SOLUTIONS



# UCLA Department of Electrical Engineering
## EE101A – Engineering Electromagnetics
## Fall 2015
## Quiz 1, October 15 2015, (20 minutes)

Name ______________________________ Student number______________________________

This is a closed book quiz – no notes or equations.

Please be neat – we cannot grade what we cannot decipher.

| | Topic | Max Points | Your points |
|---|---|---|---|
| Problem 1 | Electric Field | 50 | |
| Problem 2 | Capacitance | 50 | |
| Total | | 100 | |

Maxwell's Equations:

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

Auxillary Fields:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

In linear media:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \qquad \mathbf{D} = \varepsilon \mathbf{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$\mathbf{M} = \chi_m \mathbf{H} \qquad \mathbf{B} = \mu \mathbf{H} \qquad \mu = \mu_0 (1 + \chi_m)$$

Electrostatic Potential: $\mathbf{E} = -\nabla V$ Vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

Gradient Theorem: $\int_a^b (\nabla f) \cdot d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\int_V (\nabla \cdot \mathbf{A})\, dV = \oint_S \mathbf{A} \cdot d\mathbf{S}$

Stokes's Theorem: $\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$

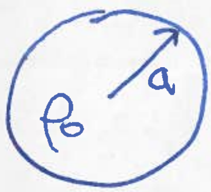
Electric energy density: $W_e = \frac{1}{2} \mathbf{E} \cdot \mathbf{D}$ or $W_e = \frac{1}{2} \varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m = \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$ or $W_m = \frac{1}{2} \mu H^2$ (in linear media)

Capacitance: $C = \frac{Q}{V}$ Inductance: $L = \frac{\Lambda}{I} = N \frac{\Phi}{I}$

1. Consider a sphere of radius $a$, that is composed of a uniform positive charge density $\rho_0$. This sphere of charge is NOT a conductor. The permittivity is $\varepsilon_0$ both inside and outside the sphere. Find the electric field inside the sphere ($R<a$) and outside the sphere ($R>a$). Don't forget to give the vector direction.

Use Gauss's Law: $\vec{E} = E_R \hat{R}$ $\oint_S \vec{E} \cdot d\vec{S} = \int_V \frac{\rho}{\varepsilon_0} dV$




Inside sphere

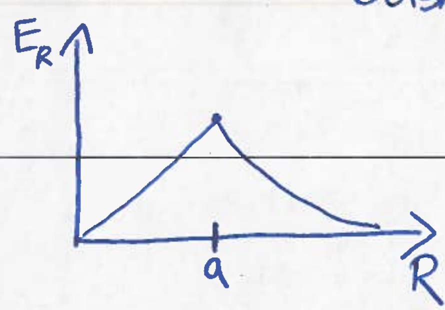
$$\oint \vec{E} \cdot d\vec{S} = \int_V \frac{\rho}{\varepsilon_0} dV$$

$$\int_0^{\pi} d\theta \int_0^{2\pi} d\phi \sin\theta \, E_R R^2 = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \sin\theta \int_0^R dR \frac{\rho_0}{\varepsilon_0}$$

$$E_R \, 4\pi R^2 = \frac{4}{3}\pi R^3 \rho_0/\varepsilon_0$$

$$\boxed{\vec{E} = \hat{R} \frac{R\rho_0}{3\varepsilon_0}} \quad R<a$$

Outside sphere




$$E_R \, 4\pi R^2 = \frac{4}{3}\pi a^3 \rho_0/\varepsilon_0$$

$$\boxed{\vec{E} = \frac{a^3 \rho_0}{3R^2\varepsilon_0} \hat{R}} \quad R>a$$

2. Consider a parallel plate capacitor with a potential difference $V_0$ applied across the plates. On the left figure, sketch the electric field lines **E** inside the dielectric, and the location and sign of the free charge. On the right side, sketch the polarization field **P**, and sketch the location and sign of the bound charge. **Please be precise and neat!**

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$