Discussion: 1C

UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination I
February 6, 2018
Duration: 1 hr 50 mins.

INSTRUCTIONS:

- The exam has 6 problems and 14 pages.
- The exam is closed-book.
- One cheat sheet of A4 size is allowed.
- Calculator is NOT allowed.
- Write your discussion session in the top-right corner. ↗↗

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Table 1: Score Table

Problem	a	b	c	d	Score	
1	4	4	4		12	12
2	2	2	8		12	12
3	6	2	4	6	18	18
4	8	8			16	9
5	10	6			16	15
6	5	5	6		16	9
Total					90	75

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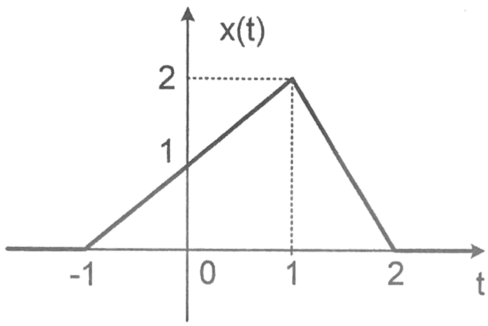
Problem 1 (12 pts) Consider the following signal $x(t)$ for (a), (b)



(a) (4 pts) Sketch even and odd decompositions $x_e(t)$ and $x_o(t)$.

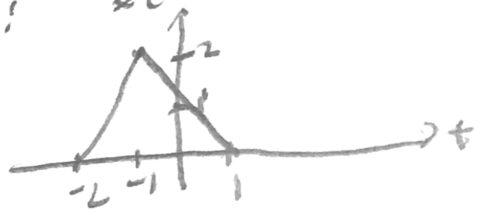
(b) (4 pts) Sketch $x(-2t + 3)$.

(c) (4 pts) Sketch $x(t/3 + 2)$.

a. $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$x(-t)$:



$x_e(t)$

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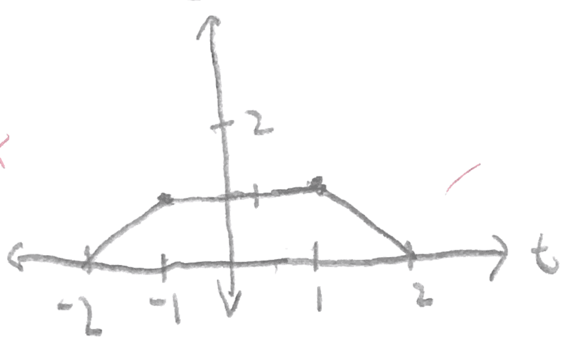
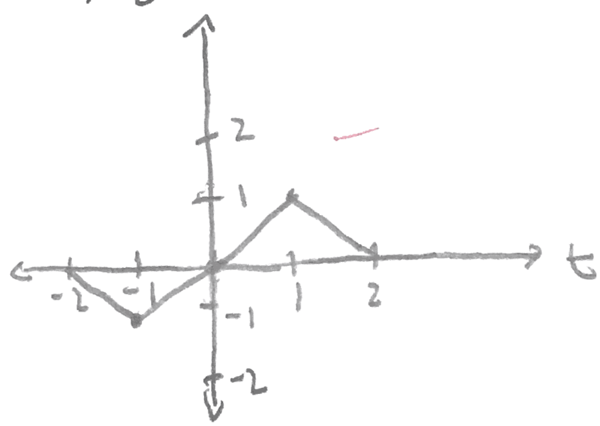
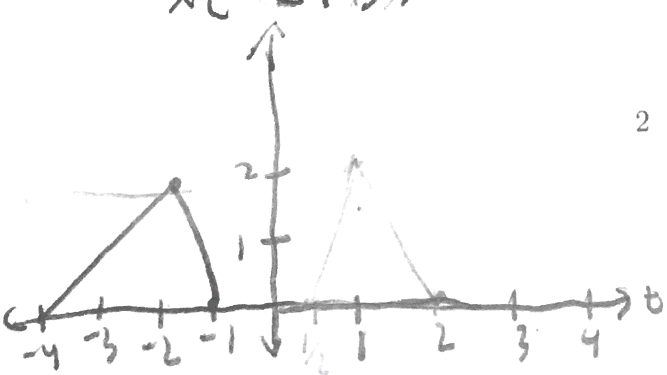


