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(not sure, it's
Friday 11:00-12:00)

UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE102: SYSTEMS & SIGNALS

Midterm Examination I
February 6, 2018
Duration: 1 hr 50 mins.

INSTRUCTIONS:

- The exam has 6 problems and 14 pages.
- The exam is closed-book.
- One cheat sheet of A4 size is allowed.
- Calculator is NOT allowed.
- Write your discussion session in the top-right corner. ↗↗

Your name ____________________

Student ID ____________________

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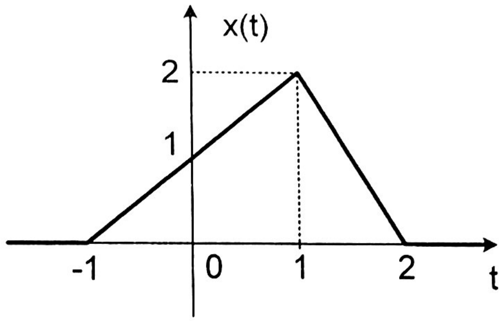
Problem	a	b	c	d	Score	
1	4	4	4		12	12
2	2	2	8		12	12
3	6	2	4	6	18	18
4	8	8			16	16
5	10	6			16	16
6	5	5	6		16	16
Total					90	90

12

Problem 1 (12 pts) Consider the following signal $x(t)$ for (a), (b)



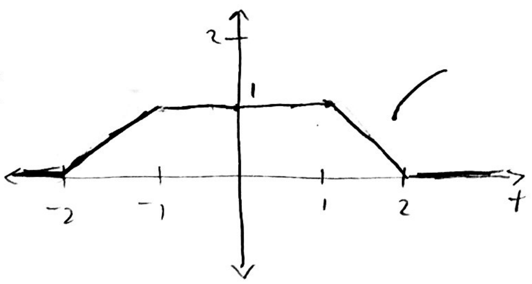
(a) (4 pts) Sketch even and odd decompositions $x_e(t)$ and $x_o(t)$.

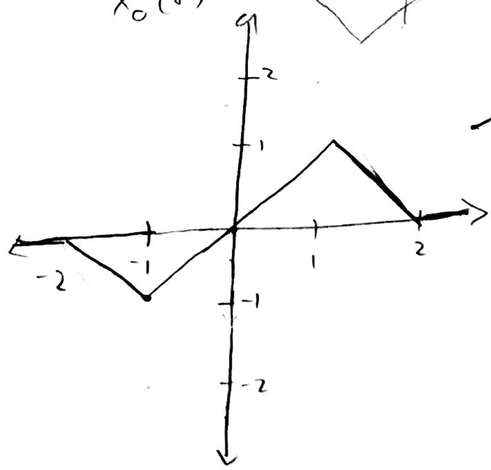
(b) (4 pts) Sketch $x(-2t+3)$.

(c) (4 pts) Sketch $x(t/3+2)$.

a) $x_e(t)$ $x_o(t)$

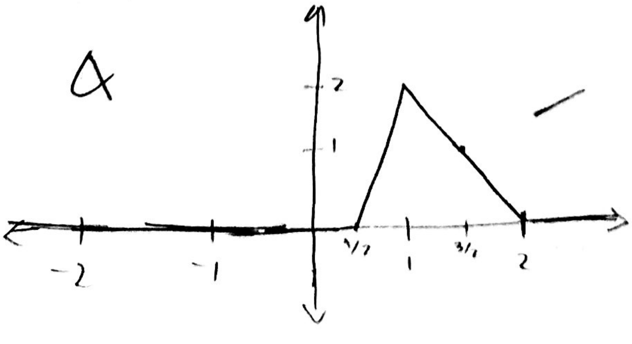
4





b) $x(-2(t-\frac{3}{2}))$

4



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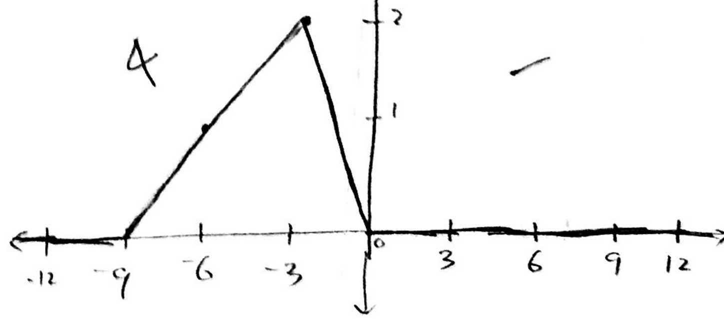
c) $x(\frac{1}{3}(t+6))$

4



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Problem 2 (12 pts) In this problem, we identify system properties from the impulse response function:

$$h(t,\tau) = e^{-(t-\tau)}u(t-\tau)u(t) \tag{1}$$

(a) (2 pts) Is the system TV or TI? Explain.

