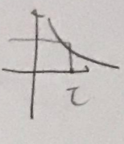


Problem 1 (7 points)

I. (2 *points*) Evaluate the following convolutions:

2

- $u(t) \star e^{-t}u(t)$, $\int_{-\infty}^{+\infty} e^{-t}u(t) \cdot u(\tau - t)\, d\tau$
- $u(t) \star (u(t-1) - u(t-2))$.

II. (5 *points*) Consider the following three LTI systems:

2

- $\mathcal{S}_1$: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ $\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$
- $\mathcal{S}_2$: LTI system with impulse response $h_2(t) = e^{-t}u(t)$.
- $\mathcal{S}_3$: LTI system with impulse response $h_3(t) = u(-2t+2) - u(-2t+4)$.

The three systems are now interconnected as shown in Figure 1. What is the impulse response $h(t)$ of the overall LTI system (i.e. from $x(t)$ to $z(t)$)? Is the overall system stable? Causal?

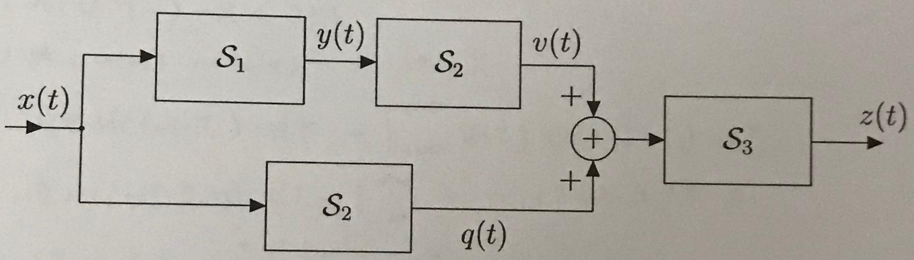


Figure 1: The LTI system in Problem 1

$h(t) = (S_1 \star S_2 + S_2) \star S_3$

$\int_{-\infty}^{+\infty} \int_{-\infty}^{t} x(t)$

$\left[u(t) \star e^{-t}u(t) + e^{-t}u(t)\right] \star \left[u(-2t+2) - u(-2t+4)\right.$

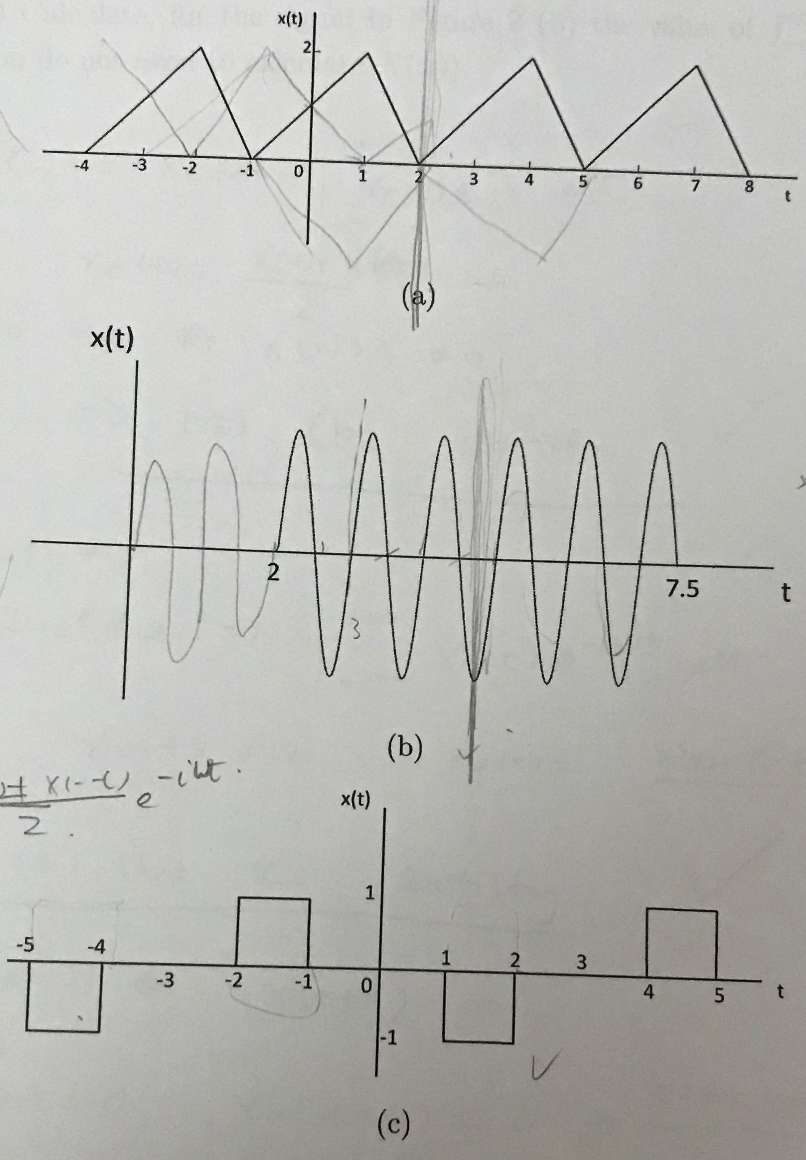
$\star\, u(-2(t-1)) - u(-2($

$\left[\frac{1}{j\omega} + \pi\delta(\omega)\right] \cdot \frac{1}{j\omega + 1}$

$= \frac{1}{-\omega^2 + j\omega}$

Problem 2 (8 points)

Figure 2 shows three real signals. All the signals have finite time support, i.e. the signals are zero at any time not shown in the figure.