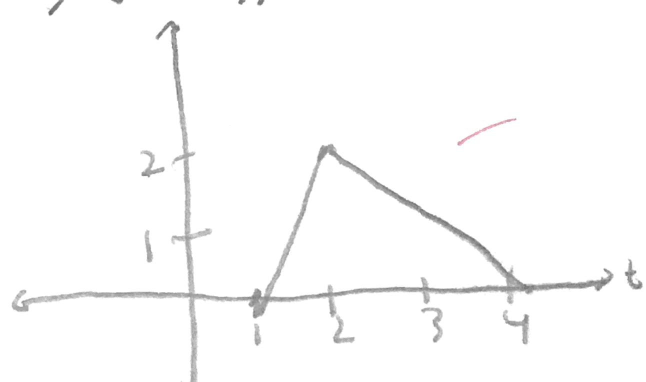
$x_o(t)$



b. $x(t+3)$

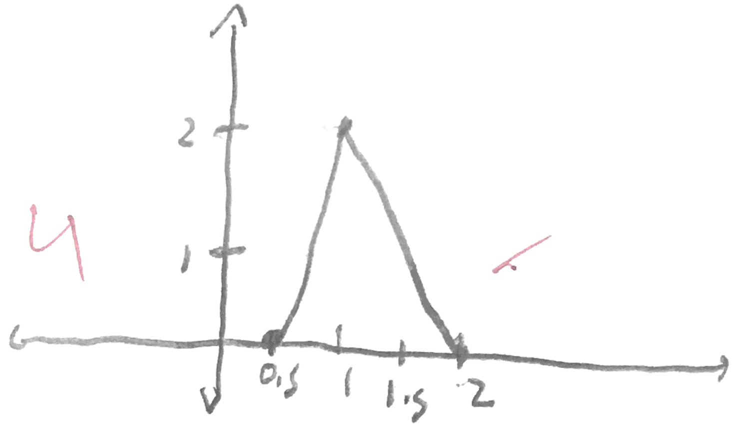


$x(-t+3)$



b.
cont.

x(-2t+3)



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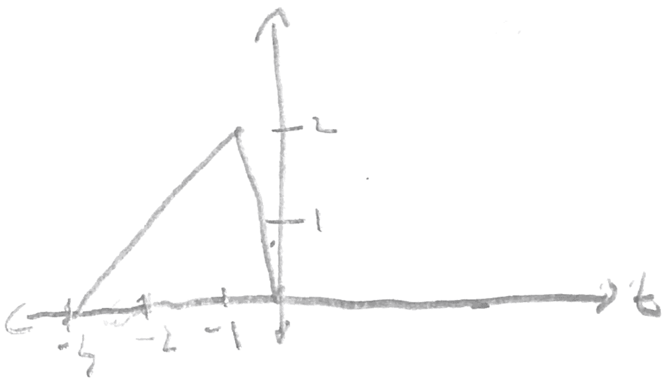
c. x(t+2)



$x(\frac{1}{3}t+2)$

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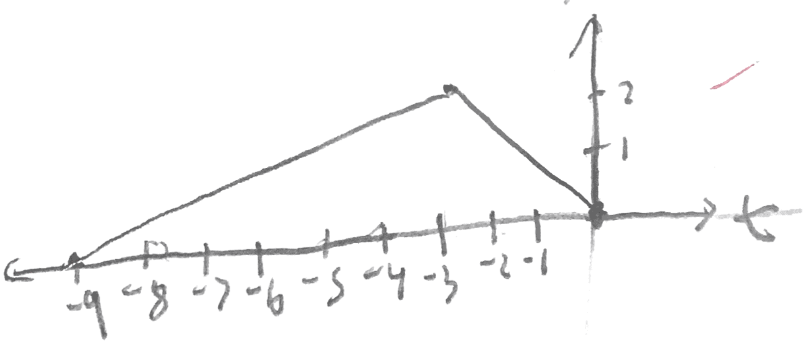

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Problem 2 (12 pts) In this problem, we identify system properties from the impulse response function:

$$h(t,\tau) = e^{-(t-\tau)}u(t-\tau)u(t) \tag{1}$$

(a) (2 pts) Is the system TV or TI? Explain.

