Problem 1 (20 points):

The magnetic field of a uniform plane wave propagating in a lossless dielectric medium is given by: $\vec{H}(z,t) = \hat{x}\frac{1}{2\pi}\cos\left(10^8 t + \frac{z}{\sqrt{2}}\right) + \hat{y}\frac{1}{4\pi}\cos\left(10^8 t + \frac{z}{\sqrt{2}}\right)\ \frac{A}{m}$

a) What is the direction of propagation of the wave?
b) Determine the frequency f and the wavelength λ.
c) Find the dielectric constant of the medium.
d) Obtain the phasor and the time domain expression for the $\vec{E}$ field.

a. $-\hat{z}$.

b. $f = \frac{10^8}{2\pi}$ Hz $= 1.59 \times 10^7$ Hz

$k = \frac{1}{\sqrt{2}}$ rad/m. $\lambda = \frac{2\pi}{k} = 2\sqrt{2}\pi$ m $= 8.89$ m.

c. $u_p = \lambda \cdot f = 2\sqrt{2}\pi \times \frac{10^8}{2\pi} = \sqrt{2} \cdot 10^8$ m/s.

$\sqrt{\varepsilon_r} = \frac{c}{u_p} = \frac{3 \times 10^8}{\sqrt{2} \times 10^8} = \frac{3}{\sqrt{2}}$.

$\varepsilon_r = \frac{9}{2}$.

d. $\tilde{E} = -\eta \hat{k} \times \tilde{H}$

$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{3/\sqrt{2}} = \frac{120\sqrt{2}\pi}{3}\ \Omega$.

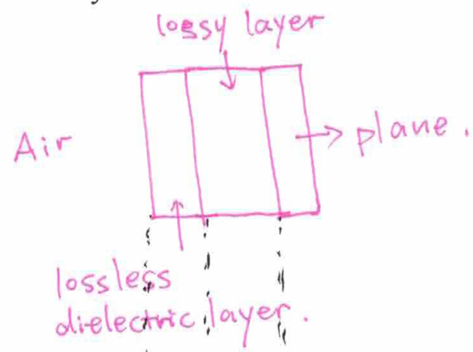
$\vec{E}(z,t) = \hat{y}\frac{120\pi \cdot \sqrt{2}}{3 \cdot 2\pi}\cos\left(10^8 t + \frac{z}{\sqrt{2}}\right) - \hat{x}\frac{120\pi\sqrt{2}}{3 \cdot 4\pi}\cos\left(10^8 t + \frac{z}{\sqrt{2}}\right)\frac{V}{m}$

$\tilde{E} = \hat{y}\frac{120\pi\sqrt{2}}{3 \cdot 2\pi} e^{jz/\sqrt{2}} - \hat{x}\frac{120\pi\sqrt{2}}{3 \cdot 4\pi} e^{jz/\sqrt{2}}$

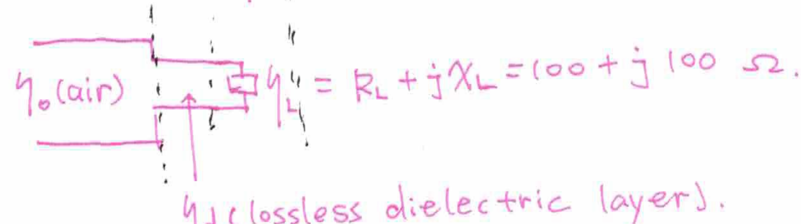
$= \hat{y}\, 20\sqrt{2}\, e^{jz/\sqrt{2}} - \hat{x} \cdot 10\sqrt{2}\, e^{jz/\sqrt{2}}\ \frac{V}{m}$

Problem 2 (30 points):

The goal of stealth technology is to make airplanes invisible to radar. One common way to achieve this goal is to cover airplanes with materials that absorb radar signals. To achieve invisibility at a 12 GHz radar frequency, we use a special absorbing layer, which is a lossy dielectric with $\eta = 100+j100\ \Omega$. However, its drawback is that there will be strong reflections at normal incidence. To eliminate these reflections, we coat the absorbing layer with another lossless dielectric, such that this dielectric coating is placed between air and the absorbing medium. Determine the width and the relative permittivity of the dielectric to eliminate these reflections.



