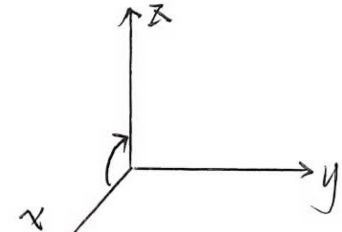


Problem 1 (40 points)

A 10 GHz left-hand circularly polarized wave with an electric field amplitude of 50 mV/m is propagating in air along the y axis. ($\varepsilon_0 \approx 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m)

a) Obtain the phasor and time domain expressions for the electric and magnetic fields of the wave.
b) Determine the average power density of the wave propagating in air.
c) Assuming that the wave is normally incident on the surface of an infinitely thick medium with a relative permittivity of 16 and conductivity of 0.01 S/m, obtain the phasor and time domain expressions for the electric and magnetic fields of the wave transmitted from air to this medium.
d) Calculate the average power density of the transmitted wave from air to this medium.
e) Determine the average power density of the wave in this medium after propagating over a 1 m distance from the air interface.

$a = 50\,\text{mV/m}$. $\omega = 2\pi f = 2\pi \times 10^{10}$ rad/s. $k = \frac{2\pi f}{c} = \frac{2\pi \times 10 \times 10^{9}}{3 \times 10^{8}} = \frac{200\pi}{3}$ rad/m

$\eta_1 = 120\pi\ \Omega$

(a) Solution 1. $\tilde{E} = a(\hat{x} + \hat{z} e^{-j\pi/2}) e^{-jky} = 50(\hat{x} - j\hat{z}) e^{-j\frac{200\pi}{3}y}$ mV/m

$\tilde{H} = \frac{1}{\eta_1} \hat{y} \times 50(\hat{x} - j\hat{z}) e^{-j\frac{200\pi}{3}y} = -\frac{5}{12\pi}(\hat{z} + j\hat{x}) e^{-j\frac{200\pi}{3}y}$ mA/m

$\vec{E} = Re[\tilde{E}e^{j\omega t}] = 50\hat{x}\cos(2\pi \times 10^{10}t - \frac{200\pi}{3}y) + 50\hat{z}\sin(2\pi \times 10^{10}t - \frac{200\pi}{3}y)$ mV/m

$\vec{H} = Re[\tilde{H}e^{j\omega t}] = -\frac{5}{12\pi}\hat{z}\cos(2\pi \times 10^{10}t - \frac{200\pi}{3}y) + \frac{5}{12\pi}\hat{x}\sin(2\pi \times 10^{10}t - \frac{200\pi}{3}y)$ mA/m

Solution 2. $\tilde{E} = 50(\hat{x}e^{j\pi/2} + \hat{z}) e^{-jky} = 50(j\hat{x} + \hat{z}) \exp(-j\frac{200\pi}{3}y)$ mV/m

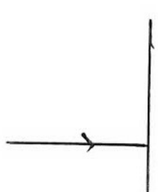
$\tilde{H} = \frac{1}{\eta_1} \hat{y} \times 50(j\hat{x} + \hat{z}) e^{-j\frac{200\pi}{3}y} = \frac{5}{12\pi}(\hat{x} - j\hat{z}) e^{-j\frac{200\pi}{3}y}$ mA/m

$\vec{E} = Re[\tilde{E}e^{j\omega t}] = -50\hat{x}\sin(2\pi \times 10^{10}t - \frac{200\pi}{3}y) + 50\hat{z}\cos(2\pi \times 10^{10}t - \frac{200\pi}{3}y)$ mV/m

$\tilde{H} = Re[\tilde{H}e^{j\omega t}] = \frac{5}{12\pi}\hat{z}\sin(2\pi \times 10^{10}t - \frac{200\pi}{3}y) + \frac{5}{12\pi}\hat{x}\cos(2\pi \times 10^{10}t - \frac{200\pi}{3}y)$ mA/m

(b) $\vec{S}_{av} = \hat{y}\frac{|\hat{E}|^2}{2\eta_1} = \hat{y}\frac{50^2 + 50^2}{2 \times 120\pi} \doteq \hat{y}\, 6.63 \times 10^{-6}$ W/m^2

(c)



For medium 2. $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\varepsilon\omega} = \frac{0.01}{8.85 \times 10^{-12} \times 16 \times 2\pi \times 10^{10}} \doteq 1.12 \times 10^{-3} < 0.01$

$\Rightarrow$ low-loss dielectric material

$\alpha_2 = \frac{\sigma}{2}\sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{10^{-2}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = \frac{10^{-2}}{2}\frac{120\pi}{\sqrt{16}} \doteq 0.47$ Np/m