(b) (2 pts) Is it C or NC? Explain.

(c) (8 pts) Find the output $y(t)$ if the input is $x(t) = (t-2)u(t-2)$.

a. $h(t,\tau)$ cannot be expressed as a function of $t-\tau$. Therefore, the system is $\boxed{\text{TV.}}$

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b. $h(t,\tau)u(t-\tau) = e^{-(t-\tau)}u(t-\tau)u(t)u(t-\tau)$

$= e^{-(t-\tau)}u(t-\tau)u(t)$ since $u(t-\tau)u(t-\tau) = u(t-\tau)$

$= h(t,\tau)$.

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This means that $h(t,\tau) \equiv 0$ for $t<\tau$, which means the $\boxed{\text{system is causal (C)}}$

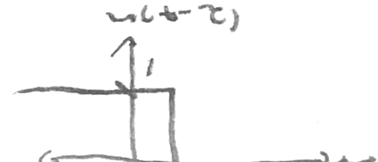
c. $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t,\tau)\,d\tau = \int_{-\infty}^{\infty} (\tau-2)u(\tau-2)e^{-(t-\tau)}u(t-\tau)u(t)\,d\tau$

$u(\tau-2)$: 0 if $\tau<2$, 1 o.w.

$= \int_2^{\infty} (\tau-2)\, e^{-(t-\tau)}u(t-\tau)u(t)\,d\tau$

if $t<2$:

$\int_2^{\infty} (\tau-2)e^{-(t-\tau)}u(t-\tau)u(t) = 0$



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Problem 3 (18 pts)
Consider IPOP relation for an LTI system S:

$$y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \left[\cos(t)\cos(\tau) + \sin(t)\sin(\tau)\right] x(\tau) d\tau$$

where $x(t)$ and $y(t)$ are input and output of the system, respectively.

(a) (6 pts) Find the impulse response function $h(t)$.

(b) (2 pts) Is the system C or NC? Provide justification.

(c) (4 pts) Is this system BIBO stable? Provide justification.

(d) (6 pts) Find Laplace transform $H(s)$ and ROC.

(Hint: Use the identity $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$)

a. $x(t) = \delta(t)$

$$h(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} [\cos(t)\cos(\tau) + \sin(t)\sin(\tau)] \delta(\tau) d\tau$$

$$= e^{-t} \int_{-\infty}^{t} e^{\tau} [\cos(t-\tau)] \delta(\tau) d\tau$$

$$= e^{-t} \int_{-\infty}^{t} e^{0} \cos(t-0) \delta(\tau) d\tau$$

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$$= e^{-t} \underbrace{\cancel{e^{0}}}_{1} \cos(t) \int_{-\infty}^{t} \delta(\tau) d\tau \Rightarrow \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$$= e^{-t} \cos(t) u(t) \qquad \boxed{h(t) = e^{-t} \cos(t) u(t)}$$

b. $\boxed{\text{The system is C (causal)}}$ This is because

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$h(t) \cdot u(t) = e^{-t} \cos(t) u(t) u(t)$
$= e^{-t} \cos(t) u(t) = h(t)$ since $u(t) \cdot u(t) = 0$

In other words, when $t < 0$, $h(t) = 0$ this means the system is causal.

c. $\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} e^{-t}\cos(t)u(t)\,dt$

