UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination II
Date: March 6, 2018
Duration: 1 hr. 50 min.

INSTRUCTIONS:

- The exam has 6 problems and 15 pages.
- The exam is closed-book.
- Two cheat sheets of A4 size are allowed.
- Calculator is NOT allowed.
- Put your discussion session in the top-right corner ↗↗

Your name: ____________________

Student ID: ____________________

Table 1: Score Table

Problem	a	b	c	d	Score	
1	5	5	5		15	8
2	5	5	5		15	15
3	10	8			18	15
4	6	6			12	12
5	5	5	5		15	7
6	8	7			15	9
Total					90	66

$e^{-s}u(t) = \frac{1}{s+1}$

8

Problem 1 (15 pts)
Given the following transfer function of an LTI system

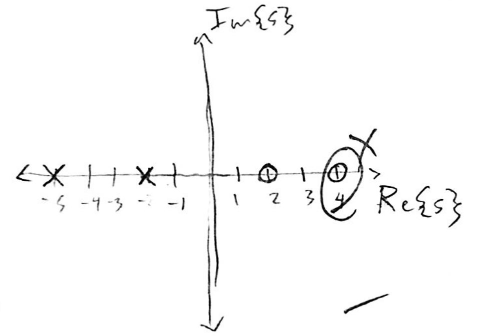
$$H(s) = \frac{s^2 - 5s + 6}{s^2 + 7s + 10} = \frac{(s-3)(s-2)}{(s+5)(s+2)}$$

(a) (5 pts) Sketch pole-zero plot of $H(s)$. State whether the system is BIBO stable or not. Explain.

(b) (5 pts) Compute the inverse Laplace transform to find a causal impulse response function $h(t)$.

(c) (5 pts) Find the steady state output $y(t)$ when the input is $x(t) = 12\cos(\sqrt{10}\ t)$.

a)



The system is BIBO stable b/c ~~the~~ the ROC is $Re\{s\} > -2$ which includes the imaginary axis, and signifies that $h(t)$ has an e^{-at} term that makes it converge to 0 as time approaches infinity

4

b) $N[H(s)]\ (s-3)(s+2-4) = (s-3)(s+2) - 4s + 12$ $\qquad 10 - \frac{49}{4} =$

$$H(s) = \frac{s-3}{s+5} - \frac{4s+12}{(s+5)(s+2)} = 1 - \frac{8}{s+5} - \left(\frac{A}{s+5} + \frac{B}{s+2}\right)$$

3

$4s + 12 = As + 2A + Bs + 5B$

$A + B = 4 \quad 2A + 5B = 12 \quad 3A = 6 \quad A = 2 \quad B = 2$

$2A + 20 - 5A = 12$

2

$$H(s) = 1 - \frac{8}{s+5} - \frac{2}{s+5} - \frac{2}{s+2} = 1 - \frac{10}{s+5} - \frac{2}{s+2}$$

$$h(t) = \delta(t) - 8e^{-5t}u(t) - 2e^{-5t}u(t) - 2e^{-2t}u(t)$$

$$\boxed{h(t) = \delta(t) - 10e^{-5t}u(t) - 2e^{-2t}u(t)}$$

c) -7

c)

$$X(s) = \frac{12s}{s^2+10} \qquad Y(s) = X(s)H(s)$$

$$Y(s) = \frac{12s}{s^2+10}\left(1 - \frac{10}{s+5} - \frac{2}{s+2}\right)$$

$$= \frac{12s}{s^2+10} - \frac{120s}{(s^2+10)(s+5)} - \frac{24s}{(s^2+10)(s+2)}$$

$$120s = As^2 + 10A + Bs^2 + 5Bs + Cs + 5C$$

$$A+B=0 \qquad 5B+C=120 \qquad 5C+10A=0$$

$$A=-B \qquad C=2B \qquad 7B=120 \qquad B=\frac{120}{7} \qquad C=\frac{240}{7} \qquad A=-\frac{120}{7}$$

