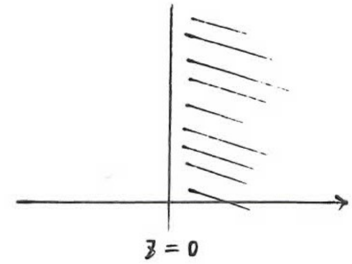


Problem 1:

A 1 GHz left-hand circularly polarized plane wave with an electric field modulus of 100 mV/m is normally incident in air upon a nonmagnetic medium with $\varepsilon_r = 2.25$, $\sigma = 10^{-4}$ S/m and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at z = 0 and t = 0.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

(a) medium 1. $\eta_1 = \eta_0 = 120\pi \ (\Omega)$ $\quad k_1 = \frac{w}{c} = \frac{2\pi \times 1 \times 10^9}{3 \times 10^8} = \frac{20\pi}{3}$ rad/m

medium 2. nonmagnetic medium $\Rightarrow \mu_2 = \mu_1 = \mu_0$

$$\frac{\sigma_2}{w\varepsilon_2} = \frac{10^{-4}}{2\pi \times 1 \times 10^9 \times 2.25 \times 8.85 \times 10^{-12}} \doteq 7.99 \times 10^{-4} \ll 1 \Rightarrow \text{low-loss medium.}$$

$$\Rightarrow \alpha_2 = \frac{\sigma_2}{2}\sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{10^{-4}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \doteq \frac{10^{-4}}{2} \frac{120\pi}{\sqrt{2.25}} \doteq 1.26 \times 10^{-2} \ \text{Np/m.}$$

$$\beta_2 = w\sqrt{\mu_2 \varepsilon_2} = w\sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = \frac{w}{c}\sqrt{\varepsilon_r} = \frac{1 \times 10^9 \times 2\pi}{3 \times 10^8}\sqrt{2.25} = 10\pi \ \text{rad/m.}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \doteq \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{2.25}} = 80\pi \ \Omega$$

LHC wave: $\tilde{E}^i = |a_0| e^{j\phi_0} (\hat{x} + \hat{y} e^{j\pi/2}) e^{-jk_1 z}$

E-field modulus: 100 mV/m, field is a positive maximum at z = 0 and t = 0

$\Rightarrow |a_0| = 100$ mV/m. $\phi_0 = 0$

$$\Rightarrow \tilde{E}^i = 100(\hat{x} + j\hat{y}) e^{-j20\pi z/3} \ \text{mV/m.}$$

(b) Reflection coefficients: $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{80\pi - 120\pi}{80\pi + 120\pi} = -0.2$

Transmission coefficients: $\tau = 1 + \Gamma = 1 - 0.2 = 0.8$

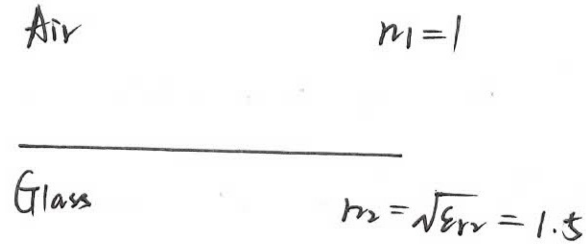
(c) $\tilde{\mathbf{E}}^r = 100\Gamma(\hat{x}+j\hat{y})e^{j20\pi z/3} = -20(\hat{x}+j\hat{y})e^{j20\pi z/3}$ mV/m.

$\tilde{\mathbf{E}}^t = 100\tau(\hat{x}+j\hat{y})e^{-\alpha_2 z - j\beta_2 z} = 80(\hat{x}+j\hat{y})e^{-1.26\times10^{-2}z - j10\pi z}$ mV/m

$\tilde{\mathbf{E}}_1 = \tilde{\mathbf{E}}^i + \tilde{\mathbf{E}}^r = 100(\hat{x}+j\hat{y})[e^{-j20\pi z/3} - 0.2e^{j20\pi z/3}]$ mV/m.

