EE 101B

Winter 2019

Wednesday, May 8, 2019

Name: ____

Student ID Number: ____

Honor Pledge:

"I have neither given nor received aid on this examination, nor have I concealed any violation of the Honor Code.

Date: ______________ Signature: ______________________________

Problem 1 (50 points)

A 100 mW parallel-polarized optical wave at a 1um wavelength in free space is incident from air onto a silicon substrate at the Brewster angle. The relative permittivity and conductivity of silicon are 16 and 0.1 S/m, respectively. The free space permittivity is $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$, and the free space permeability is $\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$.

a) Calculate the angles of the incident and transmitted beams relative to the normal to the silicon-air interface. (11)

b) Calculate the propagation constant, attenuation constant, wavenumber, and phase velocity of the optical wave in silicon. (11)

c) What is the amplitude of the transmitted electric field? Explain through calculation, how it is not a violation of the conservation of energy that the transmitted field is different from the incident field even though no wave is reflected. (10)

d) Assuming that the silicon surface is in the xy-plane, write the phasor and time-domain expressions for the electric filed and magnetic field in air and silicon. (10)

e) If the incident optical beam is a mixture of a 50 mW parallel polarized wave and a 50 mW perpendicular polarized wave, calculate the transmitted power to silicon. (8)

a) $\theta_B = \theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1}\sqrt{\frac{16\varepsilon_0}{\varepsilon_0}} = 75.96°$

Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow \theta_t = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_i\right] = 14.04°$

b) $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} \ll 1 \Rightarrow$ low-loss

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = 4.71 \frac{Np}{m}$, $\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} n_2 = 8\pi \cdot 10^6 \frac{rad}{s}$

$\gamma = \alpha + j\beta$

$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = 7.5 \cdot 10^7 \frac{m}{s}$

c) $$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{2\frac{\eta_0}{4}\cos(75.96)}{\frac{\eta_0}{4}\cos(14.04) + \eta_0 \cos(75.96)} = 0.25$$

Students who assumed $S_{\parallel}^{i} = 100 \frac{mW}{m^2}$:

$$\frac{|E_{\parallel}^{i}|^2}{2\eta_0} = 100 \Rightarrow |E^i| = \sqrt{2\eta_0 100}$$

Students who assumed $P_{\parallel}^{i} = 100mW$:

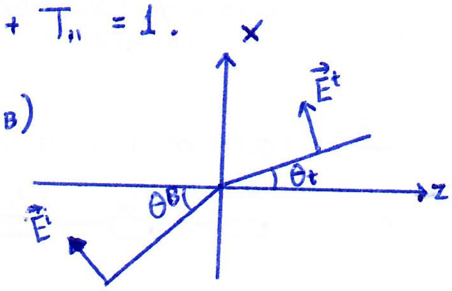
$$\frac{|E_{\parallel}^{i}|^2}{2\eta_0} A \cdot \cos\theta_i = 100 \Rightarrow |E_{\parallel}^{i}| = \sqrt{2\eta_0 A \cos\theta_i}$$

No points were deducted for this part.

$$|E^t| = \tau_{\parallel} |E^i| = 0.25\, |E^i|$$

Transmissivity: $T_{\parallel} = |\tau_{\parallel}|^2 \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i} = 1$

Energy is conserved because $|\Gamma_{\perp}|^2 + T_{\parallel} = 1$.



d)

$$\vec{E}^i = (\hat{x}\cos\theta_B - \hat{z}\sin\theta_B)\,|E^i|\, e^{-jk_0(z\cos\theta_B + x\sin\theta_B)}$$

$$\vec{H}^i = \hat{y}\,\frac{|E^i|}{\eta_0}\, e^{-jk_0(z\cos\theta_B + x\sin\theta_B)}$$

$$\vec{E}^i(z,x,t) = (\hat{z}\cos\theta_B - \hat{z}\sin\theta_B)|E^i| \cos[\omega t - k_0(z\cos\theta_B + x\sin\theta_B)]$$

$$\vec{H}^i(z,x,t) = \hat{y}\,\frac{|E^i|}{\eta_0} \cos[\omega t - k_0(z\cos\theta_B + x\sin\theta_B)]$$

Zero reflected field at Brewster angle.

