UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination
February 8, 2011

Put First Letter * of LAST Name in the corner→→↗↗
(* Otherwise Your Midterm will be LOST)

Your name:————————————————————

Instructions: Closed Book, Calculators are NOT Allowed

Good Luck!

PART 1

t-Domain Analysis

(Do NOT use Laplace Transforms)!

(Question 1) (15pts)

(i) (5pts) Plot the following function:

$$x(t) = e^{-t}U(t-1) + \delta(t+1).$$

(ii) (5pts) This function is taken to be the input to a system with IPOP relation

$$S: \quad y(t) = \int_{-\infty}^{\infty} tU(\sigma - t)x(\sigma)U(\sigma)d\sigma, \quad -\infty < t < \infty.$$

Your problem is to find $y(t)$.

iii) (5pts) S is TV/TI? C/NC?

(Question 2) (20pts)

The IPOP of system S_1 is:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau + \int_{t}^{\infty} e^{t-\tau}x(\tau)d\tau, \quad t \in (-\infty, \infty).$$

The IPOP of system S_2 is:

$$y(t) = x(t) - \int_{-\infty}^{\infty} U(t-\tau)x(\tau)d\tau, \quad t \in (-\infty, \infty).$$

(i) (5pts) Find IRFs $h_1(t,\tau)$ and $h_2(t,\tau)$.
(ii) (5pts) State the properties of S_1 and S_2: TV/TI? C/NC?
(iii) (10pts) Find IRF of the cascaded system S_{21}: $h_{21}(t,\tau)$.
(iv) (5pts) Is $h_{12}(t,\tau) = h_{21}(t,\tau)$? Why?

PART 2

s-Domain Analysis

(Use Laplace Transform!)

(Question 3) (10pts)

The input to an LTI system is

$$x(t) = U(t) - 2U(t-1) + U(t-2).$$

If the Laplace transform of the output is given by

$$Y(s) = \frac{(s+2)(1-e^{-s})^2}{s^2(s+1)^2},$$

determine the IRF of the system.

(Question 4) (10pts)

Find the Laplace Transform $F(s)$ of $f(t)$ given that

$$f(t) = \int_0^t \sin(t-\tau)\cos(t-\tau)d\tau, \quad t \geq 0.$$

(Question 5) (10pts)

The input $x(t)$ and output $y(t)$ of a linear system S are related by the equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t),\ t > 0, y(0) = 0, y^{'}(0) = 1.$$

Solve the differential equation with the given initial conditions given

$$x(t) = te^{-t}U(t).$$

PART 3

t-Domain and or s-Domain

(Use whatever method you are most comfortable with)

(Question 6) (15pts)

Given the following information regarding a system S:

$$U(t-1) \to [\mathbf{L}, \mathbf{TI}, \mathbf{C}; \mathbf{IRF} : \mathbf{h(t)U(t)}] \to \hat{g}(t)$$

Is it true that

$$\hat{g}(t+1) = \int_{-\infty}^{\infty} U(t-\tau)h(\tau)U(\tau)d\tau?$$

Please give details of your answer.
Find $h(t)$ given that

$$\hat{g}(t+1) = \sin tU(t).$$

(Question 7) (15pts)

Consider an LTI system S and a signal $x(t) = 2e^{-3t}U(t-1)$. If

$$x(t) \to y(t)$$

and

$$\frac{dx(t)}{dt} \to -3y(t) + 2e^{-2t}U(t),$$

determine the impulse response $h(t)$ of S.

Midterm solution, WINTER 2011, EE 102

1.

(i)

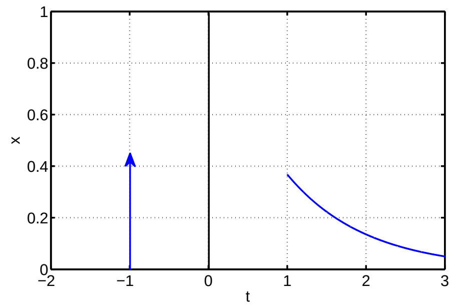

Figure 1: Q1.i

(ii)

$$y(t) = \int_{-\infty}^{\infty} tU(\sigma - t)[e^{-\sigma}U(\sigma - 1) + \delta(\sigma + 1)]U(\sigma)d\sigma$$
$$= t\int_{-\infty}^{\infty} tU(\sigma - t)e^{-\sigma}U(\sigma - 1)d\sigma.$$

It should be emphasized that the following term is zero

$$\int_{-\infty}^{\infty} tU(\sigma - t)\delta(\sigma + 1)U(\sigma)d\sigma = 0$$

For $t < 1$: $U(\sigma - t)U(\sigma - 1) = 1$ if $\sigma \geq t$ and $\sigma \geq 1$. So there are two cases

$$y(t) = t\int_{1}^{\infty} e^{-\sigma}U(\sigma - 1)d\sigma = -t[0 - e^{-1}] = te^{-1}.$$

For $t \geq 1$:

$$y(t) = t\int_{t}^{\infty} e^{-\sigma}d\sigma = te^{-t}.$$

So,

$$y(t) = te^{-1}U(1 - t) + te^{-t}U(t - 1)$$

(iii)

$$y(t-\tau) = \int_{-\infty}^{\infty} (t-\tau)U(\sigma-(t-\tau))x(\sigma)U(\sigma)d\sigma.$$

The output for a delayed input $x(t-\tau)$:

$$\int_{-\infty}^{\infty} tU(\sigma-t)x(\sigma-\tau)U(\sigma)d\sigma.$$

Using change of variables as $\epsilon = \sigma - \tau$ we have

$$\int_{-\infty}^{\infty} tU(\epsilon+\tau-t)x(\epsilon)U(\epsilon+\tau)d\epsilon \neq y(t-\tau).$$

$\therefore$ The system is TV. Besides, the system is NC as the output depends on future values of the input.

