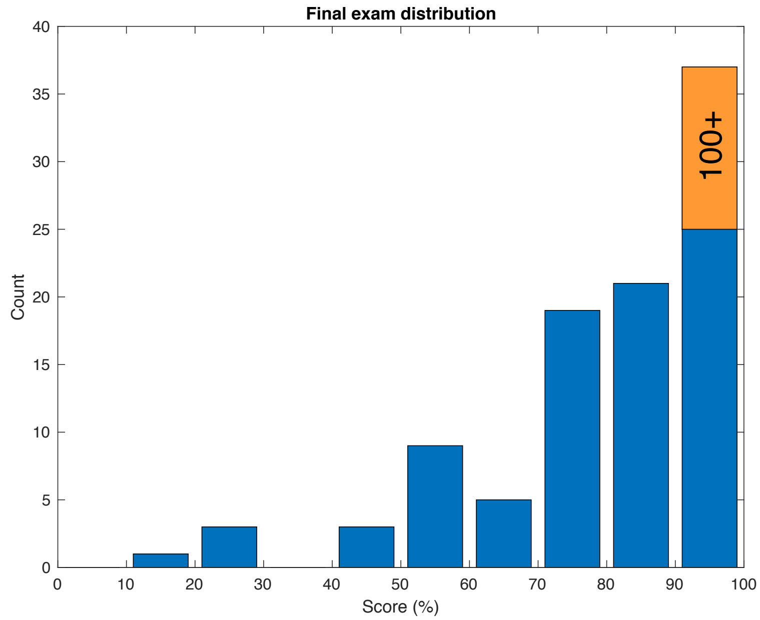
ECE 102, Fall 2018 **FINAL STATISTICS**

Department of Electrical and Computer Engineering Prof. J.C. Kao

University of California, Los Angeles TAs H. Salami & S. Shahsavari



Statistics: Mean: 80.0% (out of 100%), Standard deviation: 19.3, 25th perentile: 72.5%, Median: 84.8%, 75th percentile: 94%, Maximum score: 103.25%, Number of exams: 98

Comments:

- Regrades should be submitted if you believe we applied the rubric mistakenly. Please submit these via Gradescope. Hawraa graded questions 1, 2, and the bonus; Shadi graded questions 3 and 4; Jonathan graded questions 5 and 6.
- You demonstrated a solid understanding of material on the final. While the detailed statistics are above, I'd like to highlight that the median score was an 84.8% and the mean was 80%.
- Your final course grades were calculated using your exam and HW scores on Gradescope; we subsequently added any participation bonuses. You can calculate your letter grade by applying the scale in the syllabus. If you believe I miscalculated your final letter grade, please send me an e-mail.

ECE 102, Fall 2018 **Final Exam**

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University of California, Los Angeles TAs: H. Salami, S. Shahshavari

UCLA True Bruin academic integrity principles apply.
Open: Four pages of cheat sheet allowed.
Closed: Book, computer, internet.
11:30am-2:30pm, Haines Room 118
Tuesday, 11 Dec 2018.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.

Name: ____________________

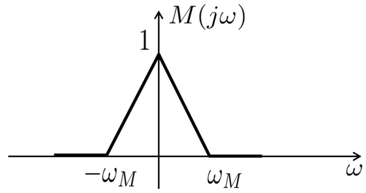
Signature: ____________________

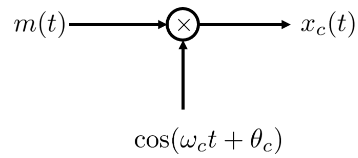
ID#: _________________

Problem 1	_____	/	25
Problem 2	_____	/	41
Problem 3	_____	/	30
Problem 4	_____	/	40
Problem 5	_____	/	20
Problem 6	_____	/	44
BONUS	_____	/	10 bonus points
Total	_____	/	200 points + 10 bonus points

Problem 1 (25 points)

Consider a bandlimited signal $m(t)$, its frequency spectrum $M(j\omega)$ is shown below. We modulate $m(t)$ with $\cos(\omega_c t + \theta_c)$, where θ_c is a constant phase but unknown:





(a) (8 points) Express $X_c(j\omega)$, the Fourier transform of $x_c(t)$, in terms of $M(j\omega)$. *Hint: use the fact that* $\cos(u) = \dfrac{e^{ju} + e^{-ju}}{2}$.