$$\vec{E}^t = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)\, \tau_{\parallel}\, |E^i|\, e^{-(\alpha+j\beta)(z\cos\theta_t + x\sin\theta_t)}$$

$$\vec{H}^t = \hat{y}\,\frac{\tau_{\parallel}|E^i|}{\eta_2}\, e^{-(\alpha+j\beta)(z\cos\theta_t + x\sin\theta_t)}$$

$$\vec{E}^t(z,x,t) = (\hat{z}\cos\theta_t - \hat{z}\sin\theta_t)\,\tau_{\parallel}|E^i|\, e^{-\alpha(z\cos\theta_t + x\sin\theta_t)} \cos[\omega t - \beta(z\cos\theta_t + x\sin\theta_t)]$$

$$\vec{H}^t = \hat{y}\,\frac{\tau_{\parallel}|E^i|}{\eta_2}\, e^{-\alpha(z\cos\theta_t + x\sin\theta_t)} \cos[\omega t - \beta(z\cos\theta_t + x\sin\theta_t)]$$

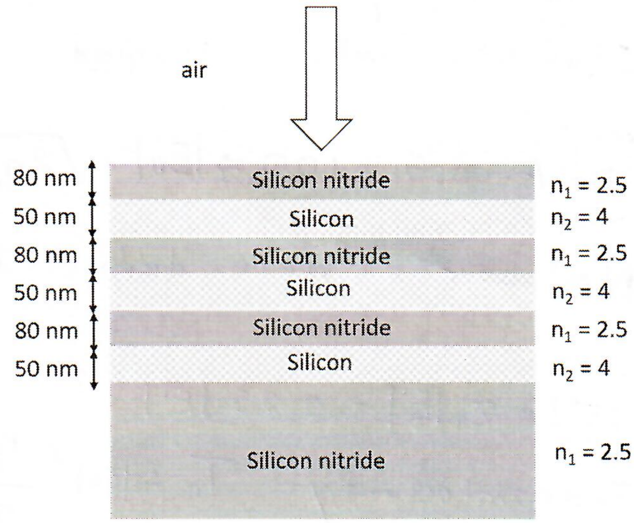
e) Parallel part is fully transmitted at Brewster angle.

$$\tau_{\perp} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = 0.1177, \quad T_{\perp} = |\tau_{\perp}|^2 \frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i} = 0.2215$$

$$P = 50mW \cdot 1 + 50mW \cdot T_{\perp} = 61\, mW$$

Problem 2 (30 points)

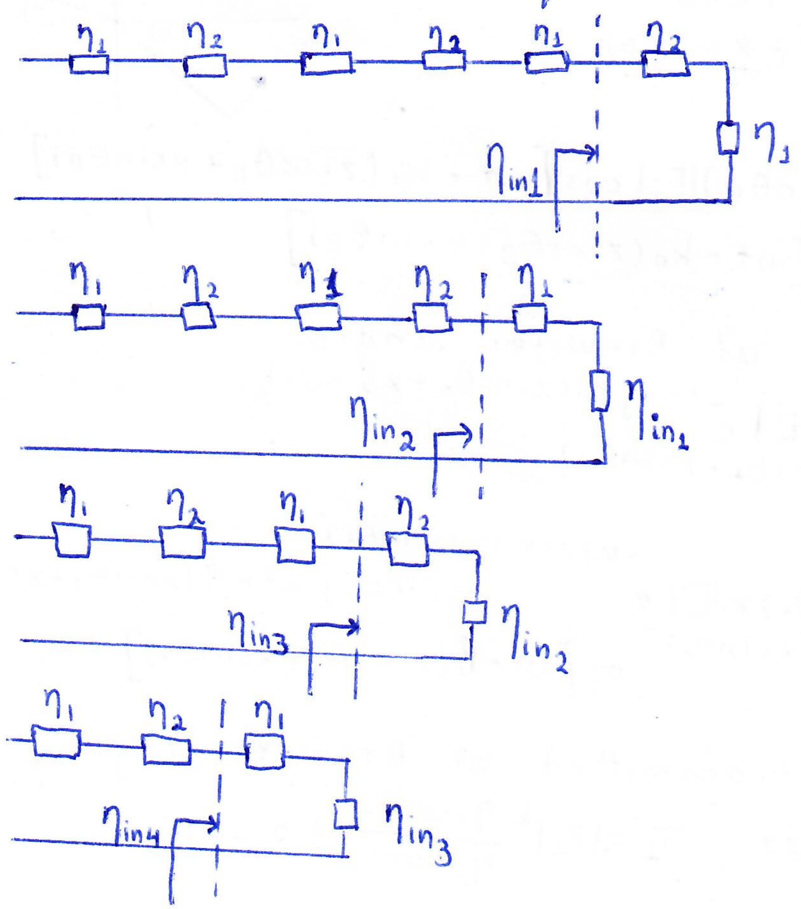
A linearly polarized optical wave at an 800 nm wavelength is normally incident on the structure shown in the figure below. The structure consists of 3 pairs of 80/50 nm silicon nitride/silicon layers placed on top of an infinitely thick silicon nitride substrate. The refractive index of silicon nitride and silicon are 2.5 and 4, respectively.



a) Calculate the percentage of the optical beam that is reflected back to air.

