UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

ECE102: SYSTEMS & SIGNALS

Midterm Examination
January 28, 2020
Duration: 1 hr 50 mins.

INSTRUCTIONS:

- The exam has 5 problems and 12 pages.
- The exam is closed-book.
- One cheat sheet of A4 size is allowed.
- Calculator is NOT allowed.
- Write your discussion session in the top-right corner. $\nearrow\nearrow$

Your name:______________________________

Student ID:______________________________

Table 1: Score Table

Problem	a	b	c	d	e	Score
1	5	5	5			15
2	5	5	5			15
3	5	5	5	5		20
4	10	10				20
5	5	5	10			20
Total						100

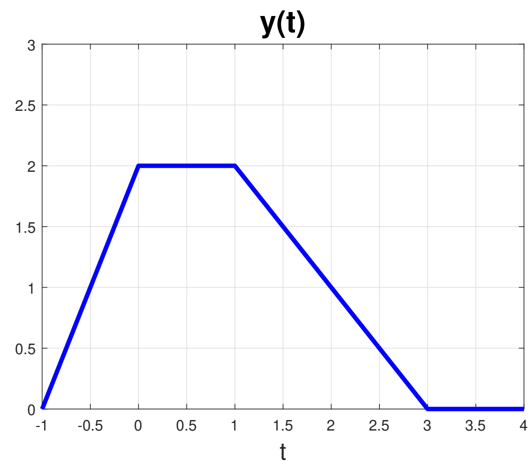
Problem 1 (15 pts)

Consider the signal $x(t) = |\cos(2\pi t)| + \sin(3\pi t)$ for (a) and (b).

(a) (5 pts) Sketch the even and odd components of $x(t)$.

(b) (5 pts) Find the fundamental period of $x(t)$.

(c) (5 pts) Consider the following signal $y(t)$. Sketch $y(-2t+3)$.



Solution:

(a)

The even part of $x(t)$ can be found using $x_e(t) = \frac{1}{2}(x(t) + x(-t)) = \frac{1}{2}(|\cos(2\pi t)| + \sin(3\pi t) + |\cos(-2\pi t)| + \sin(-3\pi t)) = |\cos(2\pi t)|$

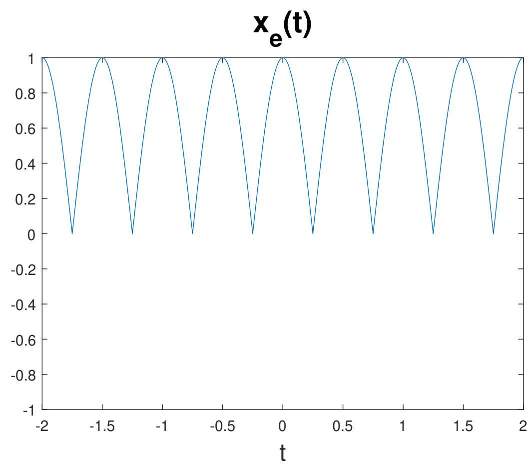
The odd part of $x(t)$ can be found using $x_o(t) = \frac{1}{2}(x(t) - x(-t)) = \frac{1}{2}(|\cos(2\pi t)| + \sin(3\pi t) - |\cos(-2\pi t)| - \sin(-3\pi t)) = \sin(3\pi t))$

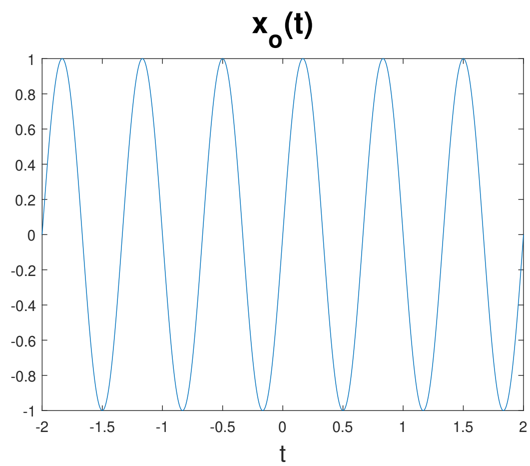
(Note: Since $x_e(t), x_o(t)$ expand the entire t axis, students only need to sketch part of the signals)

(b)

The fundamental period of $|\cos(2\pi t)|$ is 0.5 and the fundamental period of $\sin(3\pi t)$ is $\frac{2}{3}$. Therefore, the fundamental period of $x(t)$, T, satisfies