$Re = X(\omega) = \int_t \frac{x(t) + x(-t)}{2} e^{-i\omega t}$

$T = 1$

real → real

Ima

Figure 2: Problem 2

(a) Determine which, if any, of the real signals depicted in Figure 2 have Fourier transforms that satisfy each of the following conditions (and very briefly say why):

1. (1.5 *points*) $\mathcal{R}e\{X(\omega)\} \neq 0$.
2. (1.5 *points*) $\mathcal{I}m\{X(\omega)\} \neq 0$.

3. (1 *point*) There exists a real number such that $e^{ja\omega}X(\omega)$ is real.
4. (1.5 *points*) $\int_{-\infty}^{\infty} X(\omega)d\omega = 0$.
5. (1.5 *points*) $\int_{-\infty}^{\infty} \omega X(\omega)d\omega = 0$.

(b) (1 *point*) Calculate, for the signal in Figure 2 (b) the value of $\int_{-\infty}^{\infty} \cos(2\omega)X(\omega)d\omega$. (Hint: you do not need to calculate $X(\omega)$).

)

1. $Re\{X(\omega)\} = Xe(\omega) = \int_{-\infty}^{+\infty} Xe(t)e^{-i\omega t} \cdot dt$.

$Xe(t) = \frac{X(t) + X(-t)}{2}$

if $Xe(t) \neq 0$. $Re\{X(\omega)\} \neq 0$.

$\Rightarrow$ (a). (b) satisfy ✓

2. $Im\{X(\omega)\} \neq 0$

$\Rightarrow Xo(\omega) \neq 0 \Rightarrow \int_{-\infty}^{+\infty} Xo(t) \cdot e^{-i\omega t} \cdot dt$

$Xo(t) \neq 0$. $Xo(t) = \frac{X(t) - X(-t)}{2} \neq 0$.

$\Rightarrow$ (a). (b). (c) satisfy ✓

3. $e^{ja\omega}(X(\omega)) \Leftrightarrow x(t+\alpha)$

$\downarrow$ real.

$Im\{X(\omega)\} = 0$. $Xo(t+\alpha) = 0 \Rightarrow \frac{X(t+\alpha) - X(-(t+\alpha))}{2} = 0$

~~(a)~~. (b) ✓ satisfy

4. $\int_{-\infty}^{\infty} X(\omega) \cdot d\omega = 0 \Rightarrow X(\omega)$ is odd. $\Rightarrow \overset{\text{real}}{\overset{\uparrow}{X(t)}} \Rightarrow$ odd.

$X(\omega) = Re(Xe(\omega)) + Img\{Xo(\omega)\}$ $\quad Xe(t) = 0 = \frac{X(t) + X(-t)}{2}$

Problem 3 (8 points)

Consider a system where the output $y(t)$ is related to the input $x(t)$ by the following input-output equations:

$$y(t) = x(t)g(t) + x(t-2)g(t+3).$$

2 1. (3 *points*) If $g(t) = \delta(t)$, is this system linear? is it time invariant? is it causal? (Explain why).

0.5 2. (1 *point*) If $g(t) = t$ is this system stable? (Explain why).

1.5 3. (2 *points*) If $g(t) = 1$ and $x(t)$ is periodic with period $T_1 = 4$, show that $y(t)$ can be periodic with period $T_2 = 2$. Under what conditions, $y(t)$ has period $T_2 = 2$?

1.5 4. (2 *points*) Under the same assumptions of the previous part ($g(t) = 1$ and the conditions found for $y(t)$ to be periodic of period 2), what are the Fourier Series coefficients for $y(t)$, if we are given the Fourier series coefficients for $x(t)$?

$y(t) = x(t)\cdot\delta(t) + x(t-2)\delta(t+3)$

$\Rightarrow$ Not causal ; because it depends on future value X

$y(t+3) = x(t+3)\delta(t+3) + x(t+1)\delta(t+6)$

$x_1(t) = x(t+3)$: $y_1(t) = x(t+3)\delta(t) + x(t+1)g(t+3) \neq y(t+3)$

$\Rightarrow$ Not time invariant.