$$Y(s) = \frac{12s}{s^2+10} + \frac{120/7}{s+5} - \frac{\frac{120}{7}s}{s^2+10} - \frac{240/7}{s^2+10} - \frac{24s}{(s^2+10)(s+2)}$$

new A, B, C

$$24s = As^2 + 10A + Bs^2 + 2Bs + Cs + 2C$$

$$A+B=0 \qquad 10A+2C=0 \qquad 2B+C=24$$

$$2C=10B \qquad C=5B \qquad B=\frac{24}{7} \qquad C=\frac{120}{7} \qquad A=\frac{-24}{7}$$

$$Y(s) = \frac{12s}{s^2+10} + \frac{120}{7}\frac{1}{s+5} - \frac{120}{7}\frac{s}{s^2+10} - \frac{240}{7\sqrt{10}}\frac{\sqrt{10}}{s^2+10} + \frac{24}{7}\frac{1}{s+2} - \frac{24}{7}\frac{s}{s^2+10} - \frac{120}{7\sqrt{10}}\frac{\sqrt{10}}{s^2+10}$$

$$y(t) = \Big(12\cos(\sqrt{10}t) + \frac{120}{7}e^{-5t} - \frac{120}{7}\cos(\sqrt{10}t) - \frac{240}{7\sqrt{10}}\sin(\sqrt{10}t) + \frac{24}{7}e^{-2t}$$

$$- \frac{24}{7}\cos(\sqrt{10}t) - \frac{120}{7\sqrt{10}}\sin(\sqrt{10}t)\Big)u(t)$$

$$\boxed{y(t) = \left(\left(12 - \frac{144}{7}\right)\cos(\sqrt{10}t) - \frac{360}{7\sqrt{10}}\sin(\sqrt{10}t) + \frac{120}{7}e^{-5t} + \frac{24}{7}e^{-2t}\right)u(t)}$$

15

Problem 2 (15 pts)
The input-output relationship for an LTI system is given by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt} + 5x(t), t \geq 0$$

with initial conditions $y'(0) = y(0) = 0, x(0) = 0$.

(a) (5 pts) Find the transfer function $H(s)$.

(b) (5 pts) Find the impulse response function $h(t)$.

(c) (5 pts) Find the output $y(t)$ when the input is $x(t) = e^{-5(t-4)}u(t-4)$.

a) $s^2 Y(s) - s\cancel{y(0)} - \cancel{y'(0)} + 2(sY(s) - \cancel{y(0)}) + 5Y(s) = sX(s) - x(0) + 5X(s)$

5 $\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{s+5}{s^2+2s+5}}$

b) $h(t) = \mathcal{L}^{-1}[H(s)]$ $H(s) = \frac{s+5}{(s+1)^2+2^2} = \frac{s+1}{(s+1)^2+2^2} + 2\frac{2}{(s+1)^2+2^2}$

5 $\boxed{h(t) = e^{-t}u(t)\left(\cos(2t) + 2\sin(2t)\right)}$

c) $x(t+4) = e^{-5t}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+5}$

time shift by -4

$X(s) = e^{-4s}\frac{1}{s+5}$

$Y(s) = e^{-4s}\left(\frac{1}{s^2+2s+5}\right) = e^{-4s}\left(\frac{1}{2}\right)\left(\frac{2}{(s+1)^2+2^2}\right)$

5 $\boxed{y(t) = \mathcal{L}^{-1}[Y(s)] = \frac{1}{2}e^{-(t-4)}\sin(2(t-4))u(t-4)}$

15

Problem 3 (18 pts)
Consider an LTI system with impulse response function

$$h(t) = e^{-\pi(t-2)}u(t-2).$$

$\omega_0 = \pi$

When a periodic signal $x(t)$ with period 2 is applied to this system, we get the following periodic output

$$y(t) = 1 + \cos(\pi t) + \sin(3\pi t),$$

$\frac{2\pi}{3\pi}$ $\frac{2\pi}{\pi}$ $\frac{2}{3}$ 2

(a) (10 pts) Find the exponential Fourier series coefficients of the input $x(t)$.