(d) % of reflected power $= 100|\Gamma|^2 = 100\times(0.2)^2 = 4\%$

% of transmitted power $= 100|\tau|^2\dfrac{\eta_1}{\eta_2} = 100\times(0.8)^2\times\dfrac{120\pi}{80\pi} = 96\%$



Problem 2

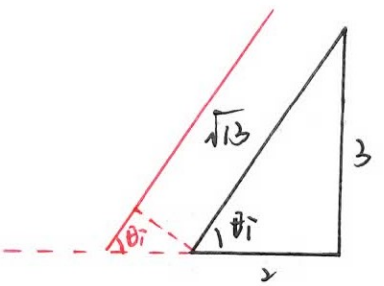
A $1\,\mathrm{mW/m^2}$ parallel-polarized electromagnetic wave is incident from air onto glass at the Brewster angle. $\varepsilon_{r-gl} = 2.25$.

a) What is the amplitude of the transmitted electric field? Explain through calculation, how it is not a violation of the conservation of energy that the transmitted field is different from the incident field even though no wave is reflected.

b) If the incident wave is a mixture of 30% parallel polarized wave and 70% perpendicular polarized wave, what portion of the incident power would be transmitted to medium 2?

(a) For parallel polarization $\tau_{\parallel} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \dfrac{2\cos\theta_i}{\cos\theta_t + \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\cos\theta_i}$ (*)

$\theta_i = \theta_B = \arctan\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \arctan 1.5$



$\Rightarrow \cos\theta_i = \dfrac{2}{\sqrt{13}}$

$\sin\theta_i = \dfrac{3}{\sqrt{13}}$

Snell's law $n_1 \sin\theta_i = n_2 \sin\theta_t$

$\Rightarrow \sin\theta_t = \dfrac{2}{\sqrt{13}} \Rightarrow \cos\theta_t = \dfrac{3}{\sqrt{13}}$. plug in eq (*). $\tau_{\parallel} = \dfrac{2}{3}$.

$S_{av}^{in} = \dfrac{{E^{in}}^2}{2\eta_1} = 1\,\mathrm{mW/m^2} = 1\times 10^{-3}\,\mathrm{W/m^2} \Rightarrow E^{in} = 0.868\,\mathrm{V/m}$

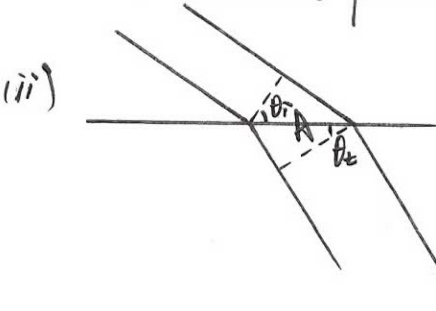
$\Rightarrow E^{tr} = \tau_{\parallel} E^{in} = 0.579\,\mathrm{V/m}$.

Conservation of energy. Two methods:

(i). Transmissivity $T_{\parallel} = |\tau_{\parallel}|^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i} = \left(\dfrac{2}{3}\right)^2 \times \sqrt{2.25} \times \dfrac{3/\sqrt{13}}{2/\sqrt{13}} = 1.$

All the incident power is transmitted into the 2nd medium. i.e. glass.

(ii)



$P_{in} = S_{av}^{in} A \cos\theta_i = \dfrac{2}{\sqrt{13}} S_{av}^{in} A$

$P_{tr} = S_{av}^{tr} A \cos\theta_{tr} = \dfrac{3}{\sqrt{13}} \dfrac{{E^{tr}}^2}{2\eta_2} A = \dfrac{3}{\sqrt{13}} \tau_{\parallel}^2 \sqrt{\varepsilon_{r2}}\, A S_{av}^{in}$

$= \dfrac{2}{\sqrt{13}} S_{av}^{in} A = P_{in}$

In conclusion, though the amplitude of the transmitted E-field is different from that of the incident electric field, all the power is transmitted into the 2nd medium. It's not a violation of the conservation of energy.

(b). When $\theta_i = \theta_B$, all the parallel polarized wave will be transmitted.