$= \int_{0}^{\infty} e^{-t}\cos(t)\,dt = e^{-t}\sin(t)\Big|_0^{\infty}$

4

$= e^{-\infty}\sin(\infty) - e^{-0}\sin(0)$

$= 0 - 0 = 0 < \infty$

Since $\int_{-\infty}^{\infty} h(t)\,dt < \infty$, the $\boxed{\text{system is BIBO stable}}$

d. $h(t) = e^{-t}\cos(t)u(t)$

Note: $L_s\{\cos(t)u(t)\} = \frac{s}{s^2+1}$

from the frequency shifting property:

if $f(t) \xrightarrow{L_s} F(s)$

then $e^{-at}f(t) \xrightarrow{L_s} F(s+a)$

therefore

$H(s) = L_s\{e^{-t}\cos(t)u(t)\} = \frac{s+1}{(s+1)^2+1} = \frac{s+1}{s^2+2s+2}$

$\boxed{H(s) = \frac{s+1}{s^2+2s+2}}$

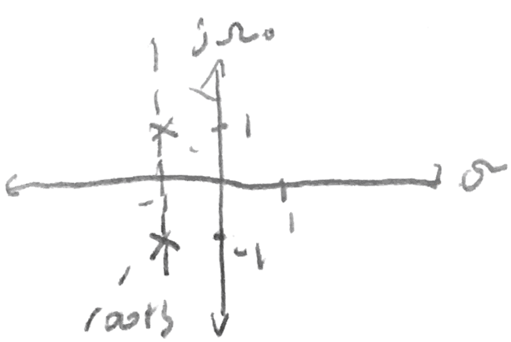
consider denominator polynomial:

s^2+2s+2 : find roots: $s = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$

6

$= \frac{-2 \pm 2j}{2} = -1 \pm j$

$s = \sigma + j\Omega_0$



rightmost root is at

$\sigma = -1$

therefore the ROC is:

$\boxed{\text{ROC: } Re\{s\} > -1}$

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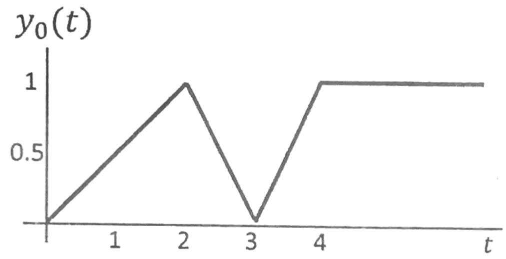
Problem 4 (16 pts) Consider an LTI system S_0 with input $x_0(t)$ and the impulse response function $h_0(t)$. The corresponding output $y_0(t)$ is shown below:



8 (a) (8 pts) Consider an LTI system S_1 with input $x_1(t) = x_0(t+2)$ and IRF $h_1(t) = h_0(t-1)$. Express the output $y_1(t)$ as a function of $y_0(t)$ and then plot it.

1 (b) (8 pts) Consider an LTI system S_2 with input $x_2(t) = x_0(-t)$ and IRF $h_2(t) = h_0(-t)$. Express the output $y_2(t)$ as a function of $y_0(t)$ and then plot it.

a. $Y_1(t) = \int_{-\infty}^{\infty} x_1(\tau) h_1(t-\tau)\, d\tau$

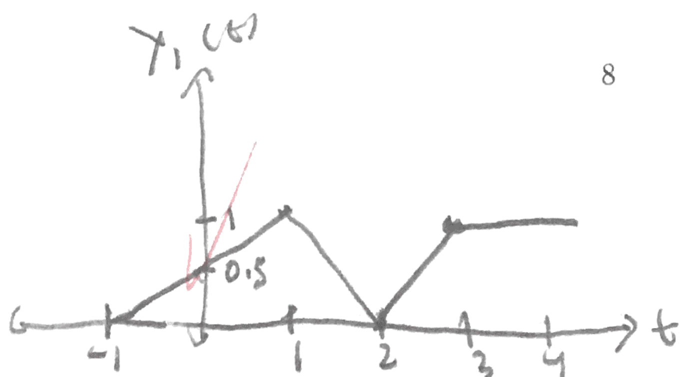
$= \int_{-\infty}^{\infty} x_0(\tau+2)\, h_0((t-1)-\tau)\, d\tau$ set $T = t-1$