(b) (8 pts) Sketch magnitude and phase spectra of X_k.
Hint: Use the following while plotting phase spectrum: $tan^{-1}(1) = \frac{\pi}{4}$, $tan^{-1}(-1) = -\frac{\pi}{4}$, $tan^{-1}(\frac{1}{3}) \approx \frac{\pi}{10}$, *and* $tan^{-1}(-\frac{1}{3}) \approx -\frac{\pi}{10}$.

a) $\left(x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\pi k t}\right) * e^{-\pi(t-2)}u(t-2) = y(t)$

$H_k = H(jk\pi) = e^{-2s}\frac{1}{s+\pi}\Big|_{s=jk\pi} = e^{-2jk\pi}\frac{1}{\pi(jk+1)}$

$= \frac{1}{\pi(jk+1)}$

$y(t) = 1 + \frac{1}{2}e^{j\pi t} + \frac{1}{2}e^{-j\pi t} + \frac{1}{2j}e^{j3\pi t} - \frac{1}{2j}e^{-j3\pi t}$

$Y_0 \quad Y_1 \quad Y_{-1} \quad Y_3 \quad Y_{-3}$

$Y_k = \sum_{k=-\infty}^{\infty} X_k H_k$ $\quad \frac{1}{H_k} = \pi(jk+1)$

$$\boxed{\begin{array}{lll} X_0 = \pi & X_1 = \frac{\pi(1+j)}{2} & X_{-1} = \frac{\pi(1-j)}{2} \\ & X_3 = \frac{\pi(1+3j)}{2j} & X_{-3} = \frac{\pi(1-3j)}{-2j} \end{array}}$$

all else are zero; $X_k = 0$ for $k \neq \{0, \pm1, \pm3\}$

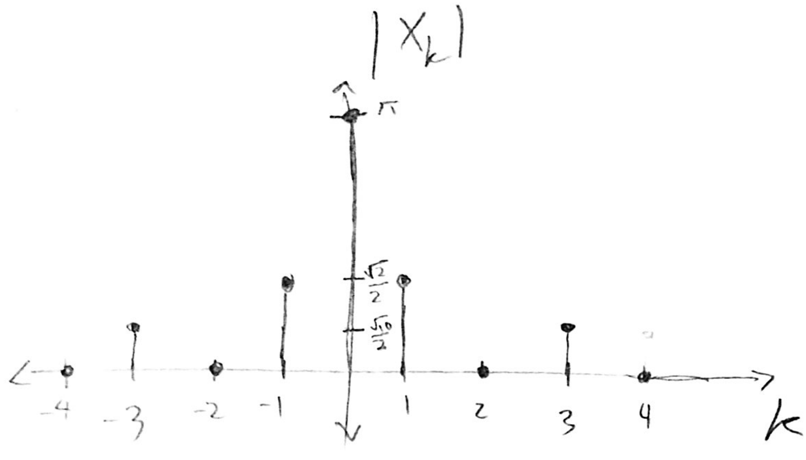
$\measuredangle X_1 = \frac{\pi}{4}$ $\quad \measuredangle X_0 = 0$
$\measuredangle X_{-1} = -\frac{\pi}{4}$ $\quad \measuredangle Y_3 = \frac{\pi}{2} - \frac{\pi}{10} = \frac{4\pi}{10}$ $\quad \measuredangle X_{-3} = -\frac{4\pi}{10}$