Solution:

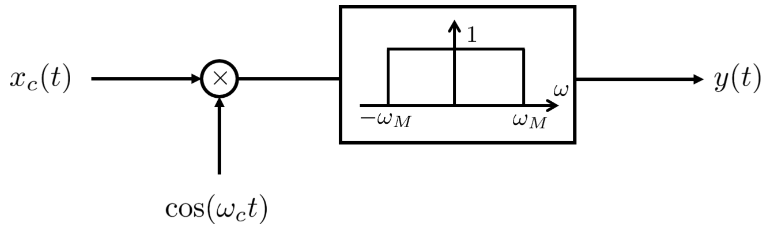
We have:

$$
\begin{aligned}
x_c(t) &= m(t)\cos(\omega_c t + \theta_c) = m(t)\frac{1}{2}\left(e^{j(\omega_c t + \theta_c)} + e^{-j(\omega_c t + \theta_c)}\right) \\
&= \frac{1}{2}e^{j\theta_c} m(t) e^{j\omega_c t} + \frac{1}{2}e^{-j\theta_c} m(t) e^{-j\omega_c t}
\end{aligned}
$$

Therefore,

$$
X_c(j\omega) = \frac{1}{2}e^{j\theta_c} M(j(\omega - \omega_c)) + \frac{1}{2}e^{-j\theta_c} M(j(\omega + \omega_c))
$$

(b) (10 points) We demodulate $x_c(t)$ as follows:



Show that $y(t) = \frac{1}{2}\cos(\theta_c)m(t)$. Assume $\omega_c \gg \omega_M$.

Solution: The input of the low pass filter:

$$x_c(t)\cos(\omega_c t)$$

Taking its Fourier transform, we obtain:

$$\begin{aligned}
&\frac{1}{2}X_c(j(\omega-\omega_c)) + \frac{1}{2}X_c(j(\omega+\omega_c)) = \\
&\qquad \frac{1}{4}e^{j\theta_c}M(j(\omega-2\omega_c)) + \frac{1}{4}e^{-j\theta_c}M(j\omega) + \frac{1}{4}e^{j\theta_c}M(j\omega) + \frac{1}{4}e^{-j\theta_c}M(j(\omega+2\omega_c)) = \\
&\qquad \frac{1}{4}e^{j\theta_c}M(j(\omega-2\omega_c)) + \frac{1}{4}\left(e^{-j\theta_c}+e^{j\theta_c}\right)M(j\omega) + \frac{1}{4}e^{-j\theta_c}M(j(\omega+2\omega_c)) = \\
&\qquad \frac{1}{4}e^{j\theta_c}M(j(\omega-2\omega_c)) + \frac{1}{2}\cos(\theta_c)M(j\omega) + \frac{1}{4}e^{-j\theta_c}M(j(\omega+2\omega_c))
\end{aligned}$$

After the low pass-filter, the term that only remains is

$$\frac{1}{2}\cos(\theta_c)M(j\omega)$$

Therefore,

$$y(t) = \frac{1}{2}\cos(\theta_c)m(t)$$

(c) (7 points) Assume that you also know $z(t) = \frac{1}{2}\sin(\theta_c)m(t)$. How can you recover $m(t)$ from $y(t)$ and $z(t)$?
Hint: $cos^2(u) + \sin^2(u) = 1$.

Solution:
To recover $m(t)$ from $z(t)$ and $y(t)$, we can compute the following:

$$2\sqrt{y^2(t) + z^2(t)}$$

This is because:

$$2\sqrt{y^2(t) + z^2(t)} = 2\sqrt{\left(\frac{1}{4}\cos^2(\theta_c)m^2(t) + \frac{1}{4}\sin^2(\theta_c)m^2(t)\right)} = 2\sqrt{\frac{1}{4}m^2(t)} = m(t)$$

