Problem 1 (40 points)

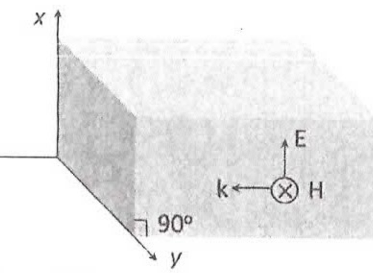
An x-polarized optical beam is propagating in z direction inside a semi-infinite dielectric slab (ε_r = 16) placed in free space. The dielectric slab has rectangular sidewalls (as shown below) with dimensions much larger than the optical beam size.

a) If the optical beam is normally incident on one of the slab sidewalls in the xy-plane, determine the propagation direction and portion of the optical power that is transmitted from the slab to air.

(1). Normal incidence. propagation direction : $+z$

(2). $\eta_D = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{16}} = 30\pi$.

$\Gamma = \frac{\eta_0 - \eta_D}{\eta_0 + \eta_D} = \frac{120\pi - 30\pi}{120\pi + 30\pi} = 0.6$. $T = 1 - R = 1 - |\Gamma|^2 = 0.64$



Thus, 64% of the optical power is transmitted from the slab to air.

b) If there is a possibility of polishing the slab sidewalls, find the slab sidewall angel, α, at which maximum optical power is transmitted from the slab to air.

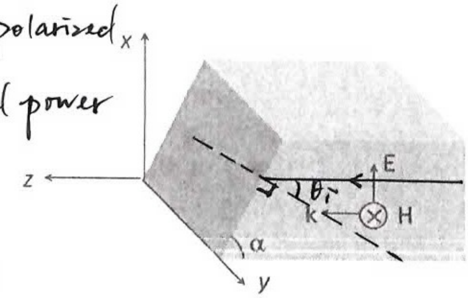
plane of incidence : $x-z$ plane. the beam is x-polarized

$\Rightarrow$ TM wave. To get maximum transmitted optical power

Set $\theta_i = \theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_0}{\varepsilon_0 \varepsilon_r}} = \tan^{-1}\sqrt{\frac{1}{16}} = 14.04°$

$\alpha = 90° - \theta_i = 75.96°$ (1.326 rad)

(check : critical angle $\theta_c = \arcsin\frac{1}{\sqrt{\varepsilon_r}} = 14.47° > \theta_i$)

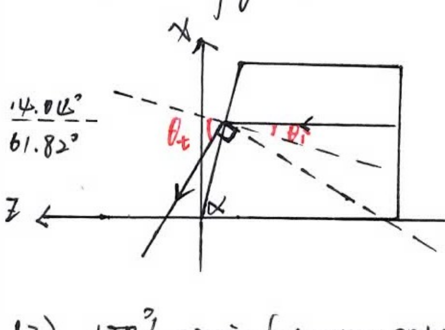


c) For part (b), determine the propagation direction and portion of the optical power that is transmitted from the slab to air.

d) For the slab sidewall angel, α, calculated in part (b) and assuming a y-polarized optical beam propagating in air in -z direction toward the slab, determine the propagation direction and portion of the optical power that is transmitted from air to the slab.

(c). (1) $n_D \sin\theta_i = n_0 \sin\theta_t \Rightarrow \theta_t = 75.96°$

shown in the figure.

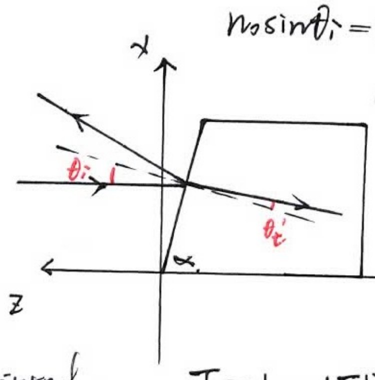


(2). 100% optical power gets transmitted.

(d) Now the beam is y-polarized. $\Rightarrow$ TE wave.

$n_0 \sin\theta_i = n_D \sin\theta_t' \Rightarrow \sin\theta_t' = \frac{1}{4\sqrt{17}} \Rightarrow \theta_t' \doteq 3.47°$

as shown in the figure.



$\Gamma = \frac{\eta_D \cos\theta_i - \eta_0 \cos\theta_t'}{\eta_D \cos\theta_i + \eta_0 \cos\theta_t'}$

$= \frac{120 - 30\sqrt{271}}{120 + 30\sqrt{271}} = -0.609$

$T = 1 - |\Gamma|^2 \doteq 0.629$. Thus, 62.9% optical power is transmitted

Problem 2 (35 points)

A plane wave traveling in medium 1 with $\varepsilon_r = 9$ is normally incident upon medium 2 with $\varepsilon_r = 2.25$. Both media are made up of non-magnetic and non-conducting materials. If the electric field of the incident wave is given by

$$E_i = \hat{z}1.5 \sin\left(2\pi \times 10^9 t - 20\pi x\right)$$

a) Obtain the time domain expressions for the electric and magnetic fields in each of the two media.

b) Determine the average power densities of the incident, transmitted, and reflected waves.

(1a) Medium #1. Incident + Reflected Wave.

