EE 101B
Winter 2019
Monday, February 11, 2019

Problem 1 (70 points)

A 1 GHz linearly polarized uniform plane wave is propagating inside a semi-infinite dielectric medium with a relative permittivity of 16, relative permeability of 1, and conductivity of 0.

a) Assuming that the wave is normally incident on the interface between the dielectric and air at $x = 0$, determine the portion of the power that is transmitted from the dielectric to air and the portion of the power that is reflected back inside the dielectric.

b) Design a coating layer (determine the thickness and relative permittivity of the coating) at the interface between air and the dielectric medium to maximize the transmitted power from dielectric to air for the wave discussed in part (a).

c) Determine the portion of the power that is transmitted from the dielectric to air for the design in part (b).

d) Design a coating layer (determine the thickness and relative permittivity of the coating) at the interface between air and the dielectric medium to minimize the transmitted power from dielectric to air for the wave discussed in part (a).

e) Determine the portion of the power that is transmitted from the dielectric to air for the design in part (d).

f) If the incident wave has an oblique incidence at the air-dielectric interface, determine the incidence angle(s) and polarization of the incident wave to maximize the transmitted power from dielectric to air.

g) Determine the portion of the power that is transmitted from the dielectric to air for the design in part (f).

h) For the wave polarization determined in part (f), determine the incident angle(s) that minimize the transmitted power from dielectric to air.

i) Determine the portion of the power that is transmitted from the dielectric to air for the design in part (h).

a) $\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{4} = 30\pi \quad \eta_2 = \eta_0 = 120\pi$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 30\pi}{120\pi + 30\pi} = 0.6$

$\frac{S^r_{av}}{S^i_{av}} = \frac{\Gamma^2}{\eta_1} = \frac{0.6^2}{} = 0.36$

$\frac{S^t_{av}}{S^i_{av}} = 1 - \frac{S^r_{av}}{S^i_{av}} = 0.64$

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b) $\eta_c = \sqrt{\eta_1 \eta_2} = 60\pi \Rightarrow \epsilon_r = 4 \Rightarrow u_p = \frac{c}{\sqrt{\epsilon_r}} = 1.5\times10^8 \Rightarrow \lambda = \frac{u_p}{f} = \frac{1.5\times10^8}{1\times10^9} = 0.15\text{m}$

$d = \frac{\lambda}{4} = \frac{0.15}{4} = 0.0375\text{m}$ or any $\frac{\lambda}{4} + \frac{n}{4}\lambda$ (quarter wavelength)

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c) $\eta_{in} = \eta_1 \Rightarrow$ no reflection, All power got transmitted, $S^t_{av}/S^i_{av} = 1$

d) $\eta_{in} = \eta_1 \quad \eta_{in} = \eta_c \frac{\eta_2 + j\eta_c \tan(k_2 d_2)}{\eta_c + j\eta_2 \tan(k_2 d_2)} = \eta_1$

$|\Gamma| = \left|\frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1}\right|$ to be as large as possible.

$\eta_{in} \to \infty$ or $\eta_{in} = 0$

$\eta_c \to \infty$, $\epsilon_r \to 0$ (not possible)

$\eta_c \to 0$, $\epsilon_r \to$ large enough.

d) $|\Gamma| = \left|\frac{y_{in} - y_1}{y_{in} + y_1}\right|$ to be as large as possible $(|\Gamma| \to 1)$, therefore, we could make y_{in} very large. or make it 0.

$y_{in} = y_c \frac{y_3 + j y_c \tan(\,)}{y_c + j y_3 \tan(\,)}$ $y_c = \sqrt{2}\frac{\alpha}{\sigma} e^{j45^\circ} = \sqrt{2} e^{j45^\circ} \sqrt{\frac{2\pi u}{\sigma}}$, we could use good conductor, $\sigma \to \infty$

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s.t. $y_c = 0$.

e) perfect conductor act as a perfect reflector, $\frac{S^t}{S_i} = 0$ then.

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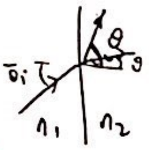
f) Maximize transmission.



$n_1 = \sqrt{\epsilon_r} = 4$

$n_2 = 1$

∥ TM polarization. $\sin\theta_B = \frac{1}{\sqrt{1+(\frac{n_1}{n_2})^2}}$ $\theta_B = 14.0362^\circ$

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g) All power transmitted, $\frac{S^t}{S^i} = 1$.