The absorbing layer absorbs radar signals.
⇒ Lossy layer can be treated as the load.



$$\eta_{in} = \eta_d \frac{\eta_L \cos\beta d + j\eta_d \sin\beta d}{\eta_d \cos\beta d + j\eta_L \sin\beta d} = \frac{\eta_0}{n_d} \frac{R_L\cos\beta d + jX_L\cos\beta d + j\frac{\eta_0}{n_d}\sin\beta d}{\frac{\eta_0}{n_d}\cos\beta d + jR_L\sin\beta d - X_L\sin\beta d} = \eta_0.$$

$$R_L\cos\beta d + jX_L\cos\beta d + j\frac{\eta_0}{n_d}\sin\beta d = \eta_0\cos\beta d + jn_dR_L\sin\beta d - n_dX_L\sin\beta d$$

Real part: $R_L\cos\beta d = \eta_0\cos\beta d - n_dX_L\sin\beta d \Rightarrow \tan\beta d = \frac{\eta_0 - R_L}{n_dX_L}$

Imaginary part: $X_L\cos\beta d + \frac{\eta_0}{n_d}\sin\beta d = n_dR_L\sin\beta d \Rightarrow \tan\beta d = \frac{X_L}{n_dR_L - \frac{\eta_0}{n_d}}$

$$\frac{\eta_0 - R_L}{n_dX_L} = \frac{X_L}{n_dR_L - \frac{\eta_0}{n_d}} = \frac{X_L \cdot n_d}{n_d^2R_L - \eta_0}.$$

$$n_d^2(\eta_0R_L - X_L^2 - R_L^2) = \eta_0^2 - R_L\eta_0$$

$$n_d = \sqrt{\frac{\eta_0(\eta_0 - R_L)}{R_L(\eta_0 - R_L) - X_L^2}} = \sqrt{\frac{377\times(377-100)}{100\times(377-100)-100^2}} = 2.43$$

$$\varepsilon_r = n_d^2 = 5.9.$$

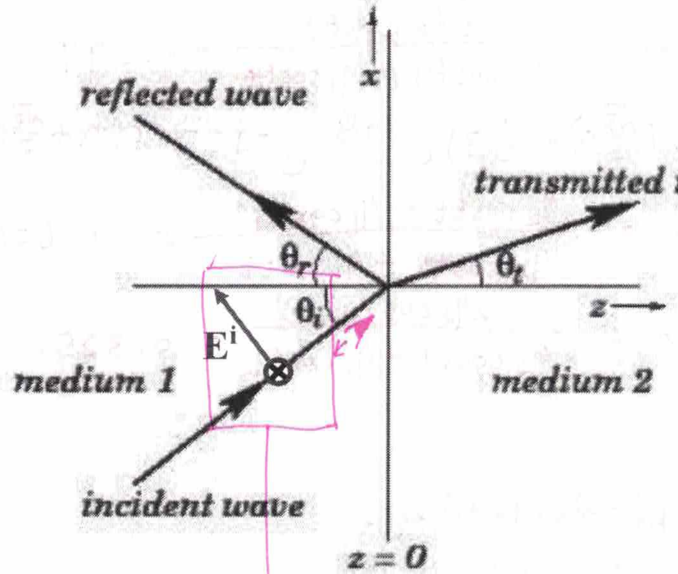
$$\tan\beta d = \frac{377-100}{2.43\times100} \approx 1. \qquad \beta d = \frac{2\pi\sqrt{\varepsilon_r}}{\lambda}d = \frac{\pi}{4}. \Rightarrow d = 0.0013m.$$

Problem 3 (40 points):

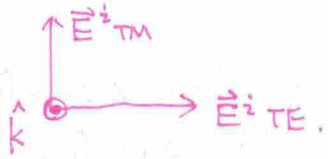
A right-hand circular polarized wave with an electric field of 2 V/m is traveling in air in +z direction.

