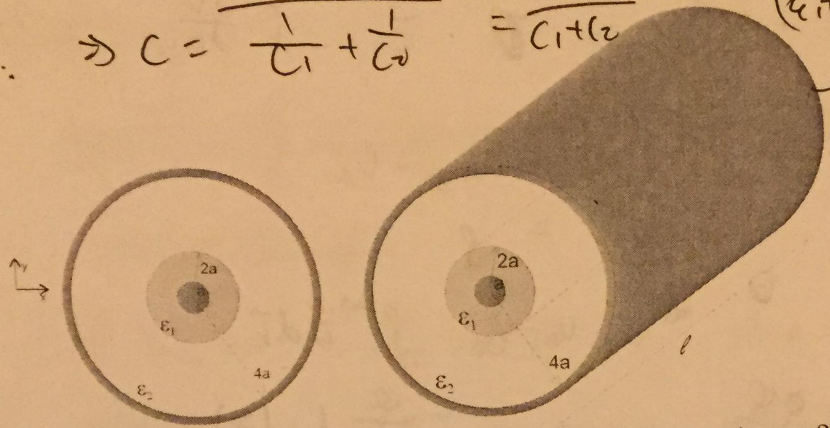
1. Coaxial capacitor (40 points)

Consider a piece of coaxial cable of length ℓ with two dielectric layers with permittivities ε_1 and ε_2. You may consider the inner conductor (radius a) and the outer conductor shell (radius $4a$) to be perfect conductors.

Answer $\therefore \Rightarrow C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1C_2}{C_1+C_2} = \frac{2\pi\varepsilon_1\varepsilon_2 l}{(\varepsilon_1+\varepsilon_2)\ln(2)}$



(a) (20 points) What is the capacitance C between the inner and outer conductors? (Your answer should be in terms of only geometric and material parameters such as ℓ, a, ε_1, ε_2)

EQS.

C_1 ε_1 $+Q$ $-Q$.

C_2 ε_2.

$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon}$

$\vec{E}(r) = \hat{r}\cdot\frac{Q}{2\pi r l\varepsilon}$

$\nabla V = -\frac{\rho}{\varepsilon}$

$\sim\sim\sim$

$V_{2a}-V_a$. $\Rightarrow -\int_a^{2a}\vec{E}\,d\vec{l} = -\frac{Q}{2\pi l\varepsilon}\int_a^{2a}\frac{1}{r}dr = -\frac{Q}{2\pi l\varepsilon}\ln(2) = \frac{Q}{2\pi l\varepsilon}\ln(\frac{1}{2})$

$\Rightarrow C_1 = |\frac{Q}{V}| = \frac{2\pi\varepsilon_1 l}{\ln(2)}$

$\Rightarrow$ similarly. $C_2 = \frac{2\pi\varepsilon_2 l}{\ln(\frac{4a}{2a})} = \frac{2\pi\varepsilon_2 l}{\ln(2)}$

$\Rightarrow$ ~~$\frac{C_1+C_2}{C_1C_2} = \frac{\ln(2)}{2\pi\varepsilon_1\varepsilon_2 l}$~~.

(b) (20 point) If a potential difference of V_0 volts is applied between the inner and outer conductor such that $V(r = a)$-$V(r = 4a) = V_0$ Volts, write an expression for the electric field inside the dielectric (i.e. between r=a and r=$4a$) in terms of V_0. There should not be an expression for charge or charge density in your final expression.

Ans. $\vec{E} = -\nabla\vec{V} \Rightarrow \vec{E} = -\hat{r}\frac{\partial V}{\partial r}$.

$\varepsilon_1 \quad \Rightarrow -\int_a^{2a} \vec{E}\,dr = V_{2a} - V_a$.

$= \rho$

$\Rightarrow \quad V_a - V_{2a} = \int_a^{2a} \vec{E}\,d\vec{r}$

$\vec{E} = \frac{Q}{2\pi r l \varepsilon}$. $\qquad = \frac{Q}{2\pi l\varepsilon_1}\ln(2)$.

$\varepsilon_2 \Rightarrow$ similarly. $\frac{V_{2a} - V_{4a}}{\cancel{V_{4a} - V_{2a}}} = \frac{Q}{2\pi l\varepsilon_2}\ln(2)$.

$V_a - V_{4a} = \frac{Q\ln(2)}{2\pi l}\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}\right) = V_0$.

$\Rightarrow Q = \frac{V_0\,\varepsilon_1\varepsilon_2\,2\pi l}{\varepsilon_1+\varepsilon_2\,\ln(2)}$.

$\Rightarrow \vec{E} = \frac{V_0}{r\ln(2)} \cdot \frac{\varepsilon_2}{(\varepsilon_1+\varepsilon_2)}\hat{r} \qquad a < r < 2a$

$\vec{E} = \frac{V_0}{r\ln(2)} \cdot \frac{\varepsilon_1}{(\varepsilon_1+\varepsilon_2)}\hat{r} \qquad 2a < r < 4a$

2. Conductors and fields (30 points)

30

(a) (10 points) Explain qualitatively/physically why the E-field must go to zero inside a perfect conductor.

