UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination
May 7, 2013

Put First Letter * of LAST Name in the corner→→↗↗
(* Otherwise Your Midterm will be LOST)

Your name:——————————————————

Instructions: Closed Book, Calculators are NOT Allowed

Good Luck!

Table 1: Score Table

	a	b	c	d	e	Max	Score
1	2	2	2	2	2	10	
2	5	5				10	
3	2	2	2	2	2	10	
4	3	3	4			10	
5	10	5				15	
6	10					10	
7	15					15	
Total						80	

(Question 1) (10pts)

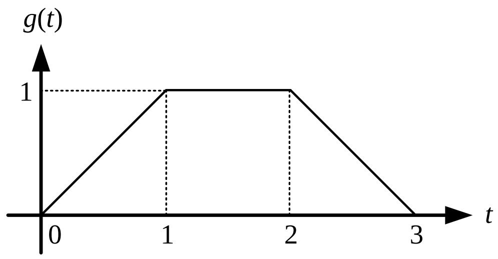
(i) (2pts) Find the even and odd parts of this function

$$g(t) = t(2 - t^2)(1 + 4t^2)$$

(ii) (2pts) Find a fundamental period of this signal.

$$g(t) = \cos(2\pi t) + \sin(3\pi t) + \cos\left(5\pi t - \frac{3\pi}{4}\right)$$

(iii) Given the signal:



Graph these signals:

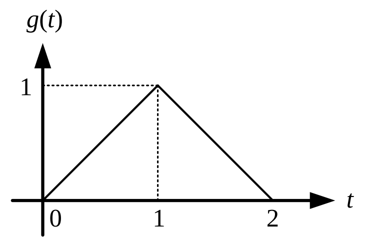
(a) (2pts) $g(2t + 3)$

(b) (2pts) $g(-2t + 3)$

(c) (2pts) $g\left(\frac{t}{2} + 1\right)$

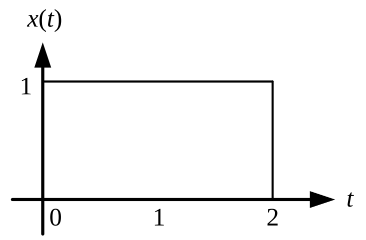
(Question 2) (10pts)

The step response of an LTI system is

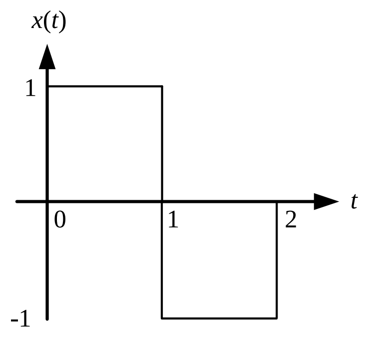


Plot the response of the system to the following inputs:

(a) (5pts)



(b) (5pts)



(Question 3) (10pts)

An LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 13y(t) = \frac{dx(t)}{dt} + 2x(t).$$

(1) (2pts) Compute the transfer function $H(s)$.
(2) (2pts) Find poles and zeros of the system.
(3) (2pts) Find impulse response function $h(t)$.
(4) (2pts) Is this system BIBO stable? Explain your answer.
(5) (2pts) Find the output $y(t)$ given $x(t) = e^{-2t}u(t)$.

(Question 4) (10pts)

The IPOP of system S is:

$$y(t) = \int_{-\infty}^{t} e^{-\tau} x(t-\tau) d\tau, \quad t \in (-\infty, \infty)$$

(i) (3pts) Find IRF $h(t, \tau)$
(ii) (3pts) State properties of S: TV/TI? C/NC?
(iii) (4pts) Find output due to $\delta(t-2) + u(t-3)$.

(Question 5) (15pts)

(1) (10pts) Using Laplace transform to find the convolution of $f_1(t)$ and $f_2(t)$, where

$$f_1(t) = e^{-3t}\cos(2t-5)u(t),$$
$$f_2(t) = e^{-2t}u(t).$$

(2) (5pts) Find $Y(s)$ of $y(t)$ given

$$y(t) = \int_0^t \sin(2(t-\sigma))\sigma cos\sigma d\sigma.$$

(Question 6) (10pts)

Let system S_1 be described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau) d\tau, \; t \in (-\infty, \infty)$$

$$x(t) \to [S_1] \to y(t)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \to [S_2] \to w(t)$$

$$w(t) = v(t) - \int_{-\infty}^{\infty} u(t-\tau) v(\tau) d\tau, \; t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2 S_1$.