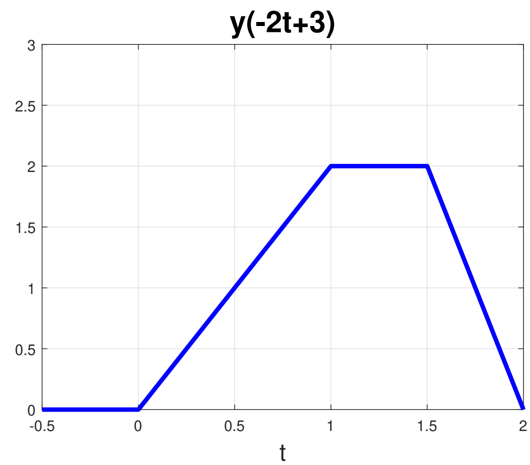
$$T = 0.5m = \frac{2}{3}n$$





where m, n are the minimum positive integer that satisfies the above equation. Then we can find $(m, n) = (4, 3)$ and $T = 2$.

(c)



Problem 2 (15 pts)

The system S is given by the following relation

$$y(t) = x(t) \times \text{sign}(x(t))$$

where $x(t)$ and $y(t)$ are the input and the output of the system, respectively. The function $\text{sign}(x(t))$ is defined as

$$\text{sign}(x(t)) = \begin{cases} 1, & x(t) \geq 0 \\ -1, & x(t) < 0 \end{cases}$$

(a) (5 pts) Is the system linear or not? Please provide justification.

(b) (5 pts) Is the system TI or TV? Please provide justification.

(c) (5 pts) Is this system C or NC? Please provide justification.

Solution:

(a)
The system is not linear. For example, let $x_1(t) = 1, x_2(t) = -1, x_3(t) = x_1(t) + x_2(t) = 0$. The outputs for $x_1(t), x_2(t), x_3(t)$ will be $y_1(t) = 1, y_2(t) = 1, y_3(t) = 0$, respectively. As we can see, $y_3(t) \neq y_1(t) + y_2(t)$.

(b)

The system is TI. The output of this system can be rewritten as $y(t) = |x(t)|$. Consider the input $x(t-t_0)$. The output will be $|x(t-t_0)| = y(t-t_0)$.

(c)
The system is causal since the output $y(t)$ doesn't depend on future inputs.

Problem 3 (20 pts)

Consider input/output (IPOP) relationship for a system S:

$$y(t) = e^{-t} \int_{-\infty}^{t} [\sin(t)\cos(\sigma) - \cos(t)\sin(\sigma)] e^{\sigma} x(\sigma) d\sigma$$

where $x(t)$ and $y(t)$ are the input and the output of the system, respectively.

(a) (5 pts) Find the impulse response function $h(t, \tau)$.

(b) (5 pts) Is the system TI or TV? Verify your answer using the impulse response of the system.

(c) (5 pts) Is the system C or NC? Verify your answer using the impulse response of the system.

(d) (5 pts) Is this system BIBO stable? Verify your answer using the impulse response of the system.

Solution:

(a)
We can rewrite the IPOP relationship as:

$$\begin{aligned} y(t) &= e^{-t} \int_{-\infty}^{t} \sin(t-\sigma) e^{\sigma} x(\sigma) d\sigma \\ &= \int_{-\infty}^{\infty} e^{-(t-\sigma)} \sin(t-\sigma) u(t-\sigma) x(\sigma) d\sigma. \end{aligned}$$

Applying impulse $x(t) = \delta(t-\tau)$ to S, we get

$$h(t, \tau) = e^{-(t-\tau)} \sin(t-\tau) u(t-\tau).$$

(b)

The system is TI since $h(t,\tau) = h(t-\tau)$. So we have $h(t) = e^{-t}\sin(t)u(t)$.

(c)

The system is causal (C) since $h(t) = 0$ for $t < 0$.

(d)

For BIBO stability we check if $h(t)$ is absolutely integrable.

$$\begin{aligned}\int_{-\infty}^{\infty} |h(t)|dt &= \int_{0}^{\infty} |e^{-t}\sin(t)|dt \\ &= \int_{0}^{\infty} e^{-t}|\sin(t)|dt \\ &\leq \int_{0}^{\infty} e^{-t}dt = 1 < \infty\end{aligned}$$

The impulse response is absolutely integrable, therefore it is BIBO stable.