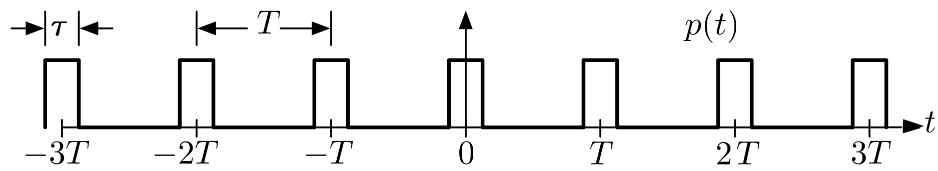
Note: We noticed that we forgot to mention in the question that $m(t) > 0$. So we assumed all of the following answers as correct: $m(t)$, $\pm m(t)$, or $|m(t)|$. We also accepted answers where the above operation was proposed to be done in the frequency domain, i.e., $\mathcal{F}^{-1}\{2\sqrt{Y(j\omega)^2 + Z(j\omega)^2}\}$. Some of you proposed the following:

$$2\left(z(t)\sin(\theta_c) + y(t)\cos(\theta_c)\right)$$

This method is mathematically valid, but it cannot be implemented to recover $m(t)$ because θ_c is unknown for us, i.e., we cannot multiply $z(t)$ or $y(t)$ by a factor that we do not know. However, we gave full credit for it.

Problem 2 (41 points)

Consider the following sequence of short rect$(\cdot)$ pulses, denoted by $p(t)$:



Each rect$(\cdot)$ pulse has width τ, and the pulses are spaced by T as diagrammed above.

(a) (14 points) Find $P(j\omega)$, the Fourier transform of $p(t)$. Express $P(j\omega)$ as a sum, and simplify where possible. *Hint: One approach is to write $p(t)$ as convolution of a rect function with an impulse train.*

Solution:

We can write $p(t)$ as follows:

$$p(t) = \text{rect}\left(\frac{t}{\tau}\right) * \delta_T(t)$$

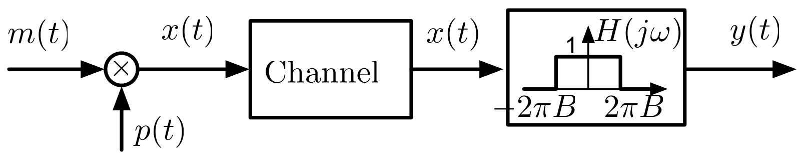
Therefore,

$$P(j\omega) = \tau\text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \cdot \omega_0\delta_{\omega_0}(\omega)$$

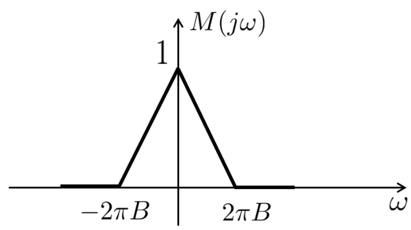
where $\omega_0 = \dfrac{2\pi}{T}$. Therefore,

$$\begin{aligned}
P(j\omega) &= \tau\text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \cdot \omega_0\delta_{\omega_0}(\omega) \\
&= \tau\omega_0\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_0) \\
&= \tau\omega_0\sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)\delta(\omega - k\omega_0) \\
&= \tau\omega_0\sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{k\omega_0\tau}{2\pi}\right)\delta(\omega - k\omega_0)
\end{aligned}$$

(b) (10 points) Consider the following system:



where the input $m(t)$ is multiplied with the rect pulse train, $p(t)$. The signal $m(t)$ is bandlimited and it has the following frequency spectrum:



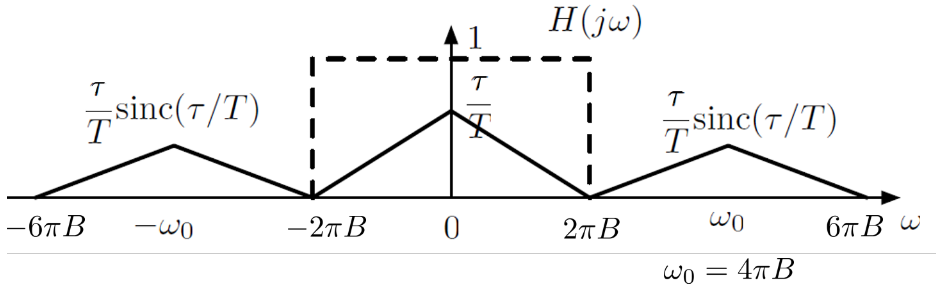
Assume that the rect$(\cdot)$ pulses are spaced by $T = \dfrac{1}{2B}$. Express the spectrum $X(j\omega)$ of $x(t)$ in terms of $M(j\omega)$.

