UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination
May 6, 2008

Put First Letter * of LAST Name in the corner→→↗↗
(* Otherwise Your Midterm will be LOST)

Your name:____

Instructions: Closed Book, Calculators are NOT Allowed

Good Luck!

PART 1

t-Domain Analysis

(Do NOT use Laplace Transforms)!

(Question 1) (15pts)

A system is described by

$$y(t) = \frac{1}{t+1}\int_0^t x(\sigma)d\sigma, t \geq 0,$$

where $x(t) = 0$ for $t < 0$.

(i) (5pts) Is the system L or NL? TV or TI? C or NC?

(ii) (5pts) Find the $y(t)$ for the input $x(t) = U(t-1)$

(iii) (5pts) Find the $y(t)$ for the input $x(t) = \delta(t-2)$

(Question 2) (15pts)

Let system S_1 be described by the IPOP relation:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau, t \in (-\infty, \infty)$$

$$x(t) \to [S_1] \to y(t)$$

and let system S_2 be described by the IPOP relation:

$$v(t) \to [S_2] \to w(t)$$

$$w(t) = v(t) - \int_{-\infty}^{\infty} U(t-\tau)v(\tau)d\tau, t \in (-\infty, \infty)$$

(i) (10pts) Compute the IRF $h_{21}(t)$ of the cascaded system $S_{21} := S_2S_1$.

(ii) (5pts) State BT–for L systems. Then write down the IPOP relation for a L, TI, NC system whose IP $x(t)$ is such that $x(t) = 0$, for $0 \leq t \leq 1$, and $x(t) \neq 0$, otherwise.

PART 2

s-Domain Analysis

(<u>This is where Laplace Transforms shine</u>)

(Question 3) (10pts)

Find the Laplace Transform of the following signals:

(i) (5pts) $e^t\delta(t-1) + e^{-t}U(t-1)$

(ii) (5pts) $e^{-2t}cos^2(t-\pi/3)U(t)$

(Question 4) (10pts)

(i) (5pts) Find $f(t)$ for the given $F(s)$:

$$F(s) = \frac{1}{(s^2+1)^2}$$

(ii) (5pts) Find the Laplace transform Y(s) of the given y(t):

$$y(t) = \int_{-\infty}^{\infty} \tau^2 sin(t-\tau)U(t-\tau)sin\tau U(\tau)d\tau, t \geq 0.$$

(Question 5) (15pts)

For the input $x(t)$ and output $y(t)$ relation given by:

$$2\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 8y(t) = \int_0^t e^{-(t-\tau)}x(\tau)d\tau,$$

for $t \geq 0$ with $y'(0) = 0$ and $y(0) = 0$. All inputs and outputs are zero before time zero.

(i) (5pts) Find the relation between $Y(s) = L_s\{y(t)\}$ and $X(s) = L_s\{x(t)\}$.

(ii) (5pts) Find the output $y(t)$ when the input is $[e^{-t}cost]U(t)$

(iii) (5pts) Find the impulse response function $h(t)$ of the system.

PART 3

t-Domain and or s-Domain

(Use whatever method you are most comfortable with)

(Question 6) (10pts)

The Laplace transform $F(s)$ of a function $f(t)$ is

$$F(s) = \frac{s-2}{s^2+3s+2}.$$

(i) (5pts) Show that $f(t)$ can be written as

$$f(t) = e^{-2t}U(t) - \int_0^t 3e^{-(t-\sigma)}e^{-2\sigma}d\sigma, t \geq 0.$$

(ii) (5pts) If the function $f(t)$ above is now the output of a LTI and Causal system corresponding to the input $e^{-t}U(t)$, find the output of the system when the input is $e^{-2t}U(t)$.

(Question 7) (15pts)

Consider the cascaded combination S_{12} of LTI, C systems S_1 and S_2 as shown below:

$$x(t) \rightarrow [S_1] \rightarrow [S_2] \rightarrow z(t)$$

where $x(t) = U(t)$ and $z(t) = (sint + cost - e^{-t})U(t)$.
(i) (5pts) Compute the IRF $h_{12}(t)$ of the cascaded system S_{12}.

(ii) (10pts) If system S_1 is described by the IPOP relation:

$$y(t) = \int_{-\infty}^t cos(t-\sigma)x(\sigma)d\sigma, t > -\infty.$$

Write down the IPOP description of system S_2.

HW6 solution, SPRING 2008, EE 102

1.

(i)It's Linear, and Causal.
(TI or TV?) Let $x(t) = \delta(t-\tau)$

$$y(t) = \frac{1}{t+1}U(t-\tau)$$

It's TV.

