SOLUTIONS

UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Winter 2014
Midterm, February 10 2014, (1:45 minutes)

Name ______________________________ Student number______________________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

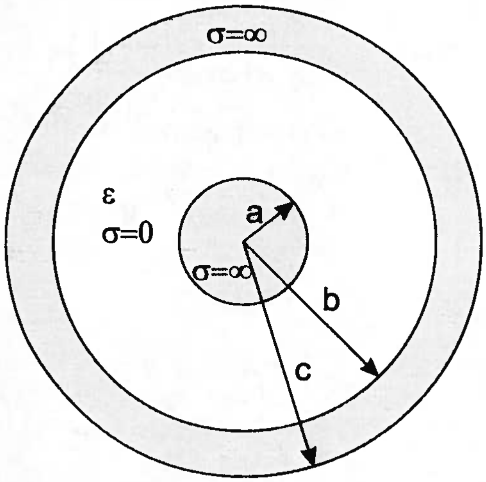
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Spherical Capacitor	60	
Problem 2	Transmission Line	40	
Total		100	

This page intentionally left blank.

1. Spherical Capacitor (60 points)



(a) (20 points) Consider a perfectly conducting metal sphere of radius a, surrounded by an uncharged dielectric of permittivity ε, and a separate spherical metal shell of inner radius b and outer radius c. What is the capacitance of this structure (i.e. between the inner and outer conductors?

Let center have charge $+Q$ & outer shell have charge $-Q$.

Charge density on center $\rho_s = \frac{Q}{4\pi a^2}$. On inner surface of outer conductor $\rho_s = \frac{-Q}{4\pi b^2}$

Inside dielectric

$$\vec{E} = \hat{R}\frac{Q}{4\pi\varepsilon R^2} \qquad V = \frac{Q}{4\pi\varepsilon R} \qquad V(a) = \frac{Q}{4\pi\varepsilon a}$$

$$V(b) = \frac{Q}{4\pi\varepsilon b}$$

$$V_0 = V(a) - V(b) = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon}\left(\frac{b-a}{ab}\right)$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}} = 4\pi\varepsilon\frac{ab}{b-a}}$$

(b) (20 points) Now consider that the dielectric in between the conductor has acquired a uniform volume charge density ρ_0. Assume that both the center conductor and shell are grounded together so that the scalar potential V=0 on each conductor.
Inside the dielectric, the scalar potential has the form of :

$$V(R) = -\frac{R^2\rho_0}{6\varepsilon} - \frac{C}{R} + D,$$

This equation satisfies Poisson's Eq in dielectric $\nabla^2 V = -\rho/\varepsilon$

where C and D are unknown coefficients. Using the information given, find values for C and D.

boundary conditions $V(a)=0$ $V(b)=0$

$$-\frac{a^2\rho_0}{6\varepsilon} - \frac{C}{a} + D = 0 \qquad -\frac{b^2\rho_0}{6\varepsilon} - \frac{C}{b} + D = 0$$

$$D = \frac{C}{a} + \frac{a^2\rho_0}{6\varepsilon}$$

$$-\frac{b^2\rho_0}{6\varepsilon} - \frac{C}{b} + \frac{C}{a} + \frac{a^2\rho_0}{6\varepsilon} = 0$$

$$-\frac{\rho_0(b^2-a^2)}{6\varepsilon} + C\left(\frac{1}{a} - \frac{1}{b}\right) = 0$$

$$C = \frac{\rho_0(b^2-a^2)}{6\varepsilon\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{\rho_0 ab(b^2-a^2)}{6\varepsilon(b-a)} = \frac{\rho_0 ab(b+a)}{6\varepsilon}$$

$$D = \frac{C}{a} + \frac{a^2\rho_0}{6\varepsilon} = \frac{\rho_0(b^2-a^2)}{6\varepsilon(1-\frac{a}{b})} + \frac{a^2\rho_0}{6\varepsilon} = \frac{\rho_0 b(b+a)}{6\varepsilon} + \frac{a^2\rho_0}{6\varepsilon}$$

$$D = \frac{\rho_0(a^2+b^2+ab)}{6\varepsilon}$$

$$V(R) = \frac{\rho_0}{6\varepsilon}\left(-R^2 - \frac{ab(b+a)}{R} + a^2+b^2+ab\right)$$

(c) (20 points) For the same conditions as in (b), what is the free surface charge density on the inner s? Conductor