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h) Total inflection.

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$\frac{\sin\theta_c}{1} = \frac{1}{4}$ $\theta_c = 14.4775^\circ$ $\theta > \theta c$

i) $\frac{S^t}{S^i} = 0$.

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$\frac{1}{2}\{\tilde{E} \times H^{*}\} = \frac{1}{2}\frac{E_0^2}{\eta}(\hat{y}-j\hat{z})(z-j\hat{y})e^{-2\alpha x}$ $\hat{z}\times\hat{y} = -\hat{x}$

$= \frac{1}{2}\frac{|E_0|^2}{\eta}(\hat{x}+\hat{x})$

$= \frac{1}{2}\frac{E_0^2}{\eta} \cdot 2\hat{x}$

$\lambda = 800 \times 10^{-9}$ $f = \frac{u_p}{\lambda}$

Problem 2 (30 points)

A right-hand circularly polarized plane wave with an 800 nm wavelength and an average power density on 1 kW/m^2 is propagating in $+x$ direction in a nonmagnetic medium with a relative permittivity of 2.56 and conductivity of 1 S/m.

$\epsilon_r = 2.56$ $\sigma = 1$ $\mu = 1$

(a) Determine the propagation constant, attenuation constant, intrinsic impedance, and phase velocity of the wave propagating in this medium.

(b) Write expressions for the electric field and magnetic field phasors.

(c) Write time-domain electric field and magnetic field.

a) $\frac{\epsilon''}{\epsilon} = \frac{\sigma}{\omega\epsilon} = 1$

800nm in the medium (not air)

$\lambda = \frac{2\pi}{\beta} \Rightarrow \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{800\times10^{-9}} = 7.85\times10^{6}$ rad/m

Assume lowloss : $u_p = \frac{c}{\sqrt{\epsilon_r}} = 1.875\times10^{8}$ m/s

$\omega = 2\pi\frac{u_p}{\lambda} = 2.34\times10^{14}$ rad/s

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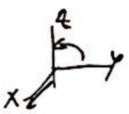
$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \frac{1}{2\pi \times 2.34\times10^{14}\times 8.85\times10^{-12}} << 1$, assumption is correct.

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}} = 117.8$ rad/m



$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{2.56}} = 75\pi$

b) $\tilde{E} = E_0(\hat{y} - j\hat{z})e^{-j\beta x}e^{-\alpha x}$

$\tilde{H} = \frac{E_0}{\eta}(\hat{z} + j\hat{y})e^{-\alpha x}e^{-j\beta x}$

at $z=0$ $S_{av} = \frac{E_0^2}{2\eta}\cdot 2 = 1K$ $E_0 = 485$ V/m

$\tilde{E} = 485(\hat{y}-j\hat{z})e^{-117.8x}e^{-j7.85\times10^{6}x}$ V/m

$\tilde{H} = \frac{485}{75\pi}(\hat{z}+j\hat{y})e^{-117.8x}e^{-j7.85\times10^{6}x}$ A/m

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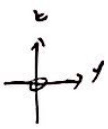

c) $\omega = 2.34\times10^{14}$.

convert back to time domain. complex number (imaginary part) should be taken into the phase.

$E = E_0(\hat{y}-j\hat{z})e^{-\alpha x}\cos(\omega t - \beta x)$ ...? $E = 485(\hat{y}-j\hat{z})\cos(2\pi\times 2.34\times10^{14}t - 7.85\times10^{6}x)e^{-117.8x}$ V/m

$H = \frac{E_0}{\eta}(\hat{z}+j\hat{y})e^{-\alpha x}\cos(\omega t - \beta x)$ ...? $H = \frac{485}{75\pi}(\hat{z}+j\hat{y})\cos(2\pi\times 2.34\times10^{14}t - 7.85\times10^{6}x)e^{-117.8x}$ A/m

$$\begin{cases} E = E_0(\hat{y}\cos(\omega t-\beta x) - \hat{z}\cos(\omega t - \beta x + \frac{\pi}{2}))e^{-\alpha x} \\ H = \frac{E_0}{\eta}(\hat{z}\cos(\omega t-\beta x) + \hat{y}\cos(\omega t-\beta x+\frac{\pi}{2}))e^{-\alpha x} \end{cases}$$