~~If $\vec{E}_{in} \neq 0$.~~

~~$\nabla \cdot \vec{E} = \frac{\rho_f}{\varepsilon}$ $\rho_f = 0$ in perfect conductor~~

~~$\Rightarrow \nabla \cdot \vec{E} = 0$~~

If $\vec{E}_{in} \neq 0$. $\vec{J}_f = \sigma \vec{E} \neq 0$ since the charge would be free to move until it cancels the external E-field. 10/10

$\vec{J}_f = 0$ $\Rightarrow$ ~~$\neq 0$~~

But $\rho_f = 0$ in perfect conductor. contradiction

$\Rightarrow \vec{E}_{in}$ must be zero.

(b) (10 points) Explain qualitatively/physically why the B-field must go to zero inside a perfect conductor.

Whenever an external B-field is applied on a perfect conductor. It will generate a Vemf that cancels out the ~~B-f~~ external $\vec{B}$ immediately. Thus $\vec{B}_{total}$ is always zero inside the conductor. 10

(c) (10 points) In a real conductor where σ is finite, we can only make approximations. Does a finite conductor act more like a perfect conductor at high or low frequencies for E-fields? How about for B-fields? Explain why.

perfect conductor for $\vec{E}$ → EQS. ✓

→ after τ_e → long time & low freq.

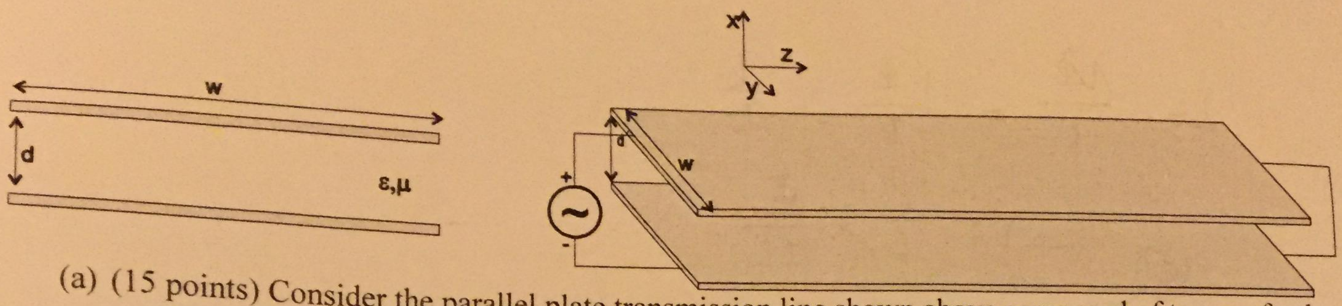
perfect conductor for $\vec{B}$ → MQS. ✓

→ ~~[illegible]~~ ~~[illegible]~~ before i induced ~~dies out~~ dies out. → during τ_m.

→ short time & ~~large~~ ~~high freq~~, high freq.

10

3. Inductance of parallel plate transmission lines (30 points)



(a) (15 points) Consider the parallel plate transmission line shown above composed of two perfectly conducting thin plates. The width is much greater than the plate separation $w \gg d$. Assume that current $+I$ is flowing on the top plate (i.e. in the z direction), and current $-I$ is flowing on the bottom plate (i.e. in the $-z$ direction) (as is implied by the circuit diagram above). Give an expression for the B-field in between the plates. Your answer should be in terms of I, and the geometric and material parameters (make sure to include the vector direction).

MDS

Due to rhr, $\vec{B} = B\cdot(-\hat{y})$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$\cancel{\vec{B} = \vec{B}_1 + \vec{B}_2 = 2\vec{B}_1 = 2B_1(-\hat{y})}$

$\cancel{\vec{B}_1 = \mu\cdot\frac{I}{w} = \frac{\mu I}{w}}$

$B\cdot l = \left(\frac{I}{w}\cdot l\right)\mu.$

$\Rightarrow B = \frac{I\mu}{w}.$

$\cancel{\Rightarrow \vec{B} = \frac{2\mu I}{w}(-\hat{y})}$

$\Rightarrow \vec{B} = \frac{\mu I}{w}(-\hat{y})$ +15

(b) (15 points) What is the inductance per unit length for this parallel plate transmission line?

$$L = \frac{\Lambda}{I} = \frac{1 \cdot \Phi}{I} = \frac{\Phi}{I}.$$

$$L' = \frac{L}{l} = \frac{\Phi}{Il}.$$

$\Phi =$

$$\cancel{\frac{\Phi}{l} = \frac{\int \oint \vec{B} \cdot d\vec{s}\, dl}{l} = \oint \vec{B} \cdot d\vec{s} = B\,dw}$$

$$\cancel{\Rightarrow L' = \frac{B\,dw}{I} = 2\mu d.}$$

$$\frac{\Phi}{l} = \frac{\oint \vec{B}\, d\vec{s}}{l} = \frac{\vec{B}\, dl}{l} = Bd.$$

$$\Rightarrow L' = \frac{Bd}{I} = \frac{\frac{\mu I}{w} d}{I} = \frac{\mu d}{w}.$$

$$\Rightarrow L' = \frac{\mu d}{w}.$$ +15