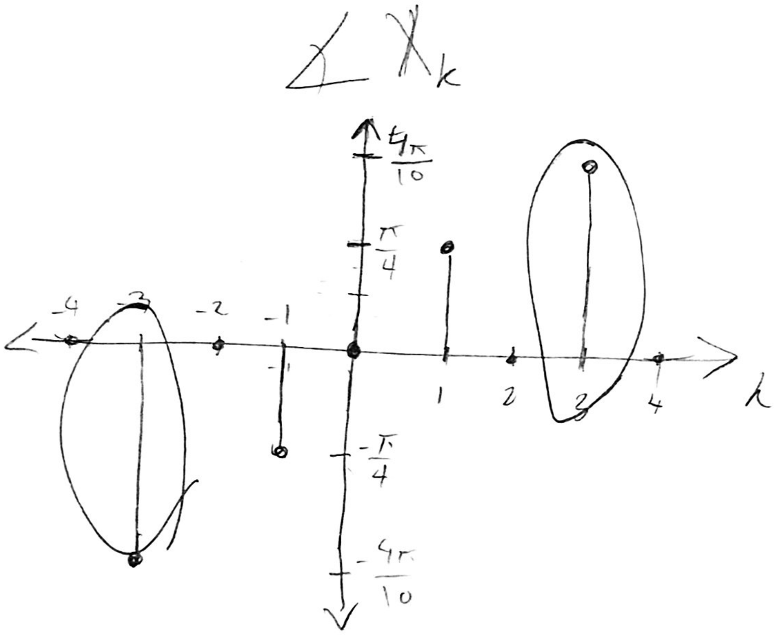
b) $\rightarrow$

$\sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\sqrt{\frac{1}{4}+\frac{9}{4}} = \sqrt{\frac{10}{4}} \quad \frac{\sqrt{10}}{2}$

b)





$T_1 = 2\pi \cdot \frac{2}{\pi} = 4$

$T_2 = 2\pi \cdot \frac{4}{\pi} = 8$

$T_0 = 8$

$\omega_0 = \frac{\pi}{4}$

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Problem 4 (12 pts) Consider the following periodic signal $x(t)$

$$x(t) = 2\cos(\pi t/2) - \sin(\pi t/4)$$

(a) (6 pts) Find the exponential Fourier series coefficients of $x(t)$.

(b) (6 pts) Find the exponential Fourier series coefficients of

$$y(t) = \frac{dx(t)}{dt}.$$

a) $x(t) = \underbrace{1}_{X_2} e^{j\frac{\pi}{2}t} + \underbrace{1}_{X_{-2}} e^{-j\frac{\pi}{2}t} \underbrace{- \frac{1}{2j}}_{X_1} e^{j\frac{\pi}{4}t} + \underbrace{\frac{1}{2j}}_{X_{-1}} e^{-j\frac{\pi}{4}t}$ $\quad \omega_0 = \frac{\pi}{4}$

$$\boxed{X_1 = -\frac{1}{2j} \quad X_{-1} = \frac{1}{2j} \quad X_2 = 1 \quad X_{-2} = 1 \quad X_k = 0 \text{ for } k \neq \{\pm 1, \pm 2\}}$$

b) $y(t) = -\pi \sin(\pi t/2) - \frac{\pi}{4}\cos(\pi t/4)$ $\quad \omega_0 = \frac{\pi}{4}$

$y(t) = \underbrace{\frac{-\pi}{2j}}_{X_2} e^{j\frac{\pi}{2}t} + \underbrace{\frac{\pi}{2j}}_{X_{-2}} e^{-j\frac{\pi}{2}t} \underbrace{- \frac{\pi}{8}}_{X_1} e^{j\frac{\pi}{4}t} \underbrace{- \frac{\pi}{8}}_{X_{-1}} e^{-j\frac{\pi}{4}t}$

$$\boxed{Y_1 = -\frac{\pi}{8} \quad Y_{-1} = -\frac{\pi}{8} \quad Y_2 = -\frac{\pi}{2j} \quad Y_{-2} = \frac{\pi}{2j} \quad Y_k = 0 \text{ for } k \neq \{\pm 1, \pm 2\}}$$

a)

$$\frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-jk\omega_0 t}\, dt = \frac{A}{T_0}\int_0^{T_0/2} e^{-jk\omega_0 t}$$

$$= \frac{A}{T_0}\left(\frac{1}{-jk\frac{2\pi}{T_0}} e^{-jk\frac{2\pi}{T_0}t}\right)\Bigg|_0^{T_0/2} = \frac{A}{T_0}\left(\frac{1}{+jk\frac{2\pi}{T_0}} + \frac{1}{jk\frac{2\pi}{T_0}}\right)$$