Solution: Since $x(t) = p(t)m(t)$, we have:

$$
\begin{aligned}
X(j\omega) &= \frac{1}{2\pi} M(j\omega) * P(j\omega) \\
&= \frac{1}{2\pi} M(j\omega) * \tau\omega_0 \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k\omega_0\tau}{2\pi}\right) \delta(\omega - k\omega_0) \\
&= \frac{\tau}{T} M(j\omega) * \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k\omega_0\tau}{2\pi}\right) \delta(\omega - k\omega_0) \\
&= \frac{\tau}{T} \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k\tau}{T}\right) M(j(\omega - k\omega_0)) \\
&= \frac{\tau}{T} \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k\tau}{T}\right) M(j(\omega - k\omega_0))
\end{aligned}
$$

where $\omega_0 = \frac{2\pi}{T} = 4\pi B$ rad/s.

(c) (10 points) Sketch $X(j\omega)$ for $-6\pi B \leq \omega \leq 6\pi B$.



Note that $\text{sinc}(0) = 1$.

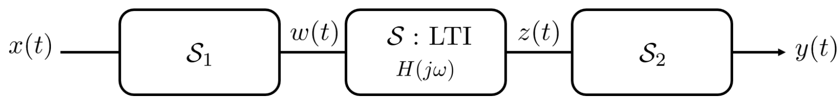
(d) (7 points) Find the spectrum of the signal at the output of the lowpass filter $Y(j\omega)$, i.e., find an expression of $Y(j\omega)$ in terms of $M(j\omega)$.

Solution: After the low pass filter, we have:

$$Y(j\omega) = \frac{\tau}{T} M(j\omega)$$

Problem 3 (30 points)

An LTI system $\mathcal{S}$ is cascaded in series with two other non-LTI systems as follows:



The system $\mathcal{S}_1$ is given by:

$$w(t) = x\left(\frac{t}{2}\right)$$

And the system $\mathcal{S}_2$ is:

$$y(t) = z(2t)$$

The system $\mathcal{S}$ has $H(j\omega)$ as its frequency response.

(This question continues on the next page.)

(a) (15 points) Find how $Y(j\omega)$ is related to $X(j\omega)$, in terms of $H(j\omega)$. Deduce the overall frequency response $H_{eq}(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$.

Solution:
We have:

$$w(t) = x\left(\frac{t}{2}\right) \rightarrow W(j\omega) = 2X(j2\omega)$$
$$z(t) = h(t) * w(t) \rightarrow Z(j\omega) = H(j\omega)W(j\omega)$$
$$y(t) = z(2t) \rightarrow Y(j\omega) = \frac{1}{2}Z\left(j\frac{\omega}{2}\right)$$

Therefore,

$$Y(j\omega) = \frac{1}{2}Z\left(j\frac{\omega}{2}\right) = \frac{1}{2}H\left(j\frac{\omega}{2}\right)W\left(j\frac{\omega}{2}\right) = H\left(j\frac{\omega}{2}\right)X\left(j\omega\right)$$

Therefore,

$$H_{eq}(j\omega) = H\left(j\frac{\omega}{2}\right)$$

(b) (15 points) If $H(j\omega)$ is given by:

$$H(j\omega) = \frac{2a - j\omega}{2a + j\omega}$$

where $a > 0$, find the impulse response $h(t)$ of the system $\mathcal{S}$. Deduce the overall impulse response $h_{eq}(t)$.

Solution: We have:

$$H(j\omega) = \frac{2a}{2a + j\omega} - \frac{j\omega}{2a + j\omega}$$

Therefore,

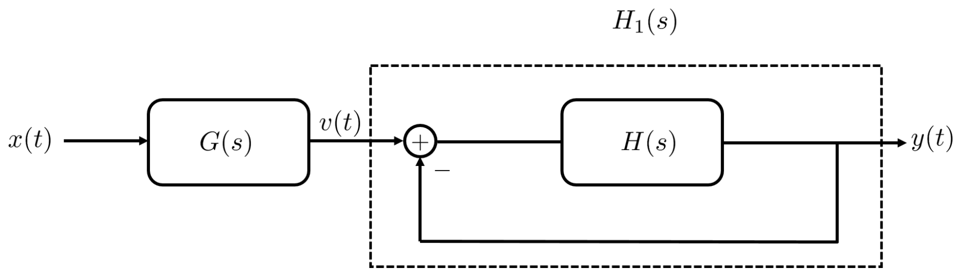
$$h(t) = 2ae^{-2at}u(t) - \frac{d}{dt}\left(e^{-2at}u(t)\right) = 2ae^{-2at}u(t) - \left(-2ae^{-2at}u(t) + \delta(t)\right) = 4ae^{-2at}u(t) - \delta(t)$$