2.

(i) Regarding $h_1(t,\tau)$:

$$y(t) = \int_{-\infty}^{\infty} U(t-\tau)x(\tau)d\tau + \int_{-\infty}^{\infty} e^{t-\tau}x(\tau)U(\tau-t)d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)[U(t-\tau) - e^{t-\tau}U(\tau-t)]d\tau$$

$$h_1(t) = U(t) + e^{t}U(-t).$$

Regarding $h_2(t,\tau)$:

$$y(t) = x(t) - \int_{-\infty}^{\infty} U(t-\tau)x(\tau)d\tau$$

$$= \int_{-\infty}^{\infty} \left(\delta(t-\tau)x(\tau) - U(t-\tau)x(\tau)\right)d\tau$$

$\therefore h_2(t) = \delta(t) - U(t).$

(ii) System 1 is TI and NC. System 2 is TI and C.

(iii)

$$h_{21}(t,\tau) = \int_{-\infty}^{\infty} h_1(t-\tau)h_2(\tau)d\tau$$

$$= \int_{-\infty}^{\infty} \left(U(t-\tau) + U(\tau - t)e^{t-\tau}\right)\left(\delta(\tau) - U(\tau)\right) d\tau.$$

For $t \geq 0$:

$$y(t) = 1 - t + 0 - 1 = -t,$$

for $t < 0$:

$$y(t) = 0 + 0 + e^t - e^t = 0.$$

$\therefore y(t) = -tU(t)$.

(iv) Yes, because the systems are LTI.

3.

$$X(s) = \frac{1}{s} - \frac{2}{s}e^{-s} + \frac{1}{s}e^{-2s}.$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+2}{s(s+1)^2} = \frac{2}{s} - \frac{2}{s+1} - \frac{1}{(s+1)^2}.$$

$$\therefore h(t) = [2 - 2e^{-t} - te^{-t}]U(t).$$

4.

We know that

$$\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$$

$$f(t) = \int_{-\infty}^{\infty} \frac{1}{2}\sin 2(t-\tau)U(\tau)d\tau \Rightarrow$$

This is a convolution integral. Therefore, $F(s) = L_s\{\frac{1}{2}\sin 2t\}L_s\{U(t)\}$

$$F(s) = \frac{1}{2}\frac{2}{s^2+4}\frac{1}{s} = \frac{1}{s(s^2+4)}.$$

5.

Considering the initial conditions, by taking Laplace transform from both sides and knowing that $X(s) = \frac{1}{(s+1)^2}$, we have

$$s^2Y(s) - 1 + 3sY(s) + 2Y(s) = X(s) \Rightarrow$$

$$Y(s) = \frac{s^2+2s+2}{(s+1)^3(s+2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3} + \frac{D}{s+2}$$

$C = 1$ and $D = -2$. Also,

$$A(s+1)^2(s+2) + B(s+1)(s+2) + (s+2) - 2(s+1)^3 = s^2 + 2s + 2$$

So, $A = 2$ and $B = -1$. Knowing that $L_s\{e^{-t}U(t)\} = \frac{1}{s+1}$, $L_s\{te^{-t}U(t)\} = \frac{1}{(s+1)^2}$ and $L_s\{t^2e^{-t}U(t)\} = \frac{2}{(s+1)^3}$, we find that

$$y(t) = (2e^{-t} - te^{-t} + \frac{1}{2}t^2e^{-t} - 2e^{-2t})U(t).$$

6.

We have

$$U(t-1) \to [S] \to \hat{g}(t)$$

Since S is TI:

$$U(t) \to [S] \to \hat{g}(t+1)$$

In other words $\hat{g}(t+1)$ is the Unit step response (USR) $g(t)$ of S:

$$\hat{g}(t+1) = g(t)$$

But you know that

$$g(t) = \int_0^t h(\tau)d\tau,\ t \geq 0$$

Therefore

$$\hat{g}(t+1) = \int_0^t h(\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)U(t-\tau)U(\tau)d\tau$$

Finally

$$h(t) = \frac{d}{dt}\hat{g}(t+1) = \frac{d}{dt}\sin tU(t)$$

or

$$h(t) = \cos tU(t) + \sin t\delta(t) = \cos tU(t).$$

7.

$$x(t) = 2e^{-3t}U(t-1) \Rightarrow X(s) = 2\frac{e^{-(s+3)}}{s+3}$$
$$x(t) \to y(t) \Rightarrow Y(s) = X(s)H(s)$$

$$\frac{dx(t)}{dt} \to -3y(t) + 2e^{-2t}U(t) \Rightarrow -3Y(s) + \frac{2}{s+2} = sX(s)H(s) \Rightarrow$$

$$H(s) = \frac{2}{(s+2)(s+3)X(s)} = \frac{e^{s+3}}{s+2} \Rightarrow h(t) = e^{-2t+1}U(t+1).$$