(b) (2 pts) Is it C or NC? Explain.

(c) (8 pts) Find the output $y(t)$ if the input is $x(t) = (t-2)u(t-2)$.

2 a) The system is TV, b/c $h(t,\tau) \neq h(t-\tau)$

b) The system is C, b/c $h(t,\tau)=0$ for $t<\tau$

2 and $h(t)=0$ for $t<0$

c) $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} (\tau-2)u(\tau-2)\, e^{-(t-\tau)} u(t-\tau)u(t)\, d\tau$

$t-2 \quad e^{\tau-t}$

$1 \quad e^{\tau-t}$

$0 \quad e^{\tau-t}$

$= u(t-2) \int_{2}^{t} (\tau-2)\, e^{-(t-\tau)}\, d\tau$

$= u(t-2)\left[(\tau-2)e^{-(t-\tau)} - e^{-(t-\tau)}\right]_2^t$

8

~~10~~

$= u(t-2)\left(t-2-1-(0-e^{-(t-2)})\right)$

$\boxed{y(t) = u(t-2)\left(e^{-(t-2)} + t - 3\right)}$

18

Problem 3 (18 pts)
Consider IPOP relation for an LTI system S:

$$y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \left[\cos(t)\cos(\tau) + \sin(t)\sin(\tau)\right] x(\tau) d\tau$$

where $x(t)$ and $y(t)$ are input and output of the system, respectively.

(a) (6 pts) Find the impulse response function $h(t)$.

(b) (2 pts) Is the system C or NC? Provide justification.

(c) (4 pts) Is this system BIBO stable? Provide justification.

(d) (6 pts) Find Laplace transform $H(s)$ and ROC.

(Hint: Use the identity $\cos(A \mp B) = \cos(A)\cos(B) + \sin(A)\sin(B)$)

a) $h(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} \left[\cos(t)\cos(\tau) + \sin(t)\sin(\tau)\right] \delta(\tau)\, d\tau$

6 by sifting property $h(t) = e^{-t} \cdot e^{0} (\cos(t) \cdot 1 + \sin(t) \cdot 0)\, u(t)$

$\boxed{h(t) = e^{-t} \cos(t)\, u(t)}$

2 b) The system is C b/c $h(t) = 0$ for $t < 0$

c) $\int_{-\infty}^{+\infty} |h(t)|\, dt = \int_{-\infty}^{+\infty} |e^{-t} \cos(t)\, u(t)|\, dt = \int_{0}^{\infty} |e^{-t} \cos(t)|\, dt$

4 $e^{-t} \approx 0$ as $t \to \infty$, so $e^{-t}\cos(t) \approx 0$ as $t \to \infty$

$\therefore \int_{0}^{\infty} |e^{-t} \cos(t)|\, dt < \infty$ $\boxed{\text{The system is BIBO stable}}$

d) $\cos(t) u(t) \xrightarrow{\mathcal{L}_s} \frac{s}{s^2+1}$

6 by the frequency shift property of laplace transforms: $\mathcal{L}_s\left[e^{-t} \cos(t) u(t)\right] = \frac{s+1}{(s+1)^2 + 1}$

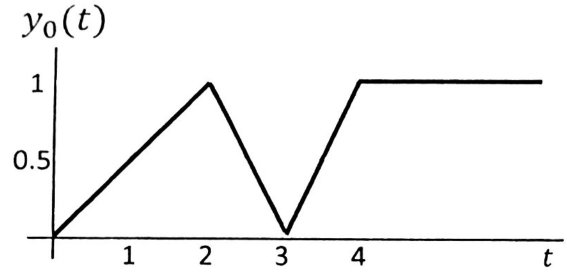
6

$\boxed{H(s) = \frac{s+1}{(s+1)^2+1}}$

$H(s) = \int_0^{\infty} e^{-st} e^{-t} \cos(t)\, dt = \int_0^{\infty} e^{-(s+1)t} \cos(t)\, dt$

$\boxed{\text{ROC: } \boxed{Re\{s\} > -1}}$ otherwise is an indefinite integral

16 **Problem 4** (16 pts) Consider an LTI system S_0 with input $x_0(t)$ and the impulse response function $h_0(t)$. The corresponding output $y_0(t)$ is shown below:



(a) (8 pts) Consider an LTI system S_1 with input $x_1(t) = x_0(t+2)$ and IRF $h_1(t) = h_0(t-1)$. Express the output $y_1(t)$ as a function of $y_0(t)$ and then plot it.