If the wave is incident on the surface of a lossless, nonmagnetic medium with $\varepsilon_r = 25$ with an angle of 30°. Determine the following:

a) The expressions for incident electric field and magnetic field in phasor domain, given that the wavelength is 6 cm.
b) The reflection and transmission coefficients.
c) The expressions for reflected and transmitted electric field and magnetic field in phasor domain.
d) The average power densities of the incident, reflected, and transmitted waves.



Looking into the propagation direction.



direction of propagation

$\hat{k} = \hat{x} \cdot \sin\theta_i + \hat{z} \cdot \cos\theta_i$

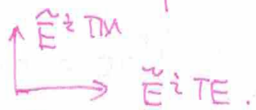
$= \frac{\hat{x}}{2} + \frac{\sqrt{3}}{2}\hat{z}$

direction of TM component: $\hat{x}\cos\theta_i - \hat{z}\sin\theta_i$

$= \frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}$

direction of TE component: $-\hat{y}$.

For RHC:



$\delta = -\frac{\pi}{2}$.

$\tilde{E} = a\left[-\hat{y} + \left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}\right)e^{-j\pi/2}\right]e^{-jk\left(\frac{x}{2} + \frac{\sqrt{3}}{2}z\right)}$

$a = 2\ V/m.$ wave number $k_1 = \frac{2\pi}{\lambda} = \frac{2\pi}{6cm} = \frac{2\pi}{0.06m} = \frac{100\pi}{3}\ \frac{rad}{m}$

$$\tilde{E}^i = 2\left[-\hat{y} + \left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}\right)e^{-j\frac{\pi}{2}}\right] e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)}$$

$$= 2\left[-\hat{y} - j\left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}\right)\right] e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)} \quad \frac{V}{m}.$$

$$\eta_1 = \frac{\eta_0}{\sqrt{1}} = 120\pi\ \Omega.$$

$$\tilde{H}^i = \frac{2}{120\pi}\left[\left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}\right) - j\hat{y}\right] e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)} \quad \frac{A}{m}.$$

(b). TE: $\tilde{E}^i_{TE} = -2\hat{y}\, e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)}.$

$$\theta t = \sin^{-1}\left(\frac{1}{\sqrt{\varepsilon_{r2}}} \times \sin\theta_i\right) = 5.74^\circ$$

$$\Gamma_{TE} = \Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta t}{\eta_2\cos\theta_i + \eta_1\cos\theta t} = \frac{24\pi\cos 30^\circ - 120\pi\cos 5.74^\circ}{24\pi\cos 30^\circ + 120\pi\cos 5.74^\circ}$$

$$= -0.703.$$

$$\tau_{TE} = \tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta t} = 0.297$$

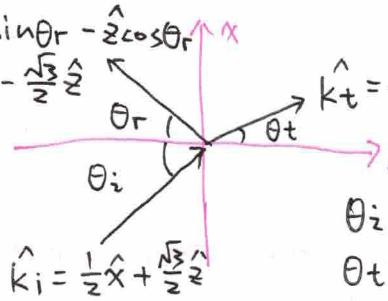
TM: $\tilde{E}^i_{TM} = 2\left(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{z}\right)e^{-j\frac{\pi}{2}}\, e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}z\right)}$

$$\Gamma_{TM} = \Gamma_{\parallel} = \frac{\eta_2\cos\theta t - \eta_1\cos\theta_i}{\eta_2\cos\theta t + \eta_1\cos\theta_i} = -0.626$$

$$\tau_{TM} = \tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta t + \eta_1\cos\theta_i} = 0.325.$$

(c).