$= \int_{-\infty}^{\infty} x_0(\tau+2)\, h_0(T-\tau)\, d\tau$

$= y_0(T+2)$ since system is LTI

$= y_0((t-1)+2) = y_0(t+1)$

$\boxed{Y_1(t) = y_0(t+1)}$



b. $y_2(t) = \int_{-\infty}^{\infty} x_2(\tau) h_2(t-\tau) d\tau$

$$= \int_{-\infty}^{\infty} x_0(-\tau) h_0(-(t-\tau)) d\tau$$

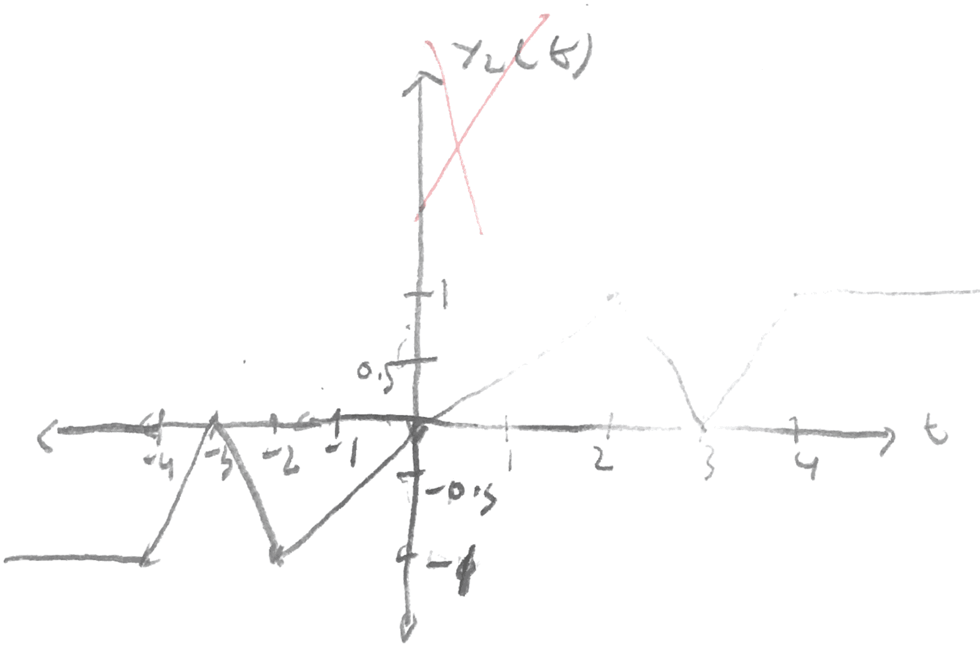
1

$$= \int_{-\infty}^{\infty} x_0(-\tau) h_0(-t+\tau) d\tau$$

let $T = -t$
if $p = -\tau$
$dp = -d\tau$

$$= -\int_{+\infty}^{-\infty} x_0(p) h_0(T-p) dp = -y_0(T) = -y_0(-t)$$

$$\boxed{y_2(t) = -y_0(-t)}$$



15 **Problem 5** (16 pts) Consider a cascade combination of two systems S_1 and S_2: $x(t)$ is input to S_1 and $y(t)$ is the output, while the output of S_2 is $z(t)$.

$$x(t) \to [S_1] \to y(t) \to [S_1] \to z(t)$$

The IPOP relation for S_1 and S_2 is:

$$S_1 : y(t) = e^{-t}x(t)u(t),$$

$$S_2 : z(t) = \int_0^t e^{-(t-\sigma)}y(\sigma)u(\sigma)d\sigma.$$

10 (a) (10 pts) Compute impulse response function $h_{12}(t,\tau)$ of the cascaded system S_1S_2.

5 (b) (6 pts) Compute the output $z(t)$ if the input is $x(t) = e^{-3t}[u(t) - u(t-3)]$.

S_1: $x(t) = \delta(t-\tau)$

a. $h_1(t,\tau) = e^{-t}\delta(t-\tau)u(t)$ signal only matters for $t \geq 0$, 0 o.w. and $\tau \geq 0$.