$x_3(t) = a x_1(t) + b x_2(t)$

$y_3(t) = (a x_1(t) + b x_2(t))\delta(t) + (a x_1(t-2) + b x_2(t-2))\delta(t+3)$

$= a[x_1(t)\delta(t) + x_1(t-2)\delta(t+3)] + b[x_2(t)\cdot\delta(t) + x_2(t-2)]\delta($

$= a\, y_1(t) + b\, y_2(t)$

$\Rightarrow$ Linear

Thus the system is linear, but not time invariant, not cau

$y(t) = x(t)\cdot t + x(t-2)(t+3)$

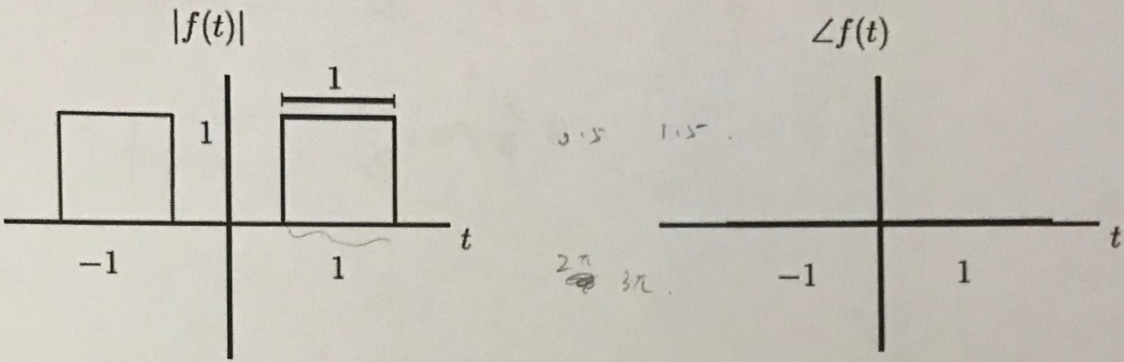
when $x(t)$ is bounded 0.5

$y(t)$ will also be bounded

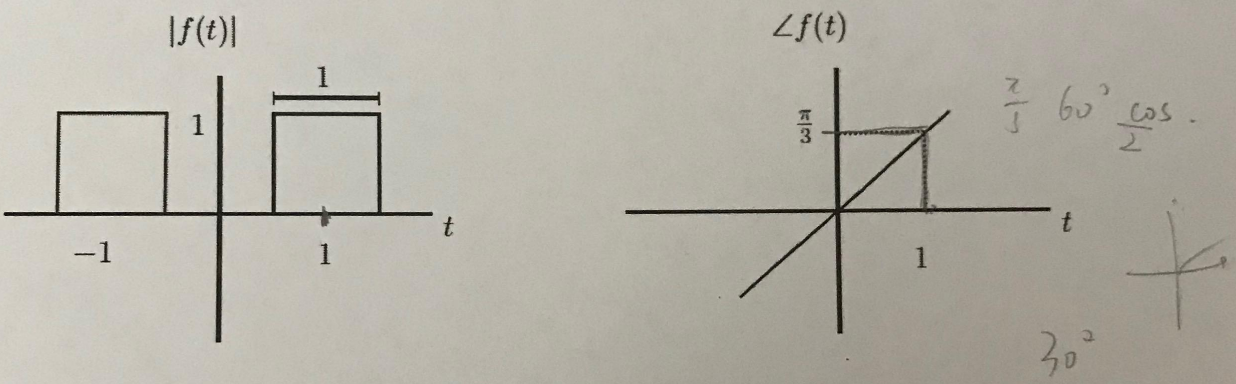
0.5 cos πt.

Problem 4 (5 points)

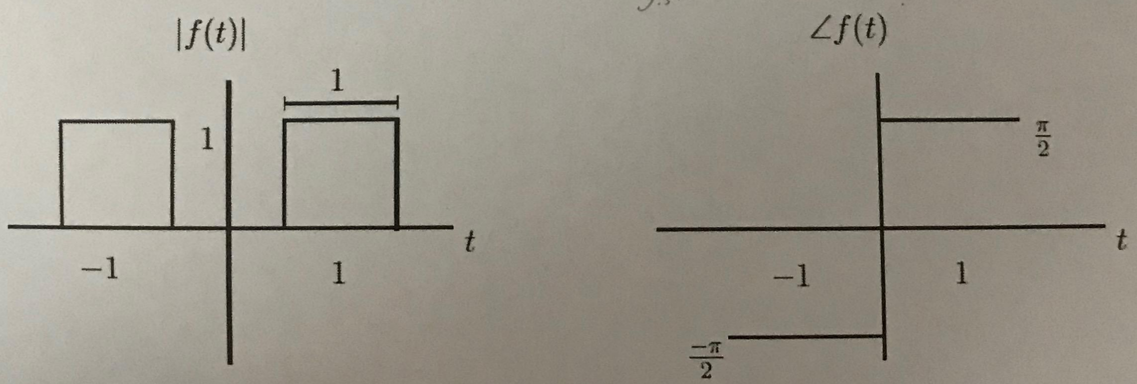
Write an expression for the following signals and their Fourier transforms. Recall that $f(t) = |f(t)|e^{i\angle f(t)}$, where $\angle f(t)$ is the phase of $f(t)$.



(a)



(b)



(c)

Figure 3: Problem 4