(Question 7) (15pts)

Calculate the following integral:

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\sigma)}(t-\sigma)u(t-\sigma)\sigma u(\sigma)d\sigma,\ t \geq 0. \tag{1}$$

Then compute the Laplace transform $Y(s)$ of $y(t)$—which you just (hopefully "correctly") calculated.

Next, compute the product:

$$\mathcal{L}_s\left\{e^{-t}tu(t)\right\} \cdot \mathcal{L}_s\left\{tu(t)\right\}.$$

Finally do you find that:

$$\mathcal{L}_s\left\{e^{-t}tu(t)\right\} \cdot \mathcal{L}_s\left\{tu(t)\right\} = Y(s)? \tag{2}$$

Why?

EE 102 Midterm Solution

QUESTION I

(i) Even: $g_e(t) = \dfrac{g(t) + g(-t)}{2}$

$$g_e(t) = \frac{(t(2-t^2)(1+4t^2)) + (-t(2-t^2)(1+4t^2))}{2}$$

$$\boxed{g_e(t) = 0}$$

Odd: $g(t) = g_e(t) + g_o(t)$

$$g_o(t) = g(t) - \cancelto{0}{g_e(t)}$$

$$\boxed{g_o(t) = t(2-t^2)(1+4t^2)}$$

(ii) $\cos(2\pi t) \rightarrow T_1 = 1$

$\sin(3\pi t) \rightarrow \sin\left(\dfrac{2\pi t}{2/3}\right) \rightarrow T_2 = \dfrac{2}{3}$

$\cos\left(5\pi t - \dfrac{3\pi}{4}\right) \rightarrow \cos\left(\dfrac{2\pi t}{2/5} - \dfrac{3\pi}{4}\right) \rightarrow T_3 = \dfrac{2}{5}$

the LCM is 2

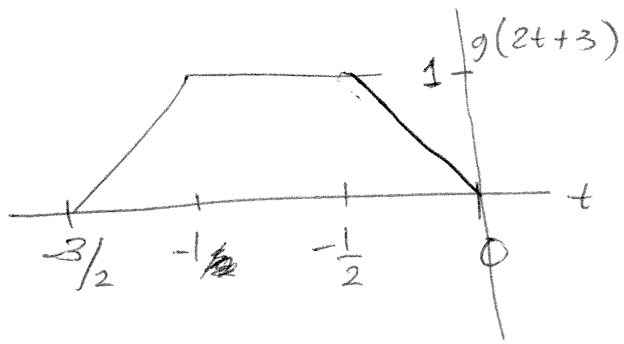
$$2(1) = 3\left(\frac{2}{3}\right) = 5\left(\frac{2}{5}\right) = 2$$

$$\boxed{T = 2}$$

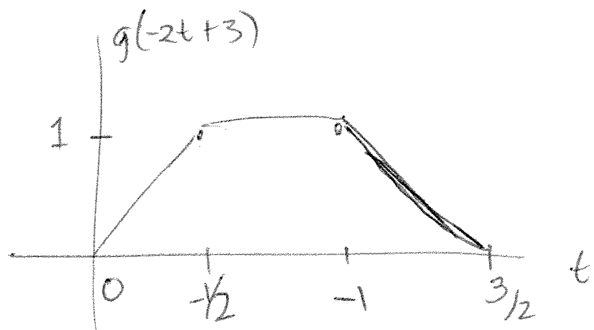
(iii) a) $g(2t+3)$:

Starting point
$2t+3=0$
$2t=-3$
$t=-3/2$

Ending Point
$2t+3=3$
$t=0$



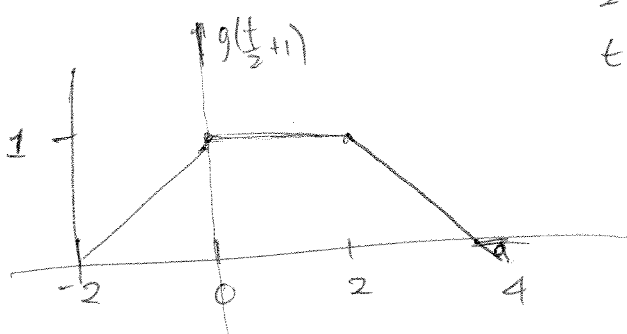
b) $g(-2t+3)$: is time reversed $g(2t+3)$ from a)



c) $g(\frac{t}{2}+1)$:

$\frac{t}{2}+1=0$
$t=-2$

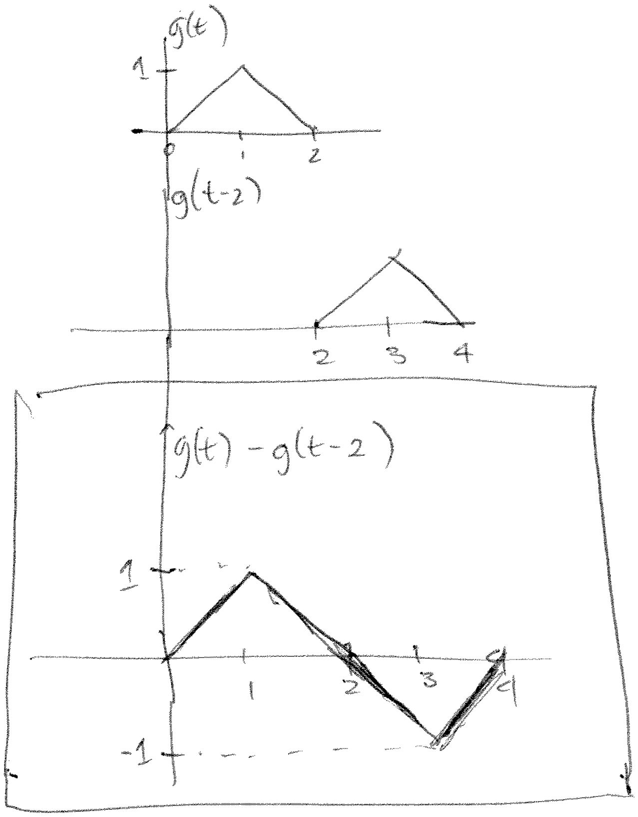
$\frac{t}{2}+1=3$
$\frac{t}{2}=2$
$t=4$



Question 2

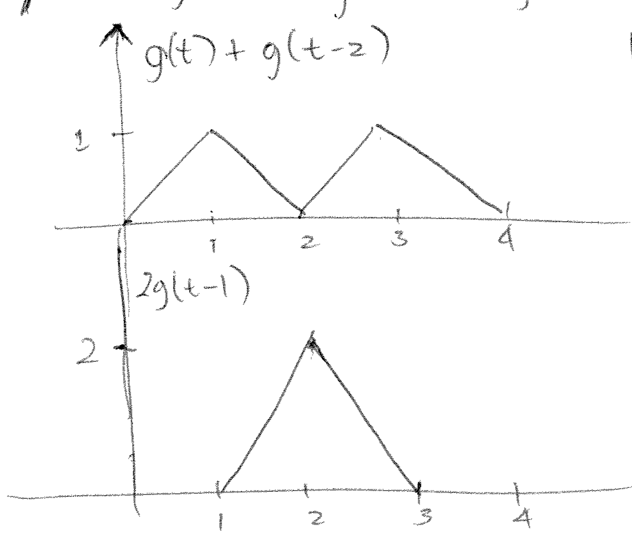
(a) $x(t) = u(t) - u(t-2)$

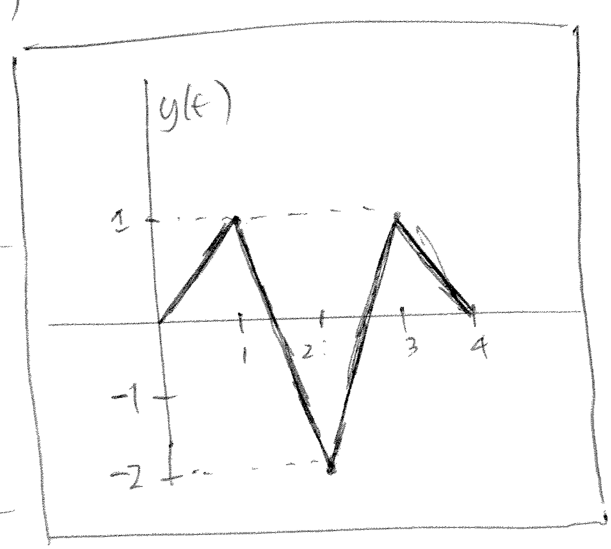
$y(t) = g(t) - g(t-2)$



(b) $x(t) = u(t) - 2u(t-1) + u(t-2)$

$y(t) = g(t) - 2g(t-1) + g(t-2)$





Question 3

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y(t) = \frac{dx(t)}{dt} + 2x(t)$$