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r1}}} = \frac{120\pi}{\sqrt{9}} = 40\pi.\quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}} = \frac{120\pi}{\sqrt{2.25}} = 80\pi.\quad \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{80\pi-40\pi}{80\pi+40\pi} = \frac{1}{3}$$

$$\vec{H_i} = \frac{1}{\eta_1}\hat{k}_i \times \vec{E_i} = \frac{1}{40\pi}\hat{x}\times\hat{z}1.5\sin(2\pi\times10^9 t - 20\pi x) = -\hat{y}\frac{3}{80\pi}\sin(2\pi\times10^9 t - 20\pi x)$$

$$\vec{E_r} = \hat{z}\frac{1}{3}\times1.5\sin(2\pi\times10^9 t + 20\pi x) = \hat{z}0.5\sin(2\pi\times10^9 t + 20\pi x)$$

$$\vec{H_r} = \frac{1}{\eta_1}\hat{k}_r \times \vec{E_r} = \frac{1}{40\pi}(-\hat{x})\times\hat{z}0.5\sin(2\pi\times10^9 t + 20\pi x) = \hat{y}\frac{1}{80\pi}\sin(2\pi\times10^9 t + 20\pi x)$$

In medium #1.

$$\vec{E_1} = \vec{E_i} + \vec{E_r} = \hat{z}1.5\sin(2\pi\times10^9 t - 20\pi x) + \hat{z}0.5\sin(2\pi\times10^9 t + 20\pi x)$$

$$\vec{H_1} = \vec{H_i} + \vec{H_r} = -\hat{y}\frac{3}{80\pi}\sin(2\pi\times10^9 t - 20\pi x) + \hat{y}\frac{1}{80\pi}\sin(2\pi\times10^9 t + 20\pi x)$$

Medium #2. Transmitted Wave.

$$\tau = 1+\Gamma = \frac{4}{3}.\quad k_2 = \frac{2\pi\sqrt{\varepsilon_{r2}}}{\lambda} = k_1\frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} = \frac{20\pi}{3}\times1.5 = 10\pi$$

In medium #2.

$$\vec{E_2} = \vec{E_t} = \frac{4}{3}\hat{z}1.5\sin(2\pi\times10^9 t - 10\pi x) = \hat{z}2\sin(2\pi\times10^9 t - 10\pi x)$$

$$\vec{H_2} = \vec{H_t} = \frac{1}{\eta_2}\hat{k}_t\times\vec{E_t} = \frac{1}{80\pi}\hat{x}\times\hat{z}2\sin(2\pi\times10^9 t - 10\pi x) = -\frac{\hat{y}}{40\pi}\sin(2\pi\times10^9 t - 10\pi x)$$

(1b)

$$\vec{S_{in}} = \hat{x}\frac{|\tilde{E_i}|^2}{2\eta_1} = \hat{x}\frac{2.25}{2\times40\pi} \doteq \hat{x}\,8.95\times10^{-3}$$

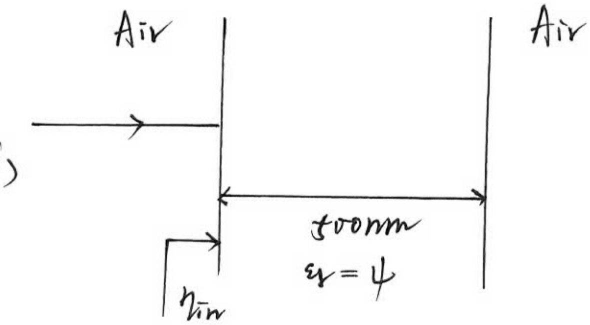
$$\vec{S_{ref}} = -\hat{x}\frac{|\tilde{E_r}|^2}{2\eta_1} = -\hat{x}\frac{0.25}{2\times40\pi} \doteq -\hat{x}\,9.95\times10^{-4}$$

$$\vec{S_{Tr}} = \vec{S_{in}} + \vec{S_{ref}} = \hat{x}\frac{|\tilde{E_t}|^2}{2\eta_2} = \hat{x}\frac{4}{2\times80\pi} \doteq \hat{x}\,7.96\times10^{-3}$$

Problem 3 (25 points)

An optical beam at a 1550 nm wavelength is normally incident on a non-magnetic dielectric slab with a thickness of 500 nm and $\varepsilon_r = 4$. Both sides of the dielectric slab are filled with air. Determine the percentage of the optical power that is reflected back from the slab.

Solve the problem with "Transmission-line analogue".



$$\eta_d = \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2}$$

$$\beta L = \frac{2\pi\sqrt{\varepsilon_r}}{\lambda}\cdot L = \frac{2\pi\cdot\sqrt{4}}{1550}\cdot 500 = \frac{40}{31}\pi \text{ (rad)}$$

$$\eta_{in} = \eta_2 \frac{\eta_0\cos\beta L + j\eta_2\sin\beta L}{\eta_2\cos\beta L + j\eta_0\sin\beta L}$$

$$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} = \frac{\eta_2\eta_0\cos\beta L + j\eta_2^2\sin\beta L - \eta_2\eta_0\cos\beta L - j\eta_0^2\sin\beta L}{\eta_2\eta_0\cos\beta L + j\eta_2^2\sin\beta L + \eta_2\eta_0\cos\beta L + j\eta_0^2\sin\beta L}$$

$$= \frac{j(\eta_2^2 - \eta_0^2)\sin\beta L}{2\eta_2\eta_0\cos\beta L + j(\eta_2^2 + \eta_0^2)\sin\beta L}$$

$$= \frac{-3j\eta_0^2}{\frac{4\eta_0^2}{\tan\beta L} + j5\eta_0^2} = \frac{-3j}{\frac{4}{\tan\beta L} + 5j}$$

With $\tan\beta L \doteq 1.29$. $\Gamma = -0.433 - 0.269j = 0.51 e^{-j2.58} = 0.51 e^{-j148^\circ}$

$\Rightarrow |\Gamma|^2 = |0.51|^2 = 0.26$.

$\Rightarrow$ 26% of the optical power is reflected back.