$h_{12}(t,\tau) = S_2[h_1(t,\tau)]$

$$h_{12}(t,\tau) = \int_0^t e^{-(t-\sigma)} h_1(\sigma,\tau)u(\sigma)\,d\sigma$$

$$= \int_0^t e^{-(t-\sigma)} e^{-\sigma}\delta(\sigma-\tau)\,u(\sigma)\,\cancel{u(\sigma)}\,d\sigma$$

$$= \int_0^t e^{-(t-\tau)} e^{-\tau}\delta(\sigma-\tau)u(\tau)\,d\sigma$$

- consider $t \geq 0$ and $\tau \geq 0$.

$$= e^{-t}u(\tau)\int_0^t \delta(\sigma-\tau)\,d\sigma = e^{-t}u(\tau)u(t-\tau)$$

$\Downarrow$ 0 if $t < \tau$, 1 if $t > \tau$

$$\boxed{h_{12}(t,\tau) = e^{-t}u(\tau)\,u(t-\tau)}$$ for $t \geq 0$, $\tau \geq 0$

b. $z(t) = \int_{-\infty}^{\infty} x(\tau)\, h_1(t,\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} e^{-3\tau} \underbrace{[u(\tau) - u(\tau-3)]}_{\substack{1 \text{ if} \\ 0 \le \tau \le 3}} e^{-t} u(\tau) u(t-\tau)\, d\tau$$

$$= \int_{0}^{3} e^{-3\tau} e^{-t} u(t-\tau)\, d\tau = INT$$

if $t < 0$

then $INT = 0$

if $0 < t < 3$

then $INT = \int_0^t e^{-3\tau} e^{-t}\, d\tau = e^{-t}\left[-\frac{e^{-3\tau}}{3} \Big|_0^t \right]$

$$= e^{-t}\left[\frac{1}{3} - \frac{e^{-3t}}{3} \right]$$

if $t > 3$

then $INT = \int_0^3 e^{-3\tau} e^{-t}\, d\tau = e^{-t}\left[-\frac{e^{-3\tau}}{3} \Big|_0^3 \right]$

$$= e^{-t}\left[\frac{1}{3} - \frac{e^{-9}}{3} \right]$$

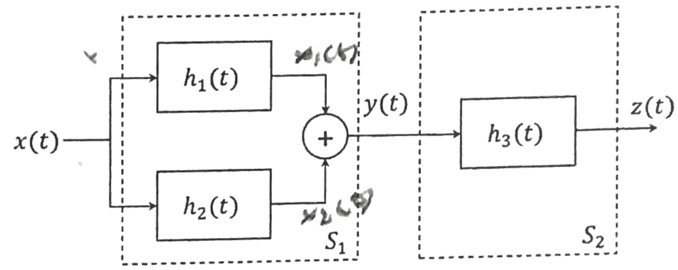
$$\cancel{z(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 - \frac{e^{-3t}}{3} & \text{if } 0 \le t \le 3 \\ 1 - \frac{e^{-9}}{3} & \text{if } t > 3 \end{cases}}$$

$$\boxed{z(t) = \left[1 - \frac{e^{-3t}}{3}\right][u(t) - u(t-3)] + \left[1 - \frac{e^{-9}}{3}\right] u(t-3)}$$

9 **Problem 6** (16 pts)
Consider a cascaded LTI system S_1S_2 as follows

$$x(t) \to [S_1] \to y(t) \to [S_2] \to z(t)$$

The cascade system is shown below.



where $h_1(t) = \delta(t-1)$, $h_2(t) = \delta(t-2)$, and $h_3(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$.
Let $x(t) = 2(u(t) - u(t-2))$, then

5 (a) (5 pts) Find the IPOP between $x(t)$ and $y(t)$. Plot $y(t)$.

3 (b) (5 pts) Write the impulse response of the cascade system S_1S_2.

