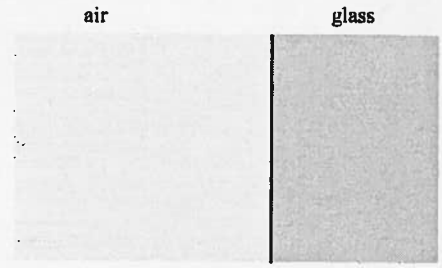
Problem 1: (40 Points)

An optical beam with 600 nm wavelength, propagating in air, is normally incident at the boundary of an infinitely thick glass substrate ($\varepsilon_{r\text{-glass}} = 2.25$, $\mu_{r\text{-glass}} = 1$, $\sigma_{glass} = 0$).

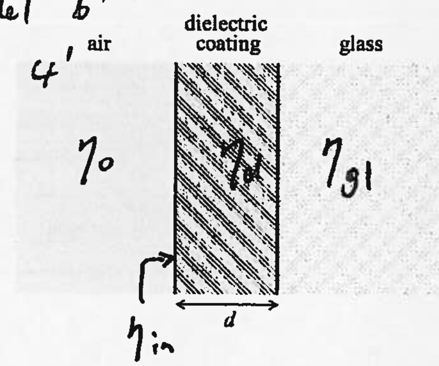


a) Determine the portion of the reflected optical power from the air-glass interface.

15

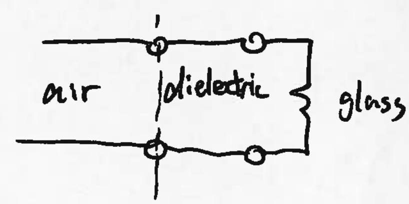
b) If the glass substrate is coated with a dielectric coating (as shown in the figure below), would it be possible to select the ~~dielectric permittivity~~ $\varepsilon_r = 13.5$ and thickness such that 64% of the incident optical power reflects back to air from the air-dielectric interface? If yes, determine the ~~dielectric permittivity~~ and thickness.

TL model 6'
answer 4'



c) Determine the portion of the reflected optical power from the interface if the glass substrate is coated with a 100 nm thick metal layer ($\sigma_{metal} \to \infty$).

15



a) $$\Gamma = \frac{1-\sqrt{\varepsilon_{r\text{-glass}}}}{1+\sqrt{\varepsilon_{r\text{-glass}}}} = \frac{1-1.5}{1+1.5} = -0.2$$

Portion of Power : $|\Gamma|^2 = 0.04$

b) $$\eta_d = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{\sqrt{13.5}} \qquad \eta_{gl} = \frac{\eta_0}{\sqrt{\varepsilon_{r\text{-glass}}}} = \frac{\eta_0}{1.5}$$

$$\eta_{in} = \eta_d \frac{\eta_{gl} + j\eta_d \tan\beta d}{\eta_d + j\eta_{gl}\tan\beta d}$$

$$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} = \frac{\eta_d \frac{\eta_{gl} + j\eta_d \tan\beta d}{\eta_d + j\eta_{gl}\tan\beta d} - \eta_0}{\eta_d \frac{\eta_{gl} + j\eta_d \tan\beta d}{\eta_d + j\eta_{gl}\tan\beta d} + \eta_0}$$

Simplify this expression,

$$\Rightarrow \Gamma = -\frac{0.0907 + j0.592\tan\beta d}{0.454 + j0.74\tan\beta d}$$

Want $|\Gamma|^2 = 0.64$

let $\beta d = \frac{\pi}{2}$

$$\Gamma = -\frac{0.592}{0.74} = -0.8$$

So that $|\Gamma|^2 = 0.64$

Thus,

$$\frac{2\pi}{\lambda_d} \cdot d = \frac{\pi}{2} + n\pi$$

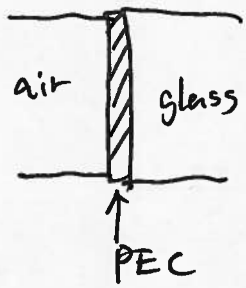
$$d = \frac{\lambda_d}{4} + n\frac{\lambda_d}{2}$$

$$d = \frac{\lambda_d}{4} = \frac{600}{4\sqrt{\varepsilon_r}} = 40.8$$

Thickness : $40.8 + n \cdot 81.6$ [nm]

$n = 0, 1, 2, \ldots$

c).



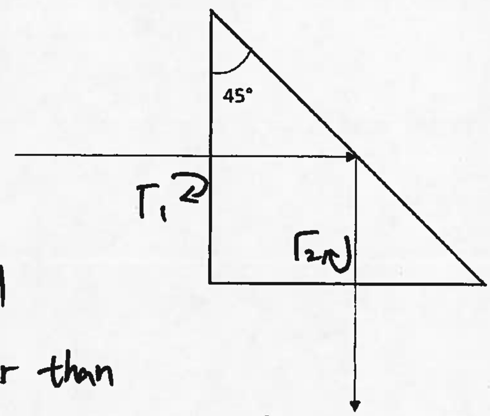
$\Gamma = -1$ for PEC

Portion of reflected power: $|\Gamma|^2 = 1$

total reflection + critical angle ~ 4' $(1-|\Gamma|^2) \sim 4$

Problem 2: (20 Points) $\Gamma_1, \Gamma_2 \sim 8'$ answer ~ 4'

A linearly polarized wave is incident on a prism, and it exits as shown in figure. The dielectric constant of the prism is 2.25, find the ratio of the exited average power density S_e to that of the incident S_i.



critical angle is

$$\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_0}{\varepsilon_r \varepsilon_0}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{2.25}}\right) = 41.8°$$

