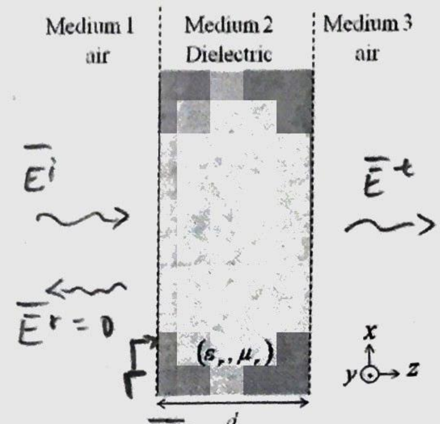
Problem 1: (35 Points)

A 3 GHz right-hand circularly polarized wave with an average power density of 1 W/m$^2$ is propagating in medium 1 along the z direction. The wave is normally incident at the boundary of air with a dielectric medium with thickness $d = 25$ mm and dielectric parameters:

$\varepsilon_r = 4.0,\ \sigma = 0,\ \mu_r = 1$




$$\lambda_d = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{100}{\sqrt{4}} = 50\,mm$$

$$d = 25\,mm = \frac{1}{2}\lambda_d$$

$\Rightarrow$ Pefect transmission $\Rightarrow \Gamma = 0 \Rightarrow \bar{E}^r = 0$

15 a) Write the phasor and time-domain expressions for the overall electric field and magnetic field in medium 1.

5 b) Determine the polarization of the wave reflected to medium 1.

No reflected field

5 c) Determine the polarization of the wave transmitted to medium 3.

RHC

5 d) Determine the average power density of the portion of the wave that is transmitted to medium 3.

$S_{av} = 1\,W/m^2$

5 e) Determine the polarization of the transmitted wave to medium 3, if the dielectric has a non-zero conductivity $\mu_r = 1,\ \varepsilon_r = 4.0,\ \sigma \neq 0$.

RHC still.

a) $$S_{AV} = \frac{|\tilde{E}_i|^2}{2\eta_0} = 1\,W/m^2$$

where for RHC, $\tilde{E}^i = (\hat{x} - j\hat{y}) A e^{-jk_0 z}$

$$\Rightarrow \frac{2A^2}{2\eta_0} = 1 \quad \Rightarrow \quad A = \sqrt{\eta_0} = 19.4$$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{2\pi}{0.1} = 20\pi\ m^{-1}$$

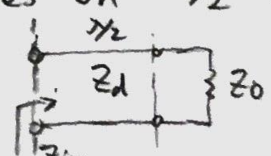
$$\tilde{E}_1 = \tilde{E}^i = (\hat{x} - j\hat{y})\sqrt{\eta_0}\, e^{-j20\pi z} \quad V/m$$

$$\tilde{H}_1 = \frac{1}{\eta_0}\hat{k}\times\tilde{E}_1 = (\hat{y} + j\hat{x})\frac{1}{\sqrt{\eta_0}}e^{-j20\pi z} \quad A/m$$

$$\bar{E}_1 = \sqrt{\eta_0}\left[\hat{x}\cos(6\pi\cdot 10^9 t - 20\pi z) + \hat{y}\sin(6\pi\cdot 10^9 - 20\pi z)\right] \ V/m$$

$$\bar{H}_1 = \frac{1}{\sqrt{\eta_0}}\left[\hat{y}\cos(6\pi\cdot 10^9 t - 20\pi z) - \hat{x}\sin(6\pi\cdot 10^9 - 20\pi z)\right] \ A/m$$

* Notes on $\lambda/2$ dielectric slab




Transmission line

$$Z_{in} = Z_d \cdot \frac{Z_0 + jZ_d \cancel{\tan\beta d}^{\,0}}{Z_d + jZ_0 \cancel{\tan\beta d}^{\,0}} \quad (\beta d = \pi)$$

$$= \cancel{Z_d}\cdot Z_0 / \cancel{Z_d} = Z_0$$

Since $Z_{in} = Z_0$, there's no reflection.

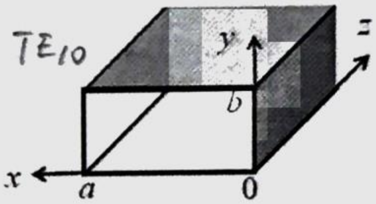
Problem 2: (35 Points)

15 a) Design a hollow metallic rectangular waveguide with waveguide cross-section dimensions ($a = 2b$) which is single-mode with a wave impedance of 500 Ω at 1 GHz.