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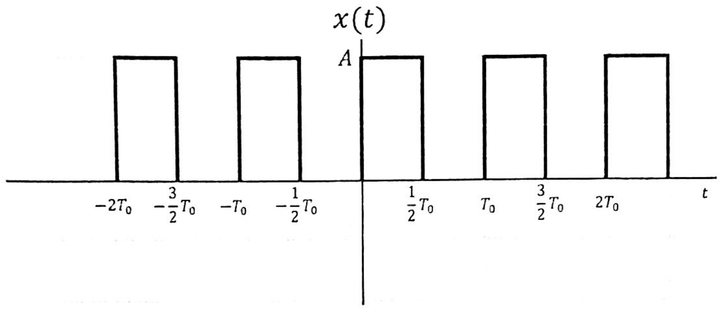
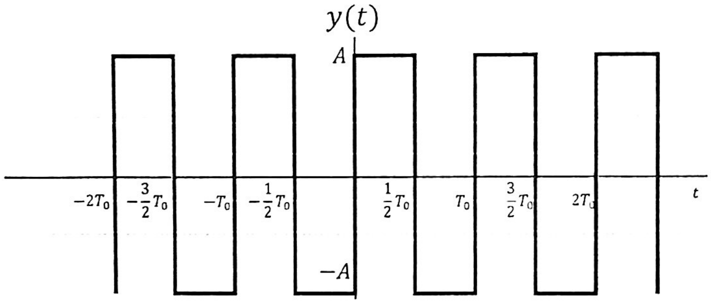
Problem 5 (15 pts) Consider the following periodic signals $x(t)$ and $y(t)$





3 (a) (5 pts) Determine the complex exponential Fourier series of $x(t)$, i.e., find the coefficients X_k that satisfy

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\omega_0 kt}$$

$$X_k = \frac{A}{T_0}\left(\frac{2T_0}{jk2\pi}\right)$$

for odd k $\boxed{X_k = \frac{A}{jk\pi}}$ X_0 ? −2

4 (b) (5 pts) Determine the trigonometric Fourier series of $x(t)$, i.e., find the coefficients a_k and b_k that satisfy

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) - b_k \sin(k\omega_0 t)$$

$\boxed{a_k = Re\{X_k\} = 0}$

$b_k = Im\{X_k\} \quad \frac{A}{k\pi}$ odd −1

0 (c) (5 pts) Using part (a), determine the complex exponential Fourier series of $y(t)$, i.e., find the coefficients Y_k that satisfy

$$y(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j\omega_0 kt}$$

$Y_k = X_k +$

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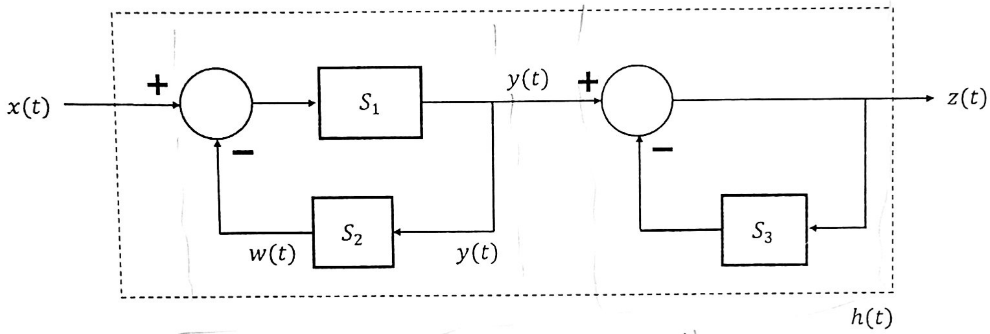
Problem 6 (15 pts) Consider the following LTI system with input $x(t)$, output $z(t)$, and impulse response function (IRF) $h(t)$. Systems S_1, S_2 and S_3 are LTI systems as well with the following characteristics