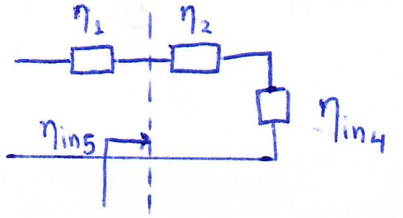
b) Calculate the percentage of the optical beam that is reflected back to air if the thickness of the silicon nitride and silicon layers are doubled.

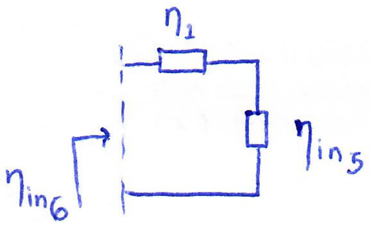
a) Transmission line equivalent



$$\eta_1 = \frac{\eta_0}{2.5}$$

$$\eta_2 = \frac{\eta_0}{4}$$





$$k_o = \frac{2\pi}{\lambda_o}$$

For silicon $kd = \frac{2\pi}{800nm} \cdot 4 \cdot 50 \cdot 50nm = \frac{\pi}{2}$

For silicon nitride:

$$kd = \frac{2\pi}{800nm} \cdot 2.5 \cdot 80 = \frac{\pi}{2}$$

So $\cos(\beta d) = 0$, $\sin(\beta d) = 1$

General Formula:

$$\eta_{in} = \eta_0 \frac{\eta_L \cos(\beta d) + \eta_a \sin(\beta d)}{\eta_0 \cos(\beta d) + j\,\eta_L \sin(\beta d)}$$

$$\eta_{in_1} = \eta_2 \cdot \frac{\eta_2}{\eta_1} = \frac{\eta_2^2}{\eta_1}$$

$$\eta_{in_2} = \eta_1 \frac{\eta_1}{\eta_{in_1}} = \eta_1^3/\eta_2^2$$

$$\eta_{in_3} = \eta_2 \frac{\eta_2}{\eta_{in_2}} = \eta_2^4/\eta_1^3$$

$$\eta_{in_4} = \eta_1 \frac{\eta_1}{\eta_{in_3}} = \eta_1^5/\eta_2^4$$

$$\eta_{in_5} = \eta_2 \frac{\eta_2}{\eta_{in_4}} = \eta_2^6/\eta_1^5$$

$$\eta_{in_6} = \eta_1 \frac{\eta_1}{\eta_{in_5}} = \eta_1^7/\eta_2^6 = \eta_0 \frac{n_2^2}{n_1^7} = \eta_0 \frac{4^6}{2.5^7} = \eta_0\, 6.7109$$

$$\Gamma = \frac{\eta_{in_6} - \eta_0}{\eta_{in_6} + \eta_0} = 0.7406, \quad R = |\Gamma|^2$$

$R = |\Gamma|^2 = 18.37\%$

b) $\beta d = \pi \Rightarrow \cos(\beta d) = -1$, $\sin(\beta d) = 0$

$\eta_{in_1} = \eta_2 \frac{\eta_1}{\eta_2} = \eta_1$, $\eta_{in_2} = \eta_{in_3} = \eta_{in_4} = \eta_{in_5} = \eta_{in_6} = \eta_1$, $\Gamma = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} = -0.4286$,

Problem 3 (20 points)

A 1-km-long optical fiber with a core refractive index of 1.55 and a cladding refractive index of 1.45 transmits digital data.

a) Determine the maximum expected time-stretch in the transmitted bits along the fiber. (13)
b) Determine the maximum usable data rate that can be transmitted through this fiber such that the time stretch in the transmitted bits does not exceed half of the data period. (7)

a) $t_{min} = \frac{\ell}{v_p} = \frac{\ell}{c} n_f$ (normal)

$t_{max} = \frac{\ell_{max}}{v_p} = \frac{\ell n_f}{c n}$

$\tau = t_{max} - t_{min} = \frac{\ell n_f}{c}\left(\frac{n_f}{n_c} - 1\right) = 3.56 \cdot 10^{-7} sec$

b) $f_p = \frac{1}{2\tau} = 1.4 \frac{Mb}{s}$