$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^{10}}{3 \times 10^{8}}\sqrt{16} \doteq 837.76$ rad/m

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{16}} = 30\pi\ \Omega.$$

$$\Gamma = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_0} = \frac{30\pi - 120\pi}{30\pi + 120\pi} = -0.6. \quad \tau = 1 + \Gamma = 0.4.$$

Solution 1. $\tilde{E} = 20(\hat{x} - j\hat{z})e^{-(0.47 + j837.76)y}$ mV/m

$\tilde{H} = -\frac{2}{3\pi}(\hat{z} + j\hat{x})e^{-(0.47 + j837.76)y}$ mA/m.

$\vec{E} = 20\hat{x}\cos(2\pi\times10^{10}t - 837.76y)e^{-0.47y} + 20\hat{z}\sin(2\pi\times10^{10}t - 837.76y)e^{-0.47y}$ mV/m

$\vec{H} = -\frac{2}{3\pi}\hat{z}\cos(2\pi\times10^{10}t - 837.76y)e^{-0.47y} + \frac{2}{3\pi}\hat{x}\sin(2\pi\times10^{10}t - 837.76y)e^{-0.47y}$ mA/m

Solution 2. $\tilde{E} = 20(\hat{x}j + \hat{z})e^{-(0.47 + j837.76)y}$ mV/m.

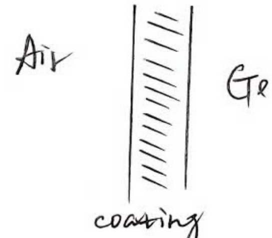
$\tilde{H} = \frac{2}{3\pi}(\hat{x} - \hat{z}j)e^{-(0.47 + j837.76)y}$ mA/m.

$\vec{E} = -20\hat{x}\sin(\omega t - 837.76y)e^{-0.47y} + 20\hat{z}\cos(2\pi\times10^{10}t - 837.76y)e^{-0.47y}$ mV/m

$\vec{H} = \frac{2}{3\pi}\hat{z}\sin(2\pi\times10^{10}t - 837.76y)e^{-0.47y} + \frac{2}{3\pi}\hat{x}\cos(2\pi\times10^{10}t - 837.76y)e^{-0.47y}$ mA/m

(d) $\vec{S}_{av} = \hat{y}\frac{|\tilde{E}(0)|^2}{2\eta_2}e^{-2\alpha y}\cos\theta_{\eta_2} = \hat{y}\frac{20^2\times2}{2\times30\pi}e^{-0.94y} = \hat{y}\,4.24\times10^{-6}e^{-0.94y}$ W/m²

(e). Set $y = 1$m. $\vec{S}_{av} \doteq \hat{y}\,1.66\times10^{-6}$ W/m².



Problem 2 (30 points)

In order to maximize the efficiency of a Germanium (n = 4) photodetector, a thin layer of coating covers the photodetector photo-absorbing facet.

a) Determine the optimum refractive index and thickness of the coating layer for a normally incident optical beam with a 1300 nm wavelength.
b) Assuming we use the coating layer thickness and refractive index calculated in part (a) calculate the portion of the optical power that is transmitted from air to Germanium for a normally incident optical beam with a 1300 nm wavelength.
c) Assuming we use the coating layer thickness and refractive index calculated in part (a) calculate the portion of the optical power that is transmitted from air to Germanium for a normally incident optical beam with an 800 nm wavelength.

(a) To maximize the efficiency $\Rightarrow R=0.\ T=1 \Rightarrow \Gamma=0$

method #1. The coating is an analogue to $\lambda/4$ transformer.

$$\Rightarrow \quad l = \frac{\lambda_2}{4} + m'\frac{\lambda_2}{2} = \frac{\lambda}{4n_2} + m'\frac{\lambda}{2n_2} = \frac{\lambda}{8} + m'\frac{\lambda}{4} = (162.5 + m'325)\ \text{nm}. \quad m'=0,1,2,\ldots$$

$$\eta_2 = \sqrt{\eta_3\eta_1} = \sqrt{\frac{\eta_1^2}{4}} = \frac{\eta_1}{2} \Rightarrow \text{refractive index } n_2 = 2$$

method #2.
$$\Gamma = \frac{\eta_{in}-\eta_1}{\eta_{in}+\eta_1} = 0 \Rightarrow \eta_{in} = \eta_2\frac{\eta_3\cos\beta l + j\eta_2\sin\beta l}{\eta_2\cos\beta l + j\eta_3\sin\beta l} = \eta_1$$

