Problem 1(5 points) The following questions are not related to each other.

1. In the following expressions, $*$ stands for the convolution operation, $u(t)$ is the unit step function and $\delta(t)$ is the delta function.

   (a) Simplify $\frac{du(t)}{dt} * (te^{-6|t+1|}\delta(t-3))$.

   (b) Calculate $\int_{-\infty}^{\infty} f(t)g(t)\,dt$ with $f(t) = u(t+3)$ and $g(t) = u(-t+5)$.

   (c) Calculate $y(0)$, where $y(t) = x_1(t) * x_2(t)$, $x_1(t) = u(t-3) - u(t-1)$ and $x_2(t) = \delta(-2-t)$.

2. Consider the signal

$$x(t) = u(t-3)$$

and the signal

$$y(t) = \begin{cases} x(t) & \text{if} \quad t \geq 0.5 \\ -x(-t) & \text{otherwise} \end{cases}.$$

Is the signal $y(t)$ even or odd or neither? Justify your answer.

1. a) $\frac{du(t)}{dt} * (te^{-6|t+1|}\delta(t-3))$

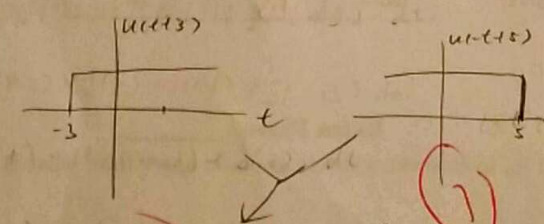
$= \delta(t) * ((3)e^{-6|3+1|}\delta(t-3))$

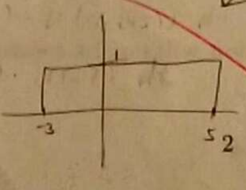
$= 3e^{-6\cdot4}\,\delta(t) * \delta(t-3)$

$= 3e^{-24}\cdot\delta(t-3)$

1

b) $\int_{-\infty}^{\infty} u(t+3)\,u(-t+5)\,dt$





1

$\int_{-\infty}^{\infty} u(t+3)\,u(-t+5)\,dt$

$= \int_{-3}^{5} 1\,dt$

$= 8$

1. $Y(t) = x_1(t) * x_2(t)$

$= [u(t-3) - u(t-1)] * \delta(-2-t)$ $\quad k = -2-t, \; t = -2-k$

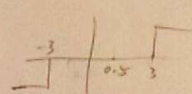
cannot do this change of variables X

$= [u(-2-k-3) - u(-2-k-1)] * \delta(k)$

$= [u(-5-k) - u(-3-k)]$ $\quad$ 0

$= u(-5-(-2-t)) - u(-3-(-2-t))$

$= u(t-3) - u(t-1)$

2. $Y(t) = \begin{cases} u(t-3) & t \geq 0.5 \\ -u(-(t+3)) & \text{other} \end{cases}$ $\rightarrow$ 

The signal is odd because it is symmetric across the origin

2

Problem 2 (5 points) The following questions are not related.

1. A LTI system has impulse response $h(t) = 3\frac{d}{dt}\delta(t) + 3\delta(t+1)$, where $\delta(t)$ is the delta function. Can you write input-output equations that describe this system?

2. A LTI system, when the input is an unknown $x(t)$, outputs the y(t) that is depicted in Fig. 1. Assume now that the input is $x_1(t)$ with corresponding output $y_1(t)$, and we know that $x_1(t) = 2x(t-1) + x(t+3)$. Calculate what is $y_1(5)$.

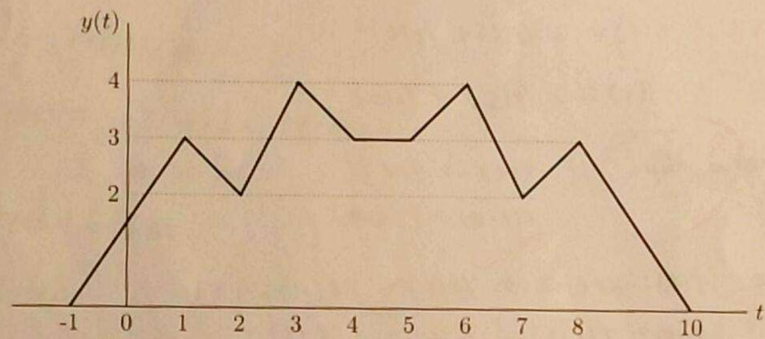


Figure 1: Response $y(t)$ for input $x(t)$

3. A system is described as depicted in Fig. 2, where is the Delay operator time shifts the input signal with the specified amount, and the Flip operator does the time reversal across Y-axis. Is this system causal? What about BIBO stability?