$\hat{k_r} = \hat{x}\cdot\sin\theta_r - \hat{z}\cos\theta_r = \frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{z}$



$\hat{k_t} = \hat{x}\sin\theta t + \hat{z}\cos\theta t = 0.1\hat{x} + 0.99\hat{z}$

$\theta_i = \theta_r = 30^\circ$

$\theta t = 5.74^\circ$

$\hat{k_i} = \frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}$

Reflected:

$$\tilde{E}^r = \tilde{E}^r_{TE} + \tilde{E}^r_{TM} \begin{cases} \tilde{E}^r_{TE} = \Gamma_{TE} \times 2(-\hat{y})\, e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)} \\ \quad = 1.406\,\hat{y}\, e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)} \\ \tilde{E}^r_{TM} = \Gamma_{TM} \times \cancel{2(-\hat{y})}\ 2\left[-j\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}\right)\right] e^{-j\frac{100\pi}{6}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)} \\ \quad = 1.252\, j\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}\right) e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)}. \end{cases}$$

$$\tilde{H}^r = \tilde{H}^r_{TE} + \tilde{H}^r_{TM} \begin{cases} \tilde{H}^r_{TE} = 0.0037\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}\right) e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)} \\ \tilde{H}^r_{TM} = -0.0033\, j\cdot\hat{y}\, e^{-j\frac{100\pi}{3}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}z\right)}. \end{cases}$$

Transmitted: wave number: $k_2 = k_1 \cdot \sqrt{\varepsilon_{r2}} = \frac{100\pi}{3} \times \sqrt{25} = \frac{500}{3}\pi$ rad/m.

intrinsic impedance: $\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r2}}} = 24\pi\ \Omega$.

$$\tilde{E}^t \begin{cases} \tilde{E}^t_{TE} = \tau_{TE} \times 2(-\hat{y})\, e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \\ \quad = -0.594 \cdot \hat{y}\, e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \\ \tilde{E}^t_{TM} = (\hat{x}\cdot\cos\theta t - \hat{z}\sin\theta t)\cdot \tau_{TM} \times 2 \cdot e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \\ \quad = 0.65\,[-j(0.99\hat{x} - 0.1\hat{z})]\, e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \end{cases}$$

$e^{-j\frac{\pi}{2}}$

$$\tilde{H}^t \begin{cases} \tilde{H}^t_{TE} = 0.00788\,(0.99\hat{x} - 0.1\hat{z})\, e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \\ \tilde{H}^t_{TM} = 0.00862\,[-j(\hat{y})]\, e^{-j\frac{500\pi}{3}(0.1x+0.99z)} \end{cases}$$

(d). Incident: $\vec{S}^i_{av} = (\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}) \frac{|E^i_0|^2}{2\eta_1} = (\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}) \frac{|E^i_{TE0}|^2 + |E^i_{TM0}|^2}{2\eta_1}$

$= (\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}) \frac{4+4}{2\times 120\pi} = (\frac{\hat{x}}{2} + \frac{\sqrt{3}}{2}\hat{z}) \frac{1}{30\pi}$

$= 0.0106\,(\frac{\hat{x}}{2} + \frac{\sqrt{3}}{2}\hat{z})\ \frac{W}{m^2}$.

Reflected: $\vec{S}^r_{av} = (\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{z})\,|\Gamma|^2 \frac{|E^i_0|^2}{2\eta_1}$

$= (\frac{\hat{x}}{2} - \frac{\sqrt{3}}{2}\hat{z}) \frac{1.406^2 + (1.252\frac{\sqrt{3}}{2})^2 + (1.252\frac{1}{2})^2}{2\eta_1}$

$= (\frac{\hat{x}}{2} - \frac{\sqrt{3}}{2}\hat{z}) \frac{3.544}{2\times 120\pi}$

$= 0.0047\,(\frac{\hat{x}}{2} - \frac{\sqrt{3}}{2}\hat{z})$.

Transmitted: $\vec{S}^t_{av} = (0.1\hat{x} + 0.99\hat{z})\,|\tau|^2 \frac{|E^i_0|^2}{2\eta_2}$

$= (0.1\hat{x} + 0.99\hat{z}) \frac{0.594^2 + (0.65\times 0.99)^2 + (0.65\times 0.1)^2}{2\times 24\pi}$

$= (0.1\hat{x} + 0.99\hat{z}) \frac{0.7712}{48\pi}$

$= 0.0051\,(0.1\hat{x} + 0.99\hat{z})$.