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{\cancel{1.5}\cos\theta_i - \frac{1}{1.5}\cos\theta_t}{\cancel{1.5}\cos\theta_i + \frac{\cos\theta_t}{1.5}} = -\frac{5}{13}$$

$$T_\perp = 1 - R_\perp = 1 - |\Gamma_\perp|^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

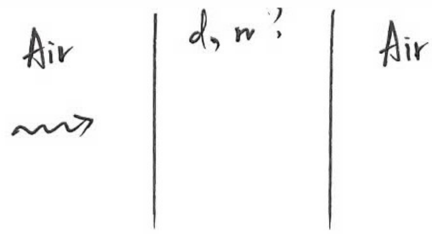
$\Rightarrow$ $\frac{144}{169}$ of the perpendicular polarized wave will be transmitted.

$\Rightarrow$ % of the incident power that would be transmitted to medium 2

$$= 30\% \times 1 + 70\% \times \frac{144}{169}$$

$$\doteq 89.6\%$$

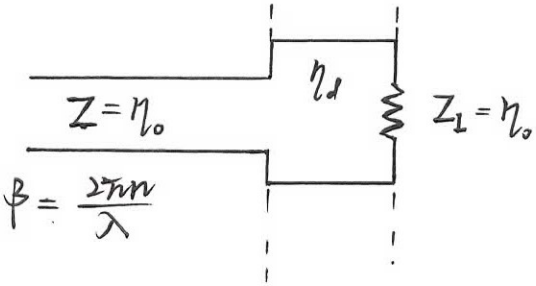
$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right)$$



Problem 3

An optical beam at a 600 nm wavelength is normally incident on a non-magnetic dielectric slab. Both sides of the dielectric slab are filled with air. Determine the thickness and refractive index of the dielectric slab that allow transmission of 75% of the incident optical beam through the slab into air.

Use the transmission-line equivalent model.



$$\eta_{in} = \eta_d \frac{\eta_0 + j\eta_d \tan\beta d}{\eta_d + j\eta_0 \tan\beta d}, \quad \eta_d = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n}, \quad \beta = \frac{2\pi n}{\lambda}$$

$$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} = \frac{\eta_d\eta_0 + j\eta_d^2 \tan\beta d - \eta_0\eta_d - j\eta_0^2 \tan\beta d}{\eta_d\eta_0 + j\eta_d^2 \tan\beta d + \eta_0\eta_d + j\eta_0^2 \tan\beta d}$$

$$= \frac{j(\eta_d^2 - \eta_0^2)\tan\beta d}{2\eta_d\eta_0 + j(\eta_d^2 + \eta_0^2)\tan\beta d} = \Gamma$$

$$\Rightarrow j(\eta_d^2 - \eta_0^2)\tan\beta d = 2\eta_d\eta_0\Gamma + j\Gamma(\eta_d^2 + \eta_0^2)\tan\beta d$$

$$\Rightarrow j(\eta_d^2 - \eta_0^2)\sin\beta d = 2\eta_d\eta_0\Gamma\cos\beta d + j\Gamma(\eta_d^2 + \eta_0^2)\sin\beta d$$

$\Rightarrow$ Real part $2\eta_d\eta_0\Gamma\cos\beta d = 0 \Rightarrow \cos\beta d = 0 \Rightarrow \beta d = \frac{\pi}{2} + m\pi$, $m = 0, 1, 2, \dots$

Imaginary part $(\eta_d^2 - \eta_0^2)\sin\beta d = \Gamma(\eta_d^2 + \eta_0^2)\sin\beta d \Leftrightarrow (1-n^2) = \Gamma(1+n^2)$

(Since $\beta d = \frac{\pi}{2} + m\pi$. $\sin\beta d = 1$ or -1)

Since $T = 75\%$, $R = 25\%$. $\Gamma = \pm 0.5$.

$$\Rightarrow n = \begin{cases} \frac{1}{\sqrt{3}} & \text{when } \Gamma = 0.5. \text{ while } u_p = \frac{c}{n} > c. \\ \sqrt{3} & \text{when } \Gamma = -0.5. \end{cases}$$

$\Rightarrow n = \sqrt{3}$

$\beta d = \frac{2\pi n}{\lambda} d = \frac{\pi}{2} + m\pi \Rightarrow d \doteq 86.6 + 173.2m$ (nm). $m = 0, 1, 2, \dots$