1. IRF of S_1 is $h_1(t) = e^{-2t}u(t)$. $H_1(s) = \frac{1}{s+2}$
2. Input-output relation for S_2 is $w(t) = \frac{d}{dt}y(t)$ $W(s) = sY(s) - y(0) = sY(s)$ $H_2(s) = s$
3. IRF of S_3 is $h_3(t) = e^{-t}u(t)$. $H_3(s) = \frac{1}{s+1}$

(Assume all the signals at $t = 0$ are zero, i.e., all initial conditions are zero).



$H_{12}(s)$ $H_4(s)$

$\frac{8}{4} - \frac{9}{4}$

5 (a) (8 pts) Compute the IRF of entire system $h(t)$ and its Laplace transform $H(s)$.

4 (b) (7 pts) Find the output $z(t)$ if the following input is applied to the system:

$$x(t) = e^{-2t}\cos(8t - 24)u(t-3) \qquad (1)$$

a) $(X(s) - Y(s)H_2(s))H_1(s) = Y(s)$ $X(s) = Y(s)\left(\frac{1}{H_1(s)} + H_2(s)\right)$

$H_{12}(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 + H_1(s)H_2(s)} = \frac{1}{(s+2)\left(1 + \frac{s}{s(s+2)}\right)} = \frac{1}{(s+2) + s}$

$H_{12}(s) = \frac{s}{s^2+2s+1} = \frac{s}{(s+1)^2}$

$Z(s) = Y(s) - Z(s)H_3(s)$ $H_4(s) = \frac{Z(s)}{Y(s)} = \frac{1}{1+H_3(s)} = \frac{1}{1+\frac{1}{s+1}} = \frac{s+1}{s+2}$

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$H(s) = H_{12}(s)H_4(s) = \frac{s(s+1)}{(s+1)^2(s+2)} = \frac{s}{(s+1)(s+2)}$

$H(s) = \frac{-1}{s+1} + \frac{2}{s+2}$

$h(t) = \mathcal{L}^{-1}[H(s)]$ $s = A(s+2) + B(s+1)$ $A + B = 1$ $2A + B = 0$ $A = -1$ $B = 2$

$h(t) = -e^{-t}u(t) + 2e^{-2t}u(t)$

b) $x(t) = e^{-6} e^{-2(t-3)} \cos(8(t-3)) u(t-3)$

$$X(s) = e^{-6}\left(\frac{s+2}{(s+2)^2+64}\right) e^{-3s}$$

$$Z(s) = \frac{s}{(s+1)(s+2)}\left(e^{-6}\left(\frac{s+2}{(s+2)^2+64}\right)e^{-3s}\right) = e^{-6}e^{-3s}\frac{s}{((s+2)^2+64)(s+1)} = e^{-6}e^{-3s}\left(\frac{A}{s+1} + \frac{Bs}{s+2} \right.$$

$$s = A((s+2)^2+64) + Bs(s+1) + C(s+1)$$

$$s = As^2 + 4As + 68A + Bs^2 + Bs + Cs + C$$

$$A + B = 0 \qquad 4A + B + C = 0 \qquad C + 68A = 5$$

$$B = -A \qquad C = -3A \qquad 65A = 5 \qquad A = \frac{1}{13} \quad B = -\frac{1}{13}$$

$$C = -\frac{3}{13}$$

$$Z(s) = \frac{e^{-6}}{13}e^{-3s}\left(\frac{1}{s+1} + -1\left(\frac{s+2}{(s+2)^2+64}\right) - \frac{1}{8}\left(\frac{8}{(s+2)^2+64}\right)\right)$$

$$\boxed{z(t) = \frac{e^{-6}}{13}\left(e^{-(t-3)} - e^{-2(t-3)}\left(\cos(8(t-3)) + \frac{1}{8}\sin(8(t-3))\right)\right)u(t-3)}$$