$$\Rightarrow \left.\begin{aligned} \eta_3\eta_2\cos\beta l &= \eta_2\eta_1\cos\beta l \\ \eta_2^2\sin\beta l &= \eta_3\eta_1\sin\beta l \end{aligned}\right\} \Rightarrow \begin{aligned} &\cos\beta l = 0 \\ &\eta_2^2 = \eta_3\eta_1 \end{aligned} \Rightarrow \begin{aligned} &\beta l = \frac{\pi}{2} + m'\pi.\ m'=0,1,2,\ldots \\ &\eta_2 = \frac{\eta_1}{2} \end{aligned}$$

$$\Rightarrow l = (162.5 + m'325)\ \text{nm}. \quad n_2 = 2$$

(b) 100%

(c).
$$\beta l = \frac{2\pi n_2}{\lambda'}\cdot\frac{\lambda}{8} = \frac{4\pi}{800}\cdot\frac{1300}{8} = \frac{13\pi}{16}$$

$$\eta_{in} = 60\pi\frac{30\pi\cos\beta l + j60\pi\sin\beta l}{60\pi\cos\beta l + j30\pi\sin\beta l} \doteq 60\pi(0.65-0.45i) \doteq 47.4\pi e^{-j34.7^\circ}\ \Omega$$

$$\Gamma = -0.47 - 0.35i = 0.53e^{-j152^\circ}. \quad T = 1 - 0.53^2 \doteq 72\%$$

Problem 3 (30 points)

A 30 GHz, TE-polarized electromagnetic wave is propagating in air with an average power density of 1 W/m^2. The wave is incident on an infinitely thick dielectric with a relative permittivity of 4, relative permeability of 1, and conductivity of 0.01 S/m at an incident angle of 45°.

a) Determine the propagation constant, attenuation constant, intrinsic impedance, and phase velocity of the transmitted wave into the dielectric.

b) Determine the average power density of the transmitted wave into the dielectric. What is the angle of the transmitted wave relative to the air/dielectric interface? Is the polarization of the transmitted wave into the dielectric TE or TM?

(a) $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{0.01}{2\pi\times30\times10^9\times8.85\times10^{-12}\times4} = 0.0015 < 0.01$ ⇒ low-loss dielectric material.

method #1. $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{0.01}{2}\frac{120\pi}{\sqrt{\varepsilon_r}} = 0.3\pi \doteq 0.942$ Np/m

$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\varepsilon_r} = 400\pi = 1256.6$ rad/m.

propagation constant $\gamma = \alpha + j\beta = 0.942 + 1256.6j$ m^{-1}

method #2. $\gamma = j\omega\sqrt{\mu\varepsilon_c} = j30\times10^9\sqrt{\mu(\varepsilon-\frac{\sigma}{\omega})} = 0.942 + 1256.6j$ m^{-1}.

$\alpha = 0.942$ Np/m

$\eta_c = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = 60\pi\ \Omega$. $u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}} = 1.5\times10^8$ m/s

(b) $n_i\sin\theta_i = n_t\sin\theta_t$. $\sin\theta_t = \frac{n_i\sin\theta_i}{n_t} = \frac{\sqrt{2}}{4} \Rightarrow \theta_t = 20.7°$. $\cos\theta_t = \frac{\sqrt{14}}{4}$

method #1

$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{120\pi\cdot\frac{\sqrt{2}}{2}}{60\pi\cdot\frac{\sqrt{2}}{2} + 120\pi\cdot\frac{\sqrt{14}}{4}} \doteq 0.549$. $\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \doteq -0.45$

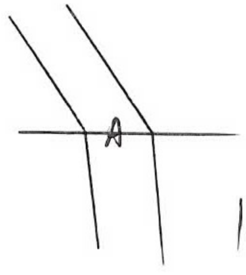
Since $\frac{|\tilde{E}|^2}{2\eta_1} = 1$ W/m^2 ⇒ $|\tilde{E}| = \sqrt{2\eta_0} = 27.5$ V/m

$|S_{tran}| = \frac{|\tau_\perp\tilde{E}|^2}{2\eta_2} \doteq 0.603$ W/m^2

$\alpha = 90° - \theta_t = 69.3°$

Polarization: TE.

method #2



$T = \frac{P_t}{P_{in}} = \frac{S_t A\cos20.7°}{S_{in} A\cos45°} = |\tau_\perp|^2\frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i} = 1 - |\Gamma_\perp|^2 \doteq 0.796$

$|S_{tran}| = T S_{in}\frac{\cos45°}{\cos20.7°} \doteq 0.602$ W/m^2.