Since $H_{eq}(j\omega) = H\left(j\frac{\omega}{2}\right)$, we have:

$$h_{eq}(t) = 2h(2t) = 8ae^{-4at}u(2t) - 2\delta(2t) = 8ae^{-4at}u(t) - \delta(t)$$

Problem 4 (40 points)

Consider the following system:



(a) (10 points) Find the transfer function $H_1(s)$ of the system that maps $v(t)$ to $y(t)$.

Solution:

$$Y(s) = H(s)(V(s) - Y(s)) \implies \frac{Y(s)}{V(s)} = \frac{H(s)}{1 + H(s)}$$

Therefore,

$$H_1(s) = \frac{H(s)}{1 + H(s)}$$

(b) (5 points) Find the overall transfer function $H_{eq}(s)$.

Solution:

$$H_{eq}(s) = G(s)H_1(s) = G(s)\frac{H(s)}{1 + H(s)}$$

(c) (10 points) How can we choose $H(s)$ in terms of $G(s)$ so that the overall system has the following impulse response $h_{eq}(t) = \delta(t)$?

Solution:

$$h_{eq}(t) = \delta(t) \to H_{eq}(s) = 1$$

Thus, we need to have:

$$G(s)\frac{H(s)}{1 + H(s)} = 1 \implies G(s)H(s) = 1 + H(s) \implies H(s) = \frac{1}{G(s) - 1}$$

(d) (15 points) Using the relation you found in part (c), find $h(t)$ if $g(t) = e^{-2t}u(t)$.

Solution:

$$G(s) = \frac{1}{s+2}$$

Therefore,

$$H(s) = \frac{1}{\frac{1}{s+2} - 1} = -\frac{s+2}{s+1} = -\left(1 + \frac{1}{s+1}\right)$$

Thus,

$$h(t) = -\delta(t) - e^{-t}u(t)$$

Problem 5 (20 points)

A system is described by the following differential equation:

$$y''(t) + 5y'(t) + 6y(t) = x'(t) + 5x(t)$$

If the input is

$$x(t) = e^{-4t}u(t-2)$$

find the output $y(t)$. Assume all initial conditions are zero.

There is additional space on the next page if needed.

Solution: Applying the Laplace trasnform to the differential equation:

$$s^2Y(s) + 5sY(s) + 6Y(s) = sX(s) + 5X(s)$$

Therefore,

$$Y(s) = \frac{s+5}{s^2+5s+6}X(s)$$

Now,

$$x(t) = e^{-8}e^{-4(t-2)}u(t-2) \implies X(s) = e^{-8}e^{-2s}\frac{1}{s+4}$$

Therefore,

$$Y(s) = e^{-8}e^{-2s}\frac{s+5}{(s+3)(s+2)(s+4)} = e^{-8}e^{-2s}\left(\frac{A}{s+3} + \frac{B}{s+2} + \frac{C}{s+4}\right)$$

where,

$$A = \left.\frac{s+5}{(s+2)(s+4)}\right|_{s=-3} = -2$$
$$B = \left.\frac{s+5}{(s+3)(s+4)}\right|_{s=-2} = 3/2$$
$$C = \left.\frac{s+5}{(s+3)(s+2)}\right|_{s=-4} = 1/2$$

Therefore,

$$y(t) = e^{-8}\left(-2e^{-3(t-2)} + \frac{3}{2}e^{-2(t-2)} + \frac{1}{2}e^{-4(t-2)}\right)u(t-2)$$

Problem 6 (44 points)

(a) (24 points) Determine if each of the following four statements is true or false. When the statement is false, a counter example is sufficient. If the statement is true, you must justify your answer to receive full credit.

i. If $x(t) * y(t) = 0$, then $x(t) = 0$ or $y(t) = 0$.