Problem 4 (20 pts)

Consider IPOP relationship for a LTI system S:

$$y(t) = e^{-3t}\int_{-\infty}^{t} 2e^{3\tau}\cos^2(t-\tau)x(\tau) - e^{3\tau}x(\tau)d\tau$$

where $x(t)$ and $y(t)$ are the input and the output of the system, respectively.

(a) (10 pts) Find the impulse response function.

(b) (10 pts) Compute the output $y(t)$ given that its input is

$$x(t) = e^{-3t}u(t).$$

Solution:

(a)

We can rewrite the IPOP relationship as:

$$\begin{aligned}
y(t) &= e^{-3t} \int_{-\infty}^{t} e^{3\tau} x(\tau)(2\cos^2(t-\tau) - 1) d\tau \\
&= e^{-3t} \int_{-\infty}^{t} e^{3\tau} x(\tau) \cos(2(t-\tau)) d\tau \\
&= \int_{-\infty}^{t} e^{-3(t-\tau)} \cos(2(t-\tau)) x(\tau) d\tau \\
&= \int_{-\infty}^{\infty} e^{-3(t-\tau)} \cos(2(t-\tau)) u(t-\tau) x(\tau) d\tau.
\end{aligned}$$

Applying impulse $x(t) = \delta(t)$ to S, we get

$$h(t) = e^{-3t} \cos(2t) u(t).$$

(b)

$$\begin{aligned}
y(t) &= h(t) * x(t) \\
&= \int_{-\infty}^{\infty} e^{-3(t-\tau)} \cos(2(t-\tau)) u(t-\tau) e^{-3\tau} u(\tau) d\tau \\
&= e^{-3t} \int_{0}^{t} \cos(2(t-\tau)) d\tau \\
&= \frac{1}{2} e^{-3t} \sin(2t) u(t)
\end{aligned}$$

Problem 5 (20 pts)

Consider a cascade of two systems $S_{12} = S_1 S_2$.
The first system S_1 is described by:

$$y(t) = \int_{-\infty}^{t} (t-\sigma)x(\sigma)d\sigma,$$

where $x(t)$ and $y(t)$ are the input and the output, respectively. The second system is described by:

$$z(t) = \int_{-\infty}^{t-1} e^{(t-\sigma+1)}y(\sigma)d\sigma,$$

where $y(t)$ and $z(t)$ and the input and the output, respectively.

(a) (5 pts) Find the impulse response function $h_1(t,\tau)$. Is the system S_1 TI or TV?

(b) (5 pts) Find the impulse response function $h_2(t,\tau)$. Is the system S_2 TI or TV?

(c) (10 pts) Find the impulse response function $h_{12}(t,\tau)$ of the cascaded system S_{12}. Is the cascaded system TI or TV?

Solution:

(a)

Rewrite the IPOP relationship of S_1 as:

$$y(t) = \int_{-\infty}^{\infty} (t-\sigma)u(t-\sigma)x(\sigma)d\sigma$$

Applying impulse $x(t) = \delta(t-\tau)$ to S_1, we get

$$h_1(t,\tau) = (t-\tau)u(t-\tau) = h_1(t-\tau).$$

S_1 is a TI system.

(b)

Rewrite the IPOP relationship of S_2 as:

$$z(t) = \int_{-\infty}^{\infty} e^{(t-\sigma+1)} u(t-1-\sigma) y(\sigma) d\sigma$$

Applying impulse $x(t) = \delta(t-\tau)$ to S_2, we get

$$h_2(t,\tau) = e^{(t-\tau+1)} u(t-\tau-1) = h_2(t-\tau).$$

S_2 is a TI system.

(b)

Apply $y(t) = h_1(t,\tau)$ as an input to S_2 to get

$$\begin{aligned}
h_{12}(t,\tau) &= \int_{-\infty}^{t-1} e^{(t-\sigma+1)} h_1(\sigma,\tau) d\sigma \\
&= \int_{-\infty}^{t-1} e^{(t-\sigma+1)} (\sigma-\tau) u(\sigma-\tau) d\sigma \\
&= \int_{\tau}^{t-1} e^{(t-\sigma+1)} (\sigma-\tau) d\sigma \\
&= \int_{\tau}^{t-1} \sigma e^{(t-\sigma+1)} - \int_{\tau}^{t-1} \tau e^{(t-\sigma+1)} d\sigma \\
&= -(t-\tau)e^2 + e^{t-\tau+1} = h_{12}(t-\tau)
\end{aligned}$$

The cascaded system is TI.