10 b) Determine group velocity, phase velocity, propagation constant and wavelength of a 1 GHz wave propagating inside the waveguide you designed.

10 c) What are the possible propagation modes at 2 GHz for the waveguide you designed?

a) Design a waveguide whose dominant mode is $TE_{10}$




$$Z_{TE} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{k\sqrt{1-(\frac{f_c}{f})^2}} = \frac{\eta_0}{\sqrt{1-(\frac{f_c}{f})^2}} = 500\,\Omega$$

$f = 1\text{GHz}$

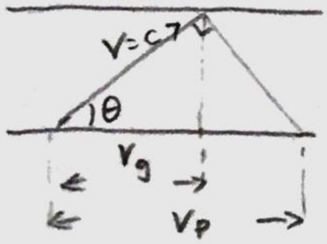
$\Rightarrow f_c = 656.9\text{ MHz}$

For $TE_{10}$, $f_c = \frac{c}{2a}$. $\quad a = \frac{c}{2f_c} = \frac{3\cdot10^8}{2\cdot656.9\cdot10^6} = \boxed{0.228\text{m}}$

$b = a/2 = \boxed{0.114\text{m}}$

b) $V_p = \frac{\omega}{\beta} = \frac{Z_{TE}}{\mu} = 3.98\cdot10^8\text{ m/s}$

Since $V_p\cdot V_g = c^2$, $\quad V_g = \frac{c^2}{V_p} = 2.26\cdot10^8\text{m/s}$





$\beta = \frac{\omega}{V_p} = \frac{2\pi\cdot10^9}{3.98\cdot10^8} = 15.79\text{m}^{-1}$ $\quad \lambda = \frac{2\pi}{\beta} = 0.398\text{m}$

$V_p = \frac{V}{\cos\theta}$ $\quad V_g = V\cos\theta$

c) $$(f_c)_{mn} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{(\frac{m\pi}{a})^2+(\frac{n\pi}{b})^2}$$

$$= \frac{c}{2}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$$

$$= \frac{c}{2}\sqrt{(\frac{m}{2b})^2+(\frac{n}{b})^2}$$

$$= \frac{c}{4b}\sqrt{m^2+4n^2}$$

| mode | $f_c$ / GHz |
|---|---|
| $TE_{10}$ | 0.657 |
| $TE_{20}$ | 1.314 |
| $TE_{01}$ | 1.314 |
| $TM_{11}$ | 1.471 |
| $TE_{11}$ | 1.471 |
| $TM_{21}$ | 1.861 |
| $TE_{21}$ | 1.861 |
| $TE_{30}$ | 1.974 |

Problem 3: (30 Points)

A 300 GHz, TE-polarized electromagnetic wave with average power density of 1 W/m$^2$ is incident at the boundary of a silicon wafer at 45° incident angle. ($\mu_{si} = \mu_0$, $\varepsilon_{r-Si} = 11.8$, $\sigma_{Si} \cong 0.01\,\text{S/m}$)

a) Determine the propagation constant, attenuation constant, intrinsic impedance, and phase velocity of the wave in silicon.

b) Write the phasor expressions for electric field and magnetic fields of the incident and reflected waves in the air and transmitted wave into silicon (you can choose arbitrary coordinates for your solution).

c) Determine the average power density and polarization of the transmitted wave into silicon substrate.

a) $\varepsilon'' = \frac{\sigma}{\omega} = \frac{0.01}{2\pi \cdot 300 \cdot 10^9} = 5.3 \cdot 10^{-15} << \varepsilon' = 11.8 \cdot \varepsilon_0$

$\Rightarrow$ Good dielectric

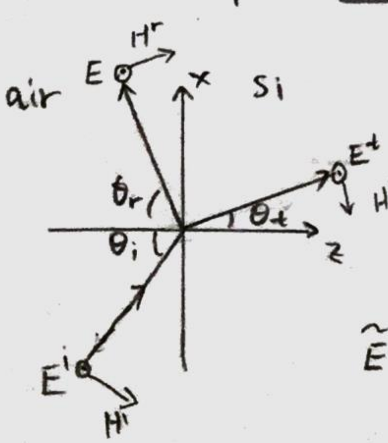
$\beta_{si} = \omega\sqrt{\mu\varepsilon} = 2\pi \cdot 300 \cdot 10^9 \sqrt{4\pi \cdot 10^{-7} \cdot 11.8 \cdot 8.854 \cdot 10^{-12}} = \underline{2.16 \cdot 10^4 \ m^{-1}}$

