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UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2015
Midterm, November 3 2015, (1:45 minutes)

Name _ _ Student number_

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

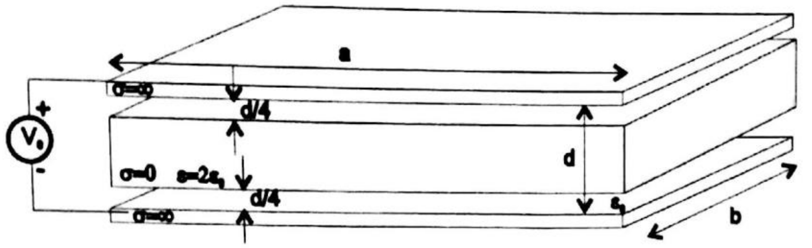
Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

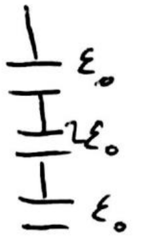
Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Capacitor	40	40
Problem 2	Conductors and fields	30	30
Problem 3	Transmission Line	30	30
Total		100	100

1. Capacitor (40 points)



10 (a) (10 points) Consider a parallel plate capacitor with the metal plates separated by d, that is partially filled with a dielectric with permittivity $\varepsilon = 2\varepsilon_0$ and thickness $d/2$. The rest of the gap (above and below) is vacuum. What is the capacitance of this capacitor?



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$C_1 = C_3$

$$C = \frac{\varepsilon A}{d}$$

so $$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{C_2}$$

so $$C_1 = \frac{\varepsilon_0 ab4}{d}$$

and $$C_2 = \frac{\varepsilon_0 ab4}{d}$$

$$\frac{1}{C_{eq}} = \frac{3d}{\varepsilon_0 ab4}$$

$$C = \frac{\varepsilon_0 ab4}{3d}$$

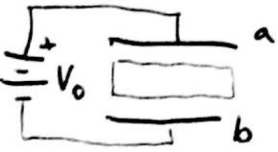
10

(b) (10 points) If a potential difference V_0 is applied to the capacitor, what is the E-field magnitude in the upper vacuum region, the dielectric, and the lower vacuum region.

let's call them region 1, 2, and 3 respectively

$|\vec{E}|$ in regions 1 and 3 should be the same

$$V_0 = V_a - V_b = \int_c \vec{E}\cdot d\vec{l} = \int_a^{a+\frac{d}{4}} \vec{E}\cdot d\vec{l} + \int_{a+\frac{d}{4}}^{a+\frac{d}{2}} \vec{E}\cdot d\vec{l} + \int_{a+\frac{d}{2}}^{b} \vec{E}\cdot d\vec{l}$$

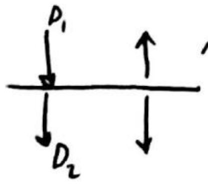


at each interface between 1, 2, 3

assign vector $\downarrow \hat{x}$

$$D_{1n} + D_{2n} = \rho_f = 0$$

surface normal vectors;



so $-\hat{x} D_1 + \hat{x} D_2 = 0$

$$D_1 = D_2$$

$$\varepsilon_0 E_1 = 2\varepsilon_0 E_2 = \varepsilon_0 E_3$$

$$E_1 = 2E_2 = E_3$$

$$V_0 = \frac{E_1 d}{4} + \frac{E_2 d}{2} + \frac{E_3 d}{4}$$

$$V_0 = \frac{E_1 d}{4} + \frac{E_1}{2}\frac{d}{2} + \frac{E_1 d}{4}$$

$$V_0 = \frac{E_1 d\, 3}{4}$$

so $E_1 = \frac{4V_0}{3d}$ $\quad E_2 = \frac{4V_0}{6d}$ $\quad E_3 = \frac{4V_0}{3d}$

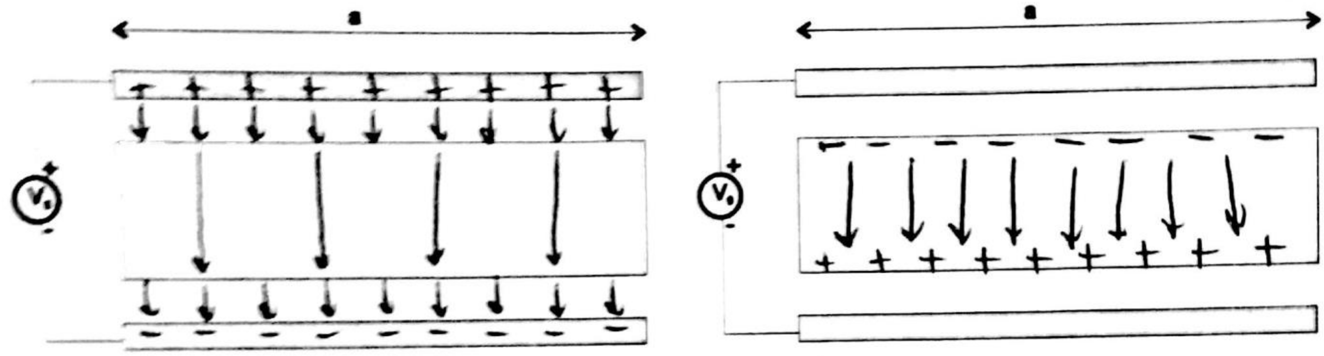
in reverse!