(ii)

$$y(t) = \frac{1}{t+1}\int_0^t U(\sigma-1)d\sigma = \frac{1}{t+1}U(t-1)(t-1)$$

(iii)

$$y(t) = \frac{1}{t+1}\int_0^t \delta(\sigma-2)d\sigma = \frac{1}{t+1}U(t-2).$$

2.

$y(t) = [e^{-t}U(t)] * x(t)$, Thus,$h_1(t) = e^{-t}U(t)$

For S_2, $h_2(t) = \delta(t) - U(t)$.

$$h_{21}(t) = \int_{-\infty}^{\infty} h_1(t-\sigma)h_2(\sigma)d\sigma = [2e^{-t}-1]U(t).$$

(ii) BT:

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)x(\tau)d\tau$$

For TI system,

$$h(t,\tau) = h(t-\tau)$$

For the property given in the question:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{0} h(t-\tau)x(\tau)d\tau + \int_{1}^{\infty} h(t-\tau)x(\tau)d\tau$$

3.

(i) Original formula= $ee^{t-1}\delta(t-1) + e^{-1}e^{-(t-1)}U(t-1) \rightarrow e^{-s}[e + e^{-1}\frac{1}{s+1}]$
(ii)$\frac{1}{2}[\frac{1}{s+2} + cos\frac{2\pi}{3}(\frac{s+2}{(s+2)^2+4}) + sin\frac{2\pi}{3}\frac{1}{(s+2)^2+4}]$

4.

(i) $F(s) = \frac{1}{2}\frac{1}{s}\frac{2s}{(s^2+1)^2} \longrightarrow \frac{1}{2}tsint * U(t)$

Thus, $f(t) = \frac{1}{2}\int_0^t \tau sin\tau d\tau = \frac{1}{2}[-tcost + sint]$

(ii) $y(t) = [t^2 sint] * [sint]$

$sint \longrightarrow \frac{1}{s^2+1}$

$t^2 sint \longrightarrow \frac{6s^2-2}{(s^2+1)^3}$

$Y(s) = \frac{6s^2-2}{(s^2+1)^4}$

5.

(i)

$$2s^2Y(s) + 10sY(s) + 8Y(s) = L[(e^{-t}) * (x(t))] = \frac{1}{s+1}X(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{2(s+1)^2(s+4)}$$

(ii)

$$L[e^{-t}cos(t)U(t)] = \frac{s+1}{(s+1)^2+1}$$

Thus,

$$Y(s) = \frac{1}{2(s+1)(s+4)(s^2+2s+2)}$$

By partial fraction,

$$Y(s) = \frac{1/6}{s+1} + \frac{-1/60}{s+4} + \frac{(1/100)s - 1/5}{s^2+2s+2}$$

By Inverse Laplace transform,

$$y(t) = U(t)[(1/6)e^{-t} + (-1/60)e^{-4t} + (1/100)e^{-t}cost + (-21/100)e^{-t}sint]$$

(iii)

$$H(s) = \frac{1}{2(s+1)^2(s+4)}$$

By partial fraction and inverse Laplace transform

$$h(t) = \frac{1}{18}e^{-4t}U(t) - \frac{1}{9}e^{-t}U(t) - \frac{2}{9}te^{-t}U(t)$$

6.

(i)

$$f(t) = U(t)[e^{-2t} - 3[e^{-t}] * [e^{-2t}]]$$

Thus,

$$F(s) = \frac{1}{s+2} - \frac{3}{s+1}\frac{1}{s+2} = \frac{s-2}{s^2+3s+2}$$

(ii) $L[e^{-t}U(t)] = \frac{1}{s+1}$
$H(s) = F(s)/X(s) = 1 - 4/(s+2)$
$L[e^{-2t}U(t)] = \frac{1}{s+2}$
$Y(s) = \frac{s-2}{(s+2)^2}$
Thus,

$$y(t) = U(t)[e^{-2t} - 4te^{-2t}]$$

7.

(i)
$X(s) = 1/s$
$Z(s) = 2s/((s+1)(s^2+1))$
$H_{12}(s) = Z(s)/X(s) = 1/(s+1) + (s-1)/(s^2+1)$
Thus,$h_{12}(t) = [e^{-t} + cost - sint]U(t)$
(ii) $H_1(s) = 1/(s^2+1)$
$H_2(s) = H_{12}(s)/H_1(s) = 2s/(s+1) \Rightarrow h_2(t) = 2[\delta(t) - e^{-t}U(t)]$
Thus,$y(t) = \int_{-\infty}^{\infty} 2[\delta(t-\tau) - e^{-(t-\tau)}U(t-\tau)]x(\tau)d\tau$