(1) assume 0 initial conditions

$$Y(s)(s^2 + 4s + 13) = X(s)(s+2)$$

$$H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{s+2}{s^2+4s+13}}$$

(2) $\boxed{\text{zeros} : -2}$

$$s^2 + 4s + 13 = 0 \Rightarrow s = \frac{-4 \pm \sqrt{4^2 - 4(13)}}{2}$$

$$s = \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm j6}{2} = -2 \pm j3$$

$\boxed{\text{poles: } -2 \pm j3}$

(3) $$H(s) = \frac{s+2}{s^2+4s+13} = \frac{s+2}{(s+2)^2 + 3^2}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} \rightarrow \boxed{h(t) = e^{-2t}\cos(3t)u(t)}$$

(4) $\boxed{\text{Yes, because all poles are in the Left Half Plane}}$

(5) $x(t) = e^{-2t}u(t)$

$$X(s) = \frac{1}{s+2}$$

$$\begin{aligned}
Y(s) &= H(s)X(s) \\
&= \left(\frac{1}{s+2}\right)\left(\frac{s+2}{s^2+4s+13}\right) \\
&= \frac{1}{s^2+4s+13} \\
&= \frac{1}{(s+2)^2+3^2} \\
&= \frac{1}{3}\left(\frac{3}{(s+2)^2+3^2}\right)
\end{aligned}$$

$$\boxed{y(t) = \frac{1}{3}\left(e^{-2t}\sin(3t)u(t)\right)}$$

(Question 4) (10pts)

The IPOP of system S is:

$$y(t) = \int_{-\infty}^{t} e^{-\tau} x(t-\tau) d\tau, \quad t \in (-\infty, \infty)$$

(i) (3pts) Find IRF $h(t, \tau)$
(ii) (3pts) State properties of S: TV/TI? C/NC?
(iii) (4pts) Find output due to $\delta(t-2) + u(t-3)$.

(i) $y(t) = \int_{-\infty}^{\infty} e^{-\tau} x(t-\tau) u(t-\tau) d\tau$

let $z = t-\tau \Rightarrow dz = -d\tau$

$y(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(\tau) x(\tau) d\tau$

$\Rightarrow h(t,\tau) = e^{-(t-\tau)} u(\tau)$

(ii) Since $y(t)$ depends on future value of $x(t) \Rightarrow$ NC

Since $h(t,\tau) \neq h(t-\tau) \Rightarrow$ TV

(iii) $y(t) = \int_{-\infty}^{t} e^{-\tau} \delta(t-2-\tau) d\tau + \int_{-\infty}^{t} e^{-\tau} u(t-3-\tau) d\tau$

$= e^{-(t-2)} + \int_{-\infty}^{t-3} e^{-\tau} d\tau$

$= e^{-(t-2)} - e^{-(t-3)} + e^{\infty} = \infty$

(Question 5) (15pts)

(1) (10pts) Using Laplace transform to find the convolution of $f_1(t)$ and $f_2(t)$, where

$$f_1(t) = e^{-3t}\cos(2t-5)u(t),$$
$$f_2(t) = e^{-2t}u(t).$$

(2) (5pts) Find $Y(s)$ of $y(t)$ given

$$y(t) = \int_0^t \sin(2(t-\sigma))\sigma cos\sigma d\sigma.$$

(1) $f_1(t) = e^{-3t}\left[\cos(2t)\cos 5 + \sin(2t)\sin 5\right]u(t)$

$\Rightarrow F_1(s) = \dfrac{\cos 5\,(s+3)}{(s+3)^2+4} + \dfrac{\frac{1}{2}\sin 5}{(s+3)^2+4}$

$F_2(s) = \dfrac{1}{s+2}$

let $f(t) = f_1(t) * f_2(t) \Rightarrow F(s) = F_1(s)\cdot F_2(s)$

$\Rightarrow F(s) = \dfrac{\cos 5\,(s+3) + \frac{1}{2}\sin 5}{(s+2)(s^2+6s+13)} = \dfrac{As+B}{s^2+6s+13} + \dfrac{C}{s+2}$