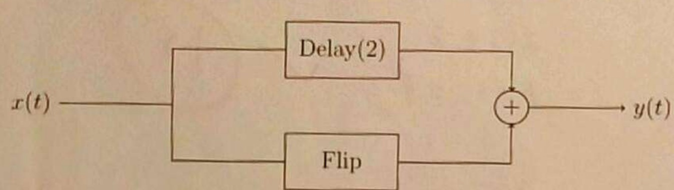


Figure 2: System for Problem 2(3).

1) $h(t) = 3\frac{d}{dt}\delta(t) + 3\delta(t+1)$

$Y(t) = 3\frac{d}{dt}x(t) + 3X(t+1)$ ✓ 1.5

2) Let $f(t) = X(t-1) \to Y_f(t)$

$g(t) = X(t+3) \to Y_g(t)$

then $X_1(t) = 2f(x) + g(x)$

Because LTI, $Y_1(t) = 2Y_f(t) + Y_g(t)$

$Y_1(5) = 2Y_f(5) + Y_g(5)$

We also know that if $X(t) \to Y(t)$,

$X(t-a) \to Y(t-a)$

therefore: $f(t) = X(t-1) \to Y_f(t) = Y(t-1)$

$g(t) = X(t+3) \to Y_g(t) = Y(t+3)$

$Y_1(5) = 2Y_f(5) + Y_g(5)$

$= 2Y(5-1) + Y(5+3)$

$= 2Y(4) + Y(8)$

$= 2 \cdot 3 + 3$

$= 6 + 3 = 9$ ✓ 1.5

3) $y(t) = x(-t) + x(t-2)$

$y(-1) = x(1) + x(-1-2)$

The system at $t=-1$ depends on information at $t=1$. System is not causal

Let $|x(t)| \le M$

$|y(t)| = |x(-t)| + |x(t-2)|$

$\le M + M$

$|y(t)| \le 2M$

System is BIBO stable.

2

Problem 3 (5 points) Assume that $x_1(t)$ and $x_2(t)$ are two periodic signals, and both of them have period T. The convolution of $x_1(t)$ and $x_2(t)$ is not well defined, because we could be collecting infinite area with the integration. Instead, for periodic signals with period T, we use what is called the "periodic convolution", that is defined as follows:

$$y(t) = (x_1 \star x_2)(t) = \int_0^T x_1(\tau)x_2(t-\tau)d\tau$$

and where $\star$ stands for periodic convolution.

1. Assume both $x_1(t)$ and $x_2(t)$ are odd signals. Is $y(t) = (x_1 \star x_2)(t)$ even, odd, or neither? Justify your answer.

2. Let $x_2(t)$ be a periodic signal with period $T_1 = 5$, and $x_1(t)$ be the periodic sampling signal

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t-4-5k),$$

where $\delta(t)$ denotes the delta function. Calculate the periodic convolution between $x_1(t)$ and $x_2(t)$ that is $(x_1 \star x_2)(t)$ as a function of $x_2(t)$.

1. $Y(-t) = (x_1 \star x_2)(-t)$

$= \int_0^T x_1(\tau)\, x_2(-t-\tau)\, d\tau$

$x_2(t-\tau) \neq x_2(-t-\tau)$, $-x_2(t-\tau) \neq x_2(-t-\tau)$

$Y(-t) \neq Y(t) \neq -Y(t)$ ++

$Y(t)$ is neither odd nor even

2) $\int_0^5 \sum_{k=-\infty}^{\infty} \delta(\tau-4-5k)\, x_2(t-\tau)\, d\tau$

$= \int_0^5 \sum_{k=-\infty}^{\infty} x_2(t-(4+5k))\,\delta(\tau-4-5k)\, d\tau$

$= \int_0^5 \sum_{k=-\infty}^{\infty} x_2(t+4-5k)\,\delta(\tau-4-5k)\, d\tau$

$= x_2(t-4-5k) \int_0^5 \sum_{k=-\infty}^{\infty} \delta(\tau-4-5k)\, d\tau$

$= x_2(t-4-5k) \cdot \sum_{k=-\infty}^{\infty} \begin{cases} 1 & \text{for } 0 \le 4+5k \le 5 \\ 0 & \text{otherwise} \end{cases} \rightarrow 1 \text{ if } k=0$

$= x_2(t-4-5k) \cdot \begin{cases} 1 & \text{if } k=0 \\ 0 & \text{other} \end{cases}$

$= x_2(t+4-5\cdot 0) + 0 \cdots$

$= x_2(t-4)$ ✓