1 (c) (6 pts) Compute and plot $z(t)$ for the specified input.

a. $x_1(t) = \int_{-\infty}^{\infty} x(\tau)\, h_1(t,\tau)\, d\tau$

$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t-1-\tau)\, d\tau$ if $p = t-1$

$= \int_{-\infty}^{\infty} x(\tau)\, \delta(p-\tau)\, d\tau = x(p)$
$= x(t-1)$

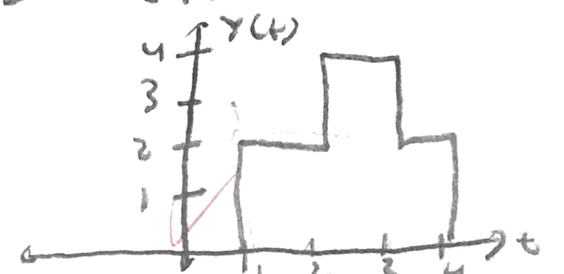
$x_2(t) = \int_{-\infty}^{\infty} x(\tau)\, h_2(t,\tau)\, d\tau$ if $p = t-2$

$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t-2-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} x(\tau)\, \delta(p-\tau)\, d\tau = x(p) = x(t-2)$

$$\boxed{y(t) = x(t-1) + x(t-2)}$$

since $x(t) = 2[u(t) - u(t-2)]$



b. $h_{1s}(t) = h_1(t) + h_2(t)$

$= \delta(t-1) + \delta(t-2)$

$h_{2s}(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

$$h_{12}(t) = \int_{-\infty}^{\infty} h_{1s}(\tau)\, h_{2s}(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} [\delta(\tau-1) + \delta(\tau-2)]\,[\delta(t-\tau-1) + \delta(t-\tau-2) + \delta(t-\tau-3)]\, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(\tau-1)\delta(t-2) + \delta(\tau-1)\delta(t-3) + \delta(\tau-1)\delta(t-4) + \delta(\tau-2)\delta(t-3) + \delta(\tau-2)\delta(t-4) + \delta(\tau-2)\delta(t-5)\, d\tau$$

$$= \delta(t-2) + 2\delta(t-3) + 2\delta(t-4) + \delta(t-5)$$

$$\boxed{h_{12}(t) = \delta(t-2) + 2\delta(t-3) + 2\delta(t-4) + \delta(t-5)}$$

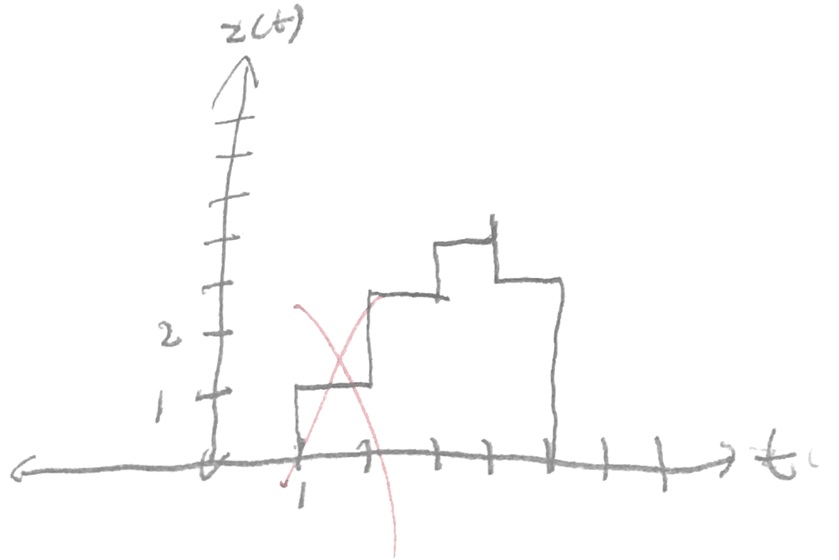
$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, h_{12}(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} 2[u(\tau) - u(\tau-2)]\, h_{12}(t-\tau)\, d\tau = 2\int_0^2 h_{12}(t-\tau)\, d\tau$$

$$= 2\int_0^2 \delta(t-\tau-2) + 2\delta(t-\tau-3) + 2\delta(t-\tau-4) + \delta(t-\tau-5)\, d\tau$$

$0 \le t-2 \le 2$

$$= \boxed{2\left[u(t-2) - u(t-4) + 2(u(t-3) - u(t-5)) + 2(u(t-4) - u(t-6)) + u(t-5) - u(t-7)\right]}$$



z(t)
2
1
1
t