Solve A, B, C $\Rightarrow \begin{cases} A = \dfrac{-\cos 5}{5} - \dfrac{2}{5}\sin 5 \\ B = \dfrac{\cos 5}{5} - \dfrac{8}{5}\sin 5 \\ C = \dfrac{\cos 5}{5} + \dfrac{2}{5}\sin 5 \end{cases}$

$\Rightarrow f(t) = A\cos(2t)e^{-3t}u(t) + \left(\dfrac{B-3A}{2}\right)\sin(2t)e^{-3t}u(t) + Ce^{-2t}u(t)$

(2) $$y(t) = \int_0^t \sin[2(t-\tau)]\, \tau \cos\tau \, d\tau$$

$$= \int_{-\infty}^{\infty} \sin[2(t-\tau)]\, u(t-\tau)\, \tau \cos\tau \, u(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

$$h(t) = \sin(2t)\, u(t)$$

$$x(t) = t\cos t\, u(t)$$

$$\Rightarrow \quad H(s) = \frac{2}{s^2+4}, \quad X(s) = \frac{s^2-1}{(s^2+1)^2}$$

$$\Rightarrow \quad Y(s) = H(s)\, X(s) = \frac{2(s^2-1)}{(s^2+4)[(s^2+1)^2]}$$

(Question 6) (10pts)

Let system S_1 be described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau) d\tau, \; t \in (-\infty, \infty)$$

$$x(t) \to [S_1] \to y(t)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \to [S_2] \to w(t)$$

$$w(t) = v(t) - \int_{-\infty}^{\infty} u(t-\tau) v(\tau) d\tau, \; t \in (-\infty, \infty)$$

Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2 S_1$.

let $v(t) = \delta(t)$

$$\Rightarrow w(t) = \delta(t) - \int_{-\infty}^{\infty} u(t-\tau)\delta(\tau) d\tau$$

$$= \delta(t) - u(t)$$

Let $x(t) = w(t)$

$$\Rightarrow y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} [\delta(\tau) - u(\tau)] d\tau$$

$$= e^{-t} - \int_{0}^{t} e^{-(t-\tau)} d\tau$$

$$= e^{-t} - e^{-t} \int_{0}^{t} e^{\tau} d\tau$$

$$= e^{-t} - e^{-t}\left[e^{\tau} \Big|_{0}^{t} \right] = \begin{cases} 2e^{-t} - 1 & \text{if } t > \\ 0 & \text{if } t < \end{cases}$$

Hence $h_{21}(t) = y(t) = (2e^{-t} - 1)u(t)$

(Question 7) (15pts)

Calculate the following integral:

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\sigma)}(t-\sigma)u(t-\sigma)\sigma u(\sigma)d\sigma,\ t \geq 0. \qquad (1)$$

Then compute the Laplace transform $Y(s)$ of $y(t)$—which you just (hopefully "correctly") calculated.

Next, compute the product:

$$\mathcal{L}_s\left\{e^{-t}tu(t)\right\} \cdot \mathcal{L}_s\left\{tu(t)\right\}.$$

Finally do you find that:

$$\mathcal{L}_s\left\{e^{-t}tu(t)\right\} \cdot \mathcal{L}_s\left\{tu(t)\right\} = Y(s)? \qquad (2)$$

Why?

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\sigma)}(t-\sigma)u(t-\sigma)\sigma u(\sigma)d\sigma$$

$$= \int_0^t e^{-(t-\sigma)}(t-\sigma)\sigma\, d\sigma$$

$$= te^{-t}\int_0^t e^{\sigma}\cdot\sigma\, d\sigma - e^{-t}\int_0^t e^{\sigma}\cdot\sigma^2 d\sigma$$

$$= te^{-t} + 2e^{-t} + t - 2 \quad \text{if } t > 0, \text{ else } y(t) = 0$$

$$\Rightarrow y(t) = (te^{-t} + 2e^{-t} + t - 2)u(t)$$

And $\Rightarrow Y(s) = \frac{1}{(s+1)^2} \cdot \frac{1}{s^2}$ 8 $= L_s\{e^{-t}tu(t)\} \cdot L_s\{tu(t)\}$