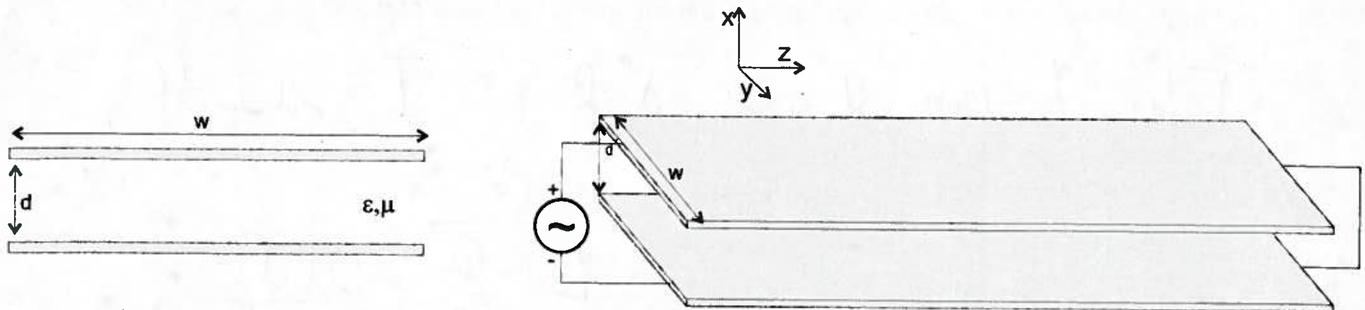
$$E = -\nabla V = -\frac{\partial V}{\partial R}\hat{R}$$

$$E = \frac{\rho_0}{6\varepsilon}\left(2R - \frac{ab(b+a)}{R^2}\right)\hat{R}$$

$$\boxed{\rho_s = \varepsilon E_R = \frac{\rho_0}{6}\left(2a - \frac{b(b+a)}{a}\right)} = \frac{\rho_0 a}{3\varepsilon} - \frac{c\varepsilon}{a^2}$$

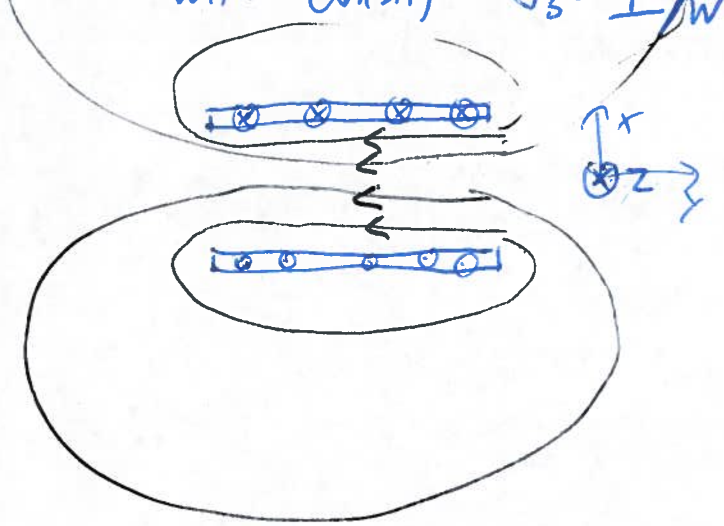
$$= \frac{\rho_0}{6a}\left(2a^2 - b^2 - ab\right) = \frac{\rho_0}{6a}(2a+b)(a-b)$$

2. Parallel plate transmission lines (40 points)



(a) (20 points) Consider the parallel plate transmission line shown above composed of two perfectly conducting thin plates. The width is much greater than the plate separation $w \gg d$. Assume that current $+I$ is flowing on the top plate (i.e. in the z direction), and current $-I$ is flowing on the bottom plate (i.e. in the $-z$ direction) (as is implied by the circuit diagram above). Give an expression for the B-field in between the plates. Your answer should be in terms of I, and the geometric and material parameters (make sure to include the vector direction).

Since the plates are thin, we treat them as surface currents with density $J_s = I/w$.



Since $w \gg d$ we can approximate the field inside the plates as due to an infinitely wide sheet current.

Bnd cond: $H_{2t} - H_{1t} = J_s$

Outside $H_{1t} \approx 0$

Inside $\vec{H} = -\hat{y} I/w$

$$\vec{B} = -\hat{y}\mu I/w$$

(b) (20 points) What is the inductance per unit length for this parallel plate transmission line?

Flux for loop of size $d \times l$ = $\Phi = \frac{\mu I d l}{w}$

$$L = \frac{\Phi}{I} = \frac{\mu d l}{w}$$

$$\boxed{\frac{L}{l} = \frac{\mu d}{w}}$$

Valid for parallel plate transmission line when $w \gg d$.

This page is left blank intentionally – use it for scrap paper.

This page is left blank intentionally – use it for scrap paper.