Thus, the reflection coefficient is $|\Gamma| = 1$

since incident angle of 45° is greater than the critical angle of $\theta_c = 41.8°$ 4'

$$\Gamma_1 = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} = \frac{1-1.5}{1+1.5} = -0.2$$ 4'

$$\Gamma_2 = -\Gamma_1 = 0.2$$ 4'

$$\frac{S_{av}^e}{S_{av}^i} = (1-|\Gamma_1|^2)(1^2)(1-|\Gamma_2|^2)$$ 4'

$$= [1-(0.2)^2]^2 = 0.96^2 = 0.9216$$ 4'

Problem 3: (40 Points)

A 6 MHz, x-polarized electromagnetic wave with average power density of $1\,W/m^2$ propagates along $\hat{k} = \frac{\sqrt{3}}{2}\hat{y} + \frac{1}{2}\hat{z}$ in air. The wave is incident at the water-air boundary at $z = 0$ ($\mu_{r-water} = 1$, $\varepsilon_{r-water} = 80$, $\sigma_{water} \cong 4$ S/m).

3' 3' 2' 2'

a) Determine the propagation constant, attenuation constant, intrinsic impedance, and phase velocity of the wave in water.

5' 5'

b) Is the polarization of the incident wave TE or TM? What is the incident angle?

$\theta_t \sim 1$ $E_0 \sim 1$
$\Gamma, \tau \sim 2$
$e^{-\gamma(\ )} \sim 2$
fields ~ 4

c) Write the phasor expressions for electric field and magnetic fields of the incident and reflected waves in the air and transmitted wave into water. 10'

d) Determine the average power density and polarization of the transmitted wave into water. 5'+5'

a) $\varepsilon'' = \frac{\sigma}{\omega} = \frac{4}{2\pi \cdot 6 \cdot 10^6} = 0.106 \cdot 10^{-6} >> \varepsilon' = 80 \cdot 8.84 \cdot 10^{-12}$

$\Rightarrow$ Good conductor

$\tilde{\gamma} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = 3.1\pi(1+j)\,m^{-1}$ or $9.7\,(1+j)\,m^{-1}$

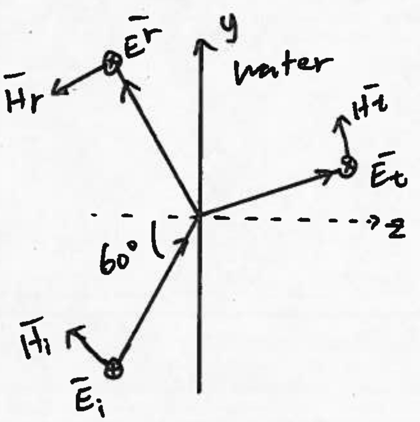
$\beta = \alpha = Re\{\tilde{\gamma}\} = 3.1\pi$ or $9.7\,m^{-1}$

$\tilde{\eta} = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$ or $\frac{\tilde{\gamma}}{\sigma}$

$= (1+j)\,2.433\ \Omega$

$V = \frac{\omega}{\beta} = \frac{2\pi \cdot 10^6 \cdot 6}{9.7} = 3.89 \cdot 10^6\,m/s$

b) TE $\quad \theta_i = \tan^{-1}\left(\frac{k_y}{k_z}\right) = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \cancel{30}\ 60^\circ$



$n_1 \sin\theta_i = n_2 \sin\theta_t$

$\sin\theta_i = \sqrt{80}\,\sin\theta_t$

$\theta_t = 5.6^\circ$

c) $$\tilde{\Gamma} = \frac{\tilde{\eta}\cos\theta_i - \eta_0\cos\theta_t}{\tilde{\eta}\cos\theta_i + \eta_0\cos\theta_t} = -0.994 + j0.006$$

$$\tilde{T} = 1 + \Gamma = 0.006 + j0.006$$

$$S^i_{av} = \frac{|E_0|^2}{2\eta_0} = 1\ W/m^2$$

$$\Rightarrow \quad E_0 = \sqrt{2\eta_0} = 27.5\ V/m$$

$$k_0 = \frac{w}{c} = \frac{2\pi \cdot 10^6 \cdot 6}{3 \cdot 10^8} = 4\pi \cdot 10^{-2}\ m^{-1}$$

$$\bar{E}_i = E_0 \hat{x} e^{-k_0(y\sin\theta_i + z\cos\theta_i)}$$

$$\bar{H}_i = \frac{E_0}{\eta_0}(-\hat{y}\cos\theta_i + \hat{z}\sin\theta_i) e^{-k_0(y\sin\theta_i + z\cos\theta_i)}$$

$$\bar{E}_r = E_0\tilde{\Gamma}\hat{y} e^{-k_0(y\sin\theta_i - z\cos\theta_i)}$$

$$\bar{H}_r = \frac{E_0\tilde{\Gamma}}{\eta_0}(\hat{y}\cos\theta_i + \hat{z}\sin\theta_i) e^{-k_0(y\sin\theta_i - z\cos\theta_i)}$$

$$\bar{E}_t = E_0\tilde{T}\hat{y} e^{-\tilde{\gamma}(y\sin\theta_t + z\cos\theta_t)}$$

$$\bar{H}_t = \frac{E_0\tilde{T}}{\tilde{\eta}}(-\hat{y}\cos\theta_t + \hat{z}\sin\theta_t) e^{-\tilde{\gamma}(y\sin\theta_t + z\cos\theta_t)}$$

d) $S_{av}^{t} = \dfrac{|T|^2 |E_0|^2}{2|\eta_c|} \cos\theta_{nc} = 0.006 \ W/m^2$

TE, $\hat{x}$ polarized