Solution:
False: We know that:

$$x(t) * y(t) \rightarrow X(j\omega)Y(j\omega)$$

Let

$$X(j\omega) = \text{rect}(\omega) \text{ and } Y(j\omega) = \text{rect}(\omega - 2)$$

We then have

$$X(j\omega)Y(j\omega) = 0$$

while $X(j\omega) \neq 0$ and $Y(j\omega) \neq 0$.

ii. If $x(t) * h(t) = x(t)$, then $h(t)$ must be an impulse, i.e., $h(t) = \delta(t)$.

Solution:
False: We have:

$$x(t) * h(t) \rightarrow X(j\omega)H(j\omega)$$

If $x(t)$ is bandlimited to $\pm\frac{1}{2}\omega_c$ and $H(j\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right)$, then

$$X(j\omega)H(j\omega) = X(j\omega)$$

However, $h(t)$ is not an impulse.

iii. A signal $x(t)$ is bandlimited where its Fourier transform $X(j\omega) = 0$ for $|\omega| > 2\pi B$ rad/s. The Nyquist rate of $\cos(4\pi Bt)x(t-2) + x(2t)$ is $6B$ Hz.

Solution:
True The fourier trasnform of the given signal:

$$\frac{1}{2}\left(e^{-j2(\omega-4\pi B)}X(j(\omega-4\pi B)) + e^{-j2(\omega+4\pi B)}X(j(\omega+4\pi B))\right) + \frac{1}{2}X(j\omega/2)$$

The highest frequency component is: $6\pi B$ rad/s or $3B$ Hz. Therefore, the Nyquist rate: $2(3B) = 6B$ Hz

iv. If $x(t) = \text{sinc}(t)$, then the energy of $x(3t+2)$ is $\frac{1}{3}$.

Solution:
True: Let $y(t) = x(3t+2)$, then the energy of $y(t)$ is given by:

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega$$

Now,

$$Y(j\omega) = \frac{1}{3} e^{j2\omega/3} X(j\omega/3) = \frac{1}{3} e^{j2\omega/3} \text{rect}(\omega/6\pi)$$

Therefore,

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega = \frac{1}{2\pi} \frac{1}{9} \int_{-3\pi}^{3\pi} 1 d\omega = \frac{6\pi}{2\pi \cdot 9} = \frac{1}{3}$$

(b) (10 points) If $y(t) = x(t) * h(t)$, then show that the following identity holds:

$$\int_{-\infty}^{\infty} y(t)dt = \left(\int_{-\infty}^{\infty} h(t)dt\right) \cdot \left(\int_{-\infty}^{\infty} x(t)dt\right)$$

Hint: One approach is to look at the integral expression for the Fourier transform when $\omega = 0$.

Solution:

We have

$$Y(j\omega) = H(j\omega)X(j\omega)$$

Therefore, if we evaluate the above equality at $\omega = 0$, we have:

$$Y(0) = H(0)X(0)$$

Now since

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt \implies Y(0) = \int_{-\infty}^{\infty} y(t)dt$$

we conclude:

$$\int_{-\infty}^{\infty} y(t)dt = \left(\int_{-\infty}^{\infty} h(t)dt\right) \cdot \left(\int_{-\infty}^{\infty} x(t)dt\right)$$

(c) (10 points) An LTI system has the following impulse response: $h(t) = e^t u(-1-t)$. Is the system stable? Is it causal?

Solution:

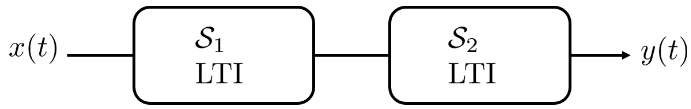
Since $u(-t-1) = 1$ for $t \leq -1$, we have $h(t) \neq 0$ for $t < 0$, therefore the system is not causal.

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{t=-\infty}^{-1} e^t dt = e^{-1} < \infty$$

The system is then stable.

BONUS (10 points)

(a) (5 points) Two LTI systems are linearly cascaded as follows:



The impulse response of the first system is $h_1(t) = e^t u(t)$ and the impulse response of the second system is $h_2(t) = e^{2t}\cos(t)$. What is the impulse response of the equivalent system $h_{eq}(t)$?