SOME USEFUL VECTOR IDENTITIES

$\mathbf{A} \cdot \mathbf{B} = AB \cos\theta_{AB}$ Scalar (or dot) product

$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB \sin\theta_{AB}$ Vector (or cross) product, $\hat{\mathbf{n}}$ normal to plane containing **A** and **B**

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

$\nabla(U + V) = \nabla U + \nabla V$

$\nabla(UV) = U\nabla V + V\nabla U$

$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$

$\nabla \cdot (U\mathbf{A}) = U\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla U$

$\nabla \times (U\mathbf{A}) = U\nabla \times \mathbf{A} + \nabla U \times \mathbf{A}$

$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times \nabla V = 0$

$\nabla \cdot \nabla V = \nabla^2 V$

$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$\int_{v} (\nabla \cdot \mathbf{A}) dv = \oint_{S} \mathbf{A} \cdot d\mathbf{s}$ Divergence theorem (S encloses v)

$\int_{S} (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_{C} \mathbf{A} \cdot d\mathbf{l}$ Stokes's theorem (S bounded by C)

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table 3-1: **Summary of vector relations.**

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A, $\lvert A\rvert$ =	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1}$ =	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l}$ =	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}$ =	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: **Coordinate transformation relations.**

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Maxwell's Equations in media:

$$\nabla\cdot\mathbf{D} = \rho_f$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}$$

Auxillary Fields: $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$, $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{M}$

In linear media: $\mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$ $\quad \mathbf{D} = \varepsilon\mathbf{E}$; $\mathbf{M} = \chi_m\mathbf{H}$ $\quad \mathbf{B} = \mu\mathbf{H}$

Ohm's law: $\mathbf{J}_f = \sigma\mathbf{E}$

Electrostatic Scalar Potential: $\mathbf{E} = -\nabla V$ $\qquad$ Vector potential: $\mathbf{B} = \nabla\times\mathbf{A}$

Electrodynamic Potential: $\mathbf{E} = -\nabla V - \dfrac{\partial\mathbf{A}}{\partial t}$

Gradient Theorem: $\displaystyle\int_a^b (\nabla f)\cdot d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\displaystyle\int_V (\nabla\cdot\mathbf{A})\,dV = \oint_S \mathbf{A}\cdot d\mathbf{S}$

Stokes's Theorem: $\displaystyle\int_S (\nabla\times\mathbf{A})\cdot d\mathbf{S} = \oint_C \mathbf{A}\cdot d\mathbf{l}$

Electric energy density: $W_e = \dfrac{1}{2}\mathbf{E}\cdot\mathbf{D}$ or $W_e = \dfrac{1}{2}\varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m = \dfrac{1}{2}\mathbf{B}\cdot\mathbf{H}$ or $W_m = \dfrac{1}{2}\mu H^2$ (in linear media)

Joule power dissipation density: $W_P = \mathbf{E}\cdot\mathbf{J}$ or $W_m = \sigma E^2$ (in Ohm's law media)

Poynting Vector: $\mathbf{S} = \mathbf{E}\times\mathbf{H}$

Time averaged Poynting vector: $\mathbf{S}_{av} = \dfrac{1}{2}\mathrm{Re}\{\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*\}$

Capacitance: $C = \dfrac{Q}{V}$

Inductance: $L = \dfrac{\Lambda}{I} = N\dfrac{\Phi}{I}$

Boundary conditions: $E_{t,2} - E_{t,1} = 0$ $\qquad H_{t,1} - H_{t,2} = J_s$; $D_{n,2} - D_{n,1} = \rho_s$ $\qquad B_{n,2} - B_{n,1} = 0$

Bound charge: $\rho_{b,v} = -\nabla\cdot\mathbf{P}$ $\qquad \rho_{b,s} = \mathbf{P}\cdot\hat{\mathbf{n}}$

Bound current: $\mathbf{J}_{b,v} = \nabla\times\mathbf{M}$ $\qquad \mathbf{J}_{b,s} = \mathbf{M}\times\hat{\mathbf{n}}$

Definition of phasor $\tilde{F}$ for time harmonic function *f*(*t*):

$$\begin{cases} f(t) = \mathrm{Re}\{\tilde{F}e^{j\omega t}\} = |F|\cos(\omega t + \phi) \\ \tan^{-1}(\phi) = \mathrm{Im}\{\tilde{F}\}/\mathrm{Re}\{\tilde{F}\} \end{cases}$$

Constants (SI units): ε_0=8.85x10^{-12} F/m (or $C^2\,N^{-1}\,m^{-2}$) $\qquad \mu_0$=4π x10^{-7} H/m (or $N\,A^{-2}$)