$\alpha_{si} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{0.01}{2}\sqrt{\frac{4\pi \cdot 10^{-7}}{11.8 \cdot 8.854 \cdot 10^{-12}}} = \underline{0.548 \ Np/m}$

$\eta_{si} = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \underline{109.7\ \Omega}$

$V_p = \frac{\omega}{\beta} = \underline{8.73 \cdot 10^7 \ m/s}$

b)




$\theta_i = \theta_r = 45°$. $\sqrt{\varepsilon_0}\sin\theta_i = \sqrt{\varepsilon_r}\sin\theta_t$

$\Rightarrow \theta_t = 11.9°$

$S_{av} = \frac{1}{2}|E_i|^2/\eta_0 \Rightarrow E_i = \sqrt{2\eta_0} = 27.46\ V/m$

$\tilde{E}^i = \hat{y}\, E_i e^{-j\beta_0(x\sin\theta_i + z\cos\theta_i)}$

$= \hat{y}\, 27.46\, e^{-j2000\pi(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}z)}\ V/m$

$\tilde{H}^i = (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\frac{E_i}{\eta_0} e^{-j\beta_0(x\sin\theta_i + z\cos\theta_i)}$

$= (-\hat{x}\frac{\sqrt{2}}{2} + \hat{z}\frac{\sqrt{2}}{2})\, 0.073\, e^{-j2000\pi(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}z)}\ A/m$

$\Gamma = \frac{\eta_{si}\cos\theta_i - \eta_0\cos\theta_t}{\eta_{si}\cos\theta_i + \eta_0\cos\theta_t} = \frac{109.7\cos 45° - 377\cos 11.9°}{109.7\cos 45° + 377\cos 11.9°} = -0.65$

$$T = \frac{2\eta_{si}\cos\theta_i}{\eta_{si}\cos\theta_i + \eta_0\cos\theta_t} = 0.35 \quad \text{or} \quad T = 1 + \Gamma = 0.35$$

$$\tilde{E}^r = \hat{y}\,\Gamma E^i\, e^{-j\beta_0(x\sin\theta_r - z\cos\theta_r)}$$
$$= \hat{y}(-17.9)\, e^{-j2000\pi(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}z)} \text{ V/m}$$

$$\tilde{H}^r = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)\frac{\Gamma E^i}{\eta_0} e^{-j\beta_0(x\sin\theta_r - z\cos\theta_r)}$$
$$= (\hat{x}\tfrac{\sqrt{2}}{2} + \hat{z}\tfrac{\sqrt{2}}{2})\underbrace{\left(-\frac{17.9}{377}\right)}_{-0.047} e^{-j2000\pi(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}z)} \text{ A/m}$$

$$\tilde{E}^t = \hat{y}\, TE_i\, e^{-\alpha(x\sin\theta_t + z\cos\theta_t)} e^{-j\beta_{si}(x\sin\theta_t + z\cos\theta_t)}$$
$$= \hat{y}\; 9.6\, e^{-0.548(0.206x + 0.979z)} e^{-j\,2.16\cdot10^4(0.206x + 0.979z)} \text{ V/m}$$

$$\tilde{H}^t = (-\hat{x}\cos\overset{11.9^\circ}{\theta_t} + \hat{z}\sin\overset{11.9^\circ}{\theta_t})\frac{TE_i}{\eta_{si}} e^{-\alpha(x\sin\theta_t + z\cos\theta_t)} e^{-j\beta_{si}(x\sin\theta_t + z\cos\theta_t)}$$
$$= (-\hat{x}\,0.979 + \hat{z}\cdot 0.206)\underbrace{\frac{9.6}{109.7}}_{0.088} e^{-0.548(0.206x + 0.979z)} e^{-j\,2.16\cdot10^4(0.206x + 0.979z)} \text{ A/m}$$

c. TE polarized

$$S_{tav} = \frac{|E^t|^2}{2\eta_2}$$
$$= \frac{|T|^2 E_i^2}{2\eta_2} = \frac{|T|^2 E_i^2 \sqrt{\varepsilon_{si}}}{2\eta_0} = 0.35^2 \cdot \sqrt{11.8} = 0.42 \text{ W/m}^2$$