SOLUTIONS

UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Winter 2016
Quiz 1, January 27 2016 (20 minutes)

Name ______________________ Student number______________________

This is a closed book quiz – no notes or equations.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Electric Field	50	
Problem 2	Capacitor and Polarization	50	
Total		100	

Maxwell's Equations:

$$\nabla \bullet \mathbf{D} = \rho_f$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

Auxillary Fields:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

In linear media:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \qquad \mathbf{D} = \varepsilon \mathbf{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$\mathbf{M} = \chi_m \mathbf{H} \qquad \mathbf{B} = \mu \mathbf{H} \qquad \mu = \mu_0 (1 + \chi_m)$$

Electrostatic Potential: $\mathbf{E} = -\nabla V$ Vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

Gradient Theorem: $\int_a^b (\nabla f) \bullet d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\int_V (\nabla \bullet \mathbf{A})\, dV = \oint_S \mathbf{A} \bullet d\mathbf{S}$

Stokes's Theorem: $\int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S} = \oint_C \mathbf{A} \bullet d\mathbf{l}$

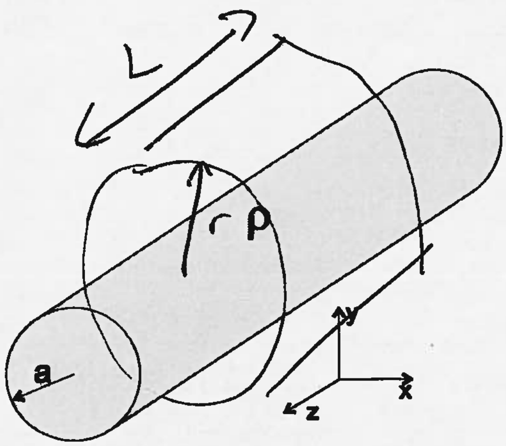
Electric energy density: $W_e = \frac{1}{2} \mathbf{E} \bullet \mathbf{D}$ or $W_e = \frac{1}{2} \varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m = \frac{1}{2} \mathbf{B} \bullet \mathbf{H}$ or $W_m = \frac{1}{2} \mu H^2$ (in linear media)

Capacitance: $C = \frac{Q}{V}$ Inductance: $L = \frac{\Lambda}{I} = N \frac{\Phi}{I}$

1. Consider an infinitely long cylinder of charge (with uniform charge density ρ) with ~~diameter~~ radius a: The permittivity is ε_0 everywhere. Find an expression for the electric field both for $r<a$ and $r>a$. Make sure to include the vector direction.

Cylindrical Gaussian surface w/ length L and radius r.



For $r<a$

$$\oint_S \vec{E}\cdot d\vec{S} = \int_V \frac{\rho}{\varepsilon_0}\,dV$$

$$2\pi r L E_r = \frac{\pi r^2 L \rho}{\varepsilon_0}$$

$$E_r = \frac{r\rho}{2\varepsilon_0}$$

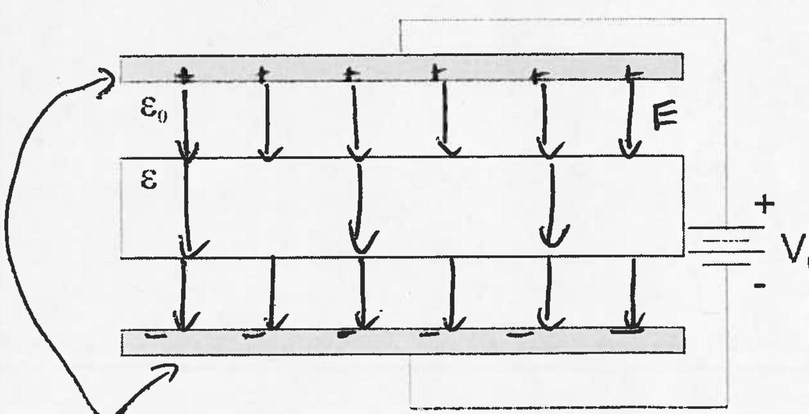
For $r>a$

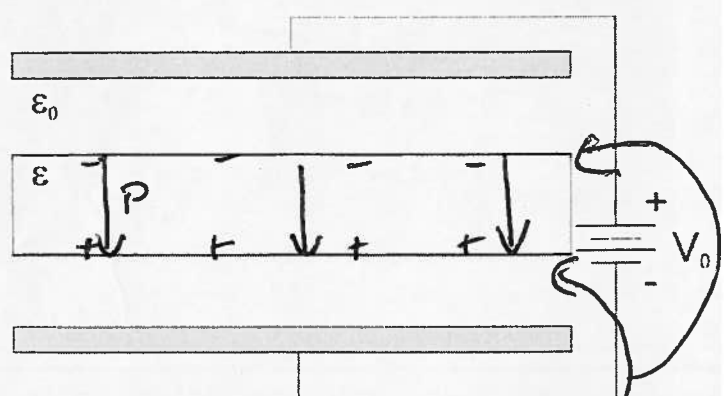
$$2\pi r L E_r = \frac{\pi a^2 L\rho}{\varepsilon_0}$$

$$E_r = \frac{a^2\rho}{2r\varepsilon_0}$$

$$\vec{E} = \begin{cases} \hat{r}\,\dfrac{r\rho}{2\varepsilon_0} & (r<a) \\ \hat{r}\,\dfrac{a^2\rho}{2r\varepsilon_0} & (r>a) \end{cases}$$

2. Consider a parallel plate capacitor with a potential difference V_0 applied across the plates (assume the plates are perfect conductors). The capacitor has a piece of perfectly insulating dielectric in between the plates, that partially fills the gap.
On the left figure, sketch the electric field lines **E** inside the dielectric, and the location and sign of the free charge. On the right figure, sketch the polarization field **P**, and sketch the location and sign of the bound charge. **Please be precise and neat!**





Free surface charge

$\varepsilon_0 E_1 = \varepsilon E_2$

$E_1 = \frac{\varepsilon}{\varepsilon_0} E_2$ E is weaker inside dielectric

Bound Surface Charge