Solution:

Since the two systems are LTI, we can switch the order so that system $\mathcal{S}_2$ comes first and $\mathcal{S}_1$ is the second system. Then in this case, computing $h_{eq}(t)$ is equivalent to compute the output of system $\mathcal{S}_1$ to input $e^{2t}\cos(t)$. To compute the output we are going to use the eigenfunction property. We have the following:

$$H_1(s) = \frac{1}{s-1}, \quad \mathrm{Re}\{s\} > 1$$

and

$$e^{2t}\cos(t) = \frac{1}{2}e^{(2+j)t} + \frac{1}{2}e^{(2-j)t}$$

Therefore,

$$\begin{aligned}
h_{eq}(t) &= \frac{1}{2}H_1(2+j)e^{(2+j)t} + \frac{1}{2}H_1(2-j)e^{(2-j)t} \\
&= \frac{1}{2}\frac{1}{1+j}e^{(2+j)t} + \frac{1}{2}\frac{1}{1-j}e^{(2-j)t} \\
&= \frac{1}{4}(1-j)e^{(2+j)t} + \frac{1}{4}(1+j)e^{(2-j)t} \\
&= \frac{1}{2}e^{2t}\left(\cos(t) + \sin(t)\right)
\end{aligned}$$

(b) (5 points) If F_s is the Nyquist rate of $x(t)$, determine in terms of F_s, the Nyquist rate of $x^3(t) * x^2(t)$.

Solution:

If F_s is the Nyquist rate of $x(t)$, then the highest frequency component of $x(t)$ is: $F_s/2$ and $x(t)$ is bandlimited to $\pm Fs/2$.

Now,

$$y(t) = x^3(t) \implies Y(j\omega) = \frac{1}{2\pi}X(j\omega) * \left(\frac{1}{2\pi}X(j\omega) * X(j\omega)\right)$$

Therefore, if $x(t)$ is bandlimited to $\pm F_s/2$, $y(t)$ is then bandlimited to $\pm 3F_s/2$.

$$z(t) = x^2(t) \implies Y(j\omega) = \left(\frac{1}{2\pi}X(j\omega) * X(j\omega)\right)$$

Therefore, if $x(t)$ is bandlimited to $\pm F_s/2$, $z(t)$ is then bandlimited to $\pm 2F_s/2$ or $\pm F_s$.

Now

$$y(t) * z(t) \rightarrow Y(j\omega)Z(j\omega)$$

This means that $y(t) * z(t)$ is bandlimited to $\pm Fs$. Therefore the Nyquist rate is $2F_s$.

Fourier Transform Tables

Property	Signal	Transform
Linearity	$\alpha\, x_1(t) + \beta\, x_2(t)$	$\alpha\, X_1(j\omega) + \beta \;\; X_2(j\omega)$
Duality	$X(t)$	$2\pi\, x(-\omega)$
Conjugate symmetry	$x(t)$ real	$X^*(j\omega) = \;\; X(-j\omega)$ Magnitude: $\lvert X(-j\omega)\rvert = \lvert X(j\omega)\rvert$ Phase: $\Theta(-\omega) = \;\; -\Theta\;(\omega)$ Real part: $X_r(-j\omega) = \;\; X_r(j\omega)$ Imaginary part: $X_i(-j\omega) = \;\; -X_i(j\omega)$
Conjugate antisymmetry	$x(t)$ imaginary	$X^*(j\omega) = -X(-j\omega)$ Magnitude: $\lvert X(-j\omega)\rvert = \lvert X(j\omega)\rvert$ Phase: $\Theta(-\omega) = \;\; -\Theta\;(\omega) \mp \pi$ Real part: $X_r(-j\omega) = \;\; -X_r(j\omega)$ Imaginary part: $X_i(-j\omega) = \;\; X_i(j\omega)$
Even signal	$x(-t) = x(t)$	$X(j\omega)$: even
Odd signal	$x(-t) = -x(t)$	$X(j\omega)$: odd
Time shifting	$x(t-\tau)$	$X(j\omega)\, e^{-j\omega\tau}$
Frequency shifting	$x(t)\; e^{j\omega_0 t}$	$X(j(\omega-\omega_0))$
Modulation property	$x(t)\;\cos(\omega_0 t)$	$\frac{1}{2}\;[X(j(\omega-\omega_0)) + \; X(j(\omega+\omega_0))]$
Time and frequency scaling	$x(at)$	$\dfrac{1}{\lvert a\rvert}\, X\left(\dfrac{j\omega}{a}\right)$
Differentiation in time	$\dfrac{d^n}{dt^n}[x(t)]$	$(j\omega)^n\, X(j\omega)$
Differentiation in frequency	$(-jt)^n\; x(t)$	$\dfrac{d^n}{d\omega^n}[X(j\omega)]$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)\, X_2(j\omega)$
Multiplication	$x_1(t)\; x_2(t)$	$\dfrac{1}{2\pi}\, X_1(j\omega) * X_2(j\omega)$
Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(j\omega)}{j\omega} + \pi\, X(0)\, \delta(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \; dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\, d\omega$	