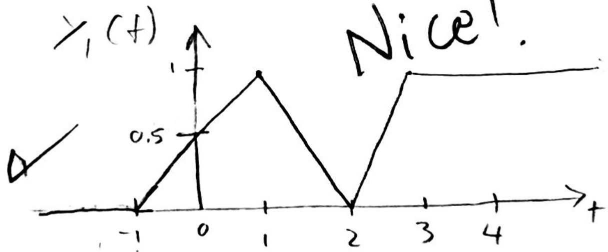
(b) (8 pts) Consider an LTI system S_2 with input $x_2(t) = x_0(-t)$ and IRF $h_2(t) = h_0(-t)$. Express the output $y_2(t)$ as a function of $y_0(t)$ and then plot it.

by the time shift property of laplace transformations

a) $X_1(s) = e^{2s} X_0(s)$ $\quad H_1(s) = e^{-s} H_0(s)$

$Y_1(s) = e^{s} X_0(s) H_0(s) = e^{s} Y(s)$ $\quad \boxed{y_1(t) = y_0(t+1)}$

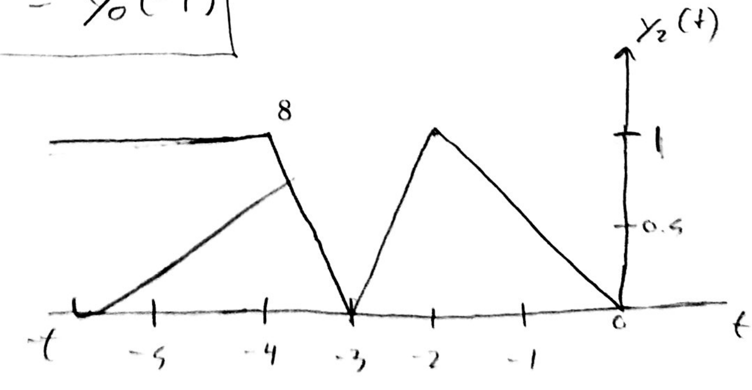
Nice!



b) $X_2(s) = X_0(-s)$ $\quad H_2(s) = H_0(-s)$ $\quad Y_2(s) = X_0(-s) H_0(-s) = Y_0(-s)$

$\boxed{y_2(t) = y_0(-t)}$

8



16

Problem 5 (16 pts) Consider a cascade combination of two systems S_1 and S_2: $x(t)$ is input to S_1 and $y(t)$ is the output, while the output of S_2 is $z(t)$.

$$x(t) \to [S_1] \to y(t) \to [S_1] \to z(t)$$

The IPOP relation for S_1 and S_2 is:

$$S_1 : y(t) = e^{-t}x(t)u(t),$$

$$S_2 : z(t) = \int_0^t e^{-(t-\sigma)}y(\sigma)u(\sigma)d\sigma.$$

(a) (10 pts) Compute impulse response function $h_{12}(t, \tau)$ of the cascaded system S_1S_2.

(b) (6 pts) Compute the output $z(t)$ if the input is $x(t) = e^{-3t}[u(t) - u(t-3)]$. $\leftarrow$ $[u(t) - u(t-3)] = u(t)\,u(3-t)$

a) $h_1(t,\tau) = e^{-t}\delta(t-\tau)u(t)$

$h_2(t,\tau) = \int_0^t e^{-(t-\sigma)}\delta(\sigma-\tau)u(\sigma)\,d\sigma = u(\tau)u(t-\tau)e^{-(t-\tau)}$

$h_{12}(t,\tau) = h_1(t,\tau) * h_2(t,\tau) = \int_{-\infty}^{+\infty} e^{-\sigma}\delta(\sigma-\tau)u(\sigma)u(\sigma)u(t-\sigma)e^{-(t-\sigma)}\,d\sigma$

$= u(t)\int_0^t e^{-\sigma}\delta(\sigma-\tau)e^{-(t-\sigma)}\,d\sigma = u(t)e^{-t}\int_0^t \delta(\sigma-\tau)\,d\sigma$

$$\boxed{h_{12}(t,\tau) = e^{-t}u(t-\tau)u(\tau)}$$

b) $z(t) = \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)u(3-\tau)e^{-t}u(t-\tau)u(\tau)\,d\tau$

$$z(t) = \begin{cases} e^{-t}\int_0^3 e^{-3\tau}\,d\tau & \text{for } t \geq 3 \quad = e^{-t}\cdot -\frac{1}{3}(e^{-9}-1) \\ e^{-t}\int_0^t e^{-3\tau}\,d\tau & \text{for } 0 \leq t < 3 \quad = e^{-t}\cdot -\frac{1}{3}(e^{-3t}-1) \\ 0 & \text{for } t < 0 \quad = 0 \end{cases}$$

10

$$z(t) = \begin{cases} -\frac{1}{3}e^{-t}(e^{-9}-1) & \text{for } t \geq 3 \\ -\frac{1}{3}e^{-t}(e^{-3t}-1) & \text{for } 0 \leq t < 3 \\ 0 & \text{for } t < 0 \end{cases}$$

[redacted]

checking a)
with Laplace

$X(s) = 2(\frac{1}{s} - e^{-2s}\frac{1}{s})$

$Y_1(s) = 2(e^{-s}\frac{1}{s} - e^{-3s}\frac{1}{s}) \quad 2(u(t-1) - u(t-3))$

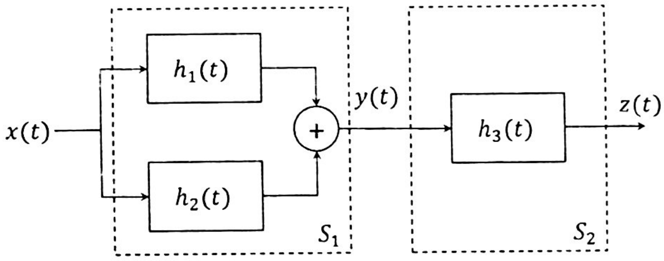
$Y_2(s) = 2(\frac{1}{s}e^{-2s} - \frac{1}{s}e^{-4s})$

16

Problem 6 (16 pts)
Consider a cascaded LTI system S_1S_2 as follows

$$x(t) \to [S_1] \to y(t) \to [S_2] \to z(t)$$

The cascade system is shown below.



$0 < \tau + 1 < 2$

where $h_1(t) = \delta(t-1)$, $h_2(t) = \delta(t-2)$, and $h_3(t) = \delta(t-1) - \delta(t-2) + \delta(t-3)$.
Let $x(t) = 2(u(t) - u(t-2))$, then

(a) (5 pts) Find the IPOP between $x(t)$ and $y(t)$. Plot $y(t)$.

(b) (5 pts) Write the impulse response of the cascade system S_1S_2.

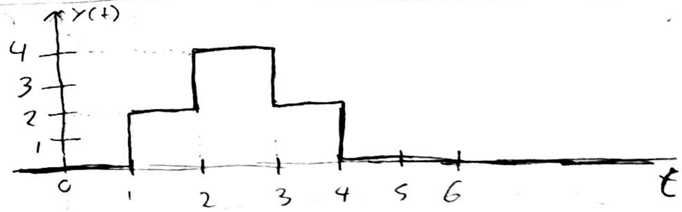
(c) (6 pts) Compute and plot $z(t)$ for the specified input.

a) $y(t,\tau) = \int_{-\infty}^{+\infty} 2(u(\tau) - u(\tau-2))\,\delta(t-1-\tau)\,d\tau + \int_{-\infty}^{\infty} 2(u(\tau) - u(\tau-2))\,\delta(t-\tau-2)\,d\tau$

sifting property $\to$ $\boxed{y(t) = 2(u(t-1) - u(t-3)) + 2(u(t-2) - u(t-4))}$

$t > t-1$ holds true for all t, no $u(\sim)$ to multiply $y(t)$ by

confirmed by Laplace



b) $h_{12}(t) = \mathcal{L}_s^{-1}\left[(H_1(s) + H_2(s))H_3(s)\right]$

$H_1(s) = e^{-s}$ $\quad H_2(s) = e^{-2s}$ $\quad H_3(s) = e^{-s} - e^{-2s} + e^{-3s}$

$H_{12}(s) = (e^{-s} + e^{-2s})(e^{-s} - e^{-2s} + e^{-3s}) = e^{-2s} - e^{-3s} + e^{-4s} + e^{-3s} - e^{-4s} + e^{-5s}$

$H_{12}(s) = e^{-2s} + e^{-5s}$ $\quad \boxed{h_{12}(t) = \delta(t-2) + \delta(t-5)}$

c) on back $\to$

c) $X(s) = 2\left(\frac{1}{s} - e^{-2s}\frac{1}{s}\right)$

$H_{12}(s) = e^{-2s} + e^{-5s}$

$Z(s) = 2\left(\frac{1}{s} - e^{-2s}\frac{1}{s}\right)\left(e^{-2s} + e^{-5s}\right) = 2\left(\frac{1}{s}e^{-2s} - \frac{1}{s}e^{-4s} + \frac{1}{s}e^{-5s} - \frac{1}{s}e^{-7s}\right)$

tv $z(t) = 2\left(u(t-2) - u(t-4) + u(t-5) - u(t-7)\right)$