Table 4.4 – Fourier transform properties.

Additional properties:		
	x(t): even and real	$X(j\omega)$: even and real
	x(t): odd and real	$X(j\omega)$: odd and imaginary
	x(t): even and imaginary	$X(j\omega)$: even and imaginary
	x(t): odd and imaginary	$X(j\omega)$: odd and real

Name	Signal	Transform
Rectangular pulse	$x(t) = A\,rect(t/\tau)$	$X(j\omega) = A\tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
Triangular pulse	$x(t) = A\,\Lambda(t/\tau)$	$X(j\omega) = A\tau\,\text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$
Right-sided exponential	$x(t) = e^{-at}\,u(t)$	$X(j\omega) = \frac{1}{a + j\omega}$
Two-sided exponential	$x(t) = e^{-a\|t\|}$	$X(j\omega) = \frac{2a}{a^2 + \omega^2}$
Signum function	$x(t) = \text{sgn}(t)$	$X(j\omega) = \frac{2}{j\omega}$
Unit impulse	$x(t) = \delta(t)$	$X(j\omega) = 1$
Sinc function	$x(t) = \text{sinc}(t)$	$X(j\omega) = rect\left(\frac{\omega}{2\pi}\right)$
Constant-amplitude signal	$x(t) = 1,\ \text{all } t$	$X(j\omega) = 2\pi\,\delta(\omega)$
	$x(t) = \frac{1}{\pi t}$	$X(j\omega) = -j\,\text{sgn}(\omega)$
Unit-step function	$x(t) = u(t)$	$X(j\omega) = \pi\,\delta(\omega) + \frac{1}{j\omega}$
Modulated pulse	$x(t) = rect\left(\frac{t}{\tau}\right)\cos(\omega_0 t)$	$X(j\omega) = \frac{\tau}{2}\,\text{sinc}\left(\frac{(\omega-\omega_0)\,\tau}{2\pi}\right) + \frac{\tau}{2}\,\text{sinc}\left(\frac{(\omega+\omega_0)\,\tau}{2\pi}\right)$

Note:

$$\text{sinc}(\alpha) = \frac{\sin(\pi\alpha)}{\pi\alpha}$$

$$\text{rect}(t/\tau) = u(t + \tau/2) - u(t - \tau/2)$$

Table 4.5 – Some Fourier transform pairs.

LAPLACE TRANSFORM

1. Some Laplace transform pairs

Signal	Transform	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} > 0$
$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathcal{R}e\{s\} > 0$
$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathcal{R}e\{s\} > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathcal{R}e\{s\} > -a$
$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathcal{R}e\{s\} > -a$
$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathcal{R}e\{s\} > -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathcal{R}e\{s\} > -a$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\mathcal{R}e\{s\} > 0$

2. Laplace transform properties

Signal	Transform	ROC
$x(t)$	$X(s)$	R_x
$x_1(t)$	$X_1(s)$	R_1
$x_2(t)$	$X_2(s)$	R_2
$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
$x(t - t_0)$	$e^{-st_0}X(s)$	R_x
$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R_x (s is in the ROC if $s - s_0 \in R_x$)
$x(at)$, $a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$	Scaled version of R_x (s is in the ROC if $s/a \in R_x$)
$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
$\int_0^t x(\tau)d\tau$	$\frac{X(s)}{s}$	At least $R_x \cap \{\mathcal{R}e\{s\} > 0\}$
$\frac{d}{dt}x(t)$	$sX(s) - x(0)$	At least R_x
$\frac{d^2}{dt^2}x(t)$	$s^2X(s) - sx(0) - x'(0)$	At least R_x
$tx(t)$	$-\frac{d}{ds}X(s)$	R_x