$$V_0 = \frac{4V_0}{3d}\left(\frac{d}{4}\right) + \frac{4V_0}{6d}\left(\frac{d}{2}\right) + \frac{4V_0}{3d}\left(\frac{d}{4}\right)$$

$$V_0 = \frac{V_0}{3} + \frac{V_0}{3} + \frac{V_0}{3} \qquad V_0 = V_0$$

10

(c) (10 points) On the diagrams below, on the left side of the picture, sketch the E-field vectors, and the location and sign of the free charge. On the right side of the picture, sketch the polarization field P vectors and the location and sign of the bound charge. Remember to pay attention to the relative strengths of the fields (stronger field means larger density of field vectors). not length



$$C = \frac{A\varepsilon}{d} = \frac{Q}{Ed}$$

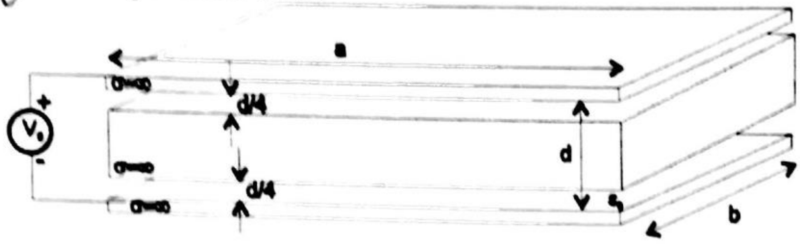
$$A\varepsilon = \frac{Q}{E} \qquad E = \frac{Q}{A\varepsilon}$$

$$E \propto \frac{Q}{\varepsilon}$$

$\uparrow\varepsilon$, $\downarrow E$ and according to part (b)

10

(d) (10 points) Now consider that the dielectric is replaced by a perfect conductor σ=∞. What is the capacitance now?



The perfect conductor screens ~~perfect~~ perfectly any magnetic fied, and it has no electric field within, due to ~~polarization~~ movement of charge that will cause an opposing electric field within to match the external field.

since $C = \frac{Q}{V} = \frac{\oint \rho_s \cdot d\vec{V}}{\int_c \vec{E} \cdot d\vec{l}}$

$V_a - V_b = \int_c \vec{E} \cdot d\vec{l}$ it is known that voltage will not drop within the ~~capacitor~~ conductor, and the E field outside the conductor is, in this case where all E field is normal to the conductor, unchanged, leaving $\int_c \vec{E} \cdot d\vec{l}$ to be only over half the conductor gap.

$C = \frac{Q}{V_a - V_b} = \frac{Q}{\frac{|\vec{E}|d}{2}}$

"The same E field," but for only half the distance

C is double the value without the dielectric

$C = \frac{2Q}{|\vec{E}|d}$

without dielectric: $C = \frac{A\varepsilon_0}{d}$

with perfect conductor: $C = \frac{2A\varepsilon_0}{d}$

where area $A = ab$

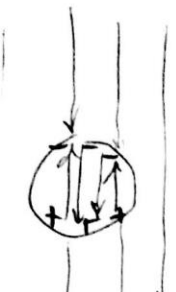
$C = \frac{2ab\varepsilon_0}{d}$

30

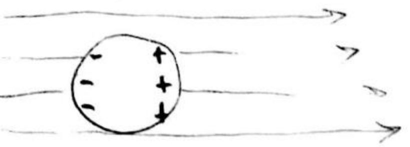
2. Conductors and fields (30 points)

(a) (10 points) Explain qualitatively/physically why the E-field must go to zero inside a perfect conductor.

As explained in the previous question, but to elaborate, Charges are free to move about the surface of the perfect conductor. An external E field will cause the charges on the conductor to move in such a way that creates an equal magnitude and opposite direction E field within the conductor.



The charges stop moving to one side, and stay, when the net electric field is zero at that time.



"Charges moved by the external field create an equal magnitude and opposing direction E field within the conductor" 10/10

(b) (10 points) Explain qualitatively/physically why the B-field must go to zero inside a perfect conductor.

B field must go to zero inside a perfect conductor because the field will induce a lasting current that creates an opposing B field. The net B field within the conductor is zero. unlike imperfect conductors, and insulators, This current has nothing to slow it down, to reduce it, so the opposition field is maintained. 10/10

(c) (10 points) In a real conductor where σ is finite, we can only make approximations. Does a finite conductor act more like a perfect conductor at high or low frequencies for E-fields? How about for B-fields? Explain why

E fields

at high frequencies, transient behavior dominates. steady state behavior

$\frac{\varepsilon}{\sigma} = T_e \propto \frac{1}{\sigma}$ High frequency can be interpreted as the time $t << T_e$ before the incident effect inverts.

charges move quickly about the surface, but do not cancel E field instantaneously.

as a capacitor acts, in high freq it does not build the opposing E field quick enough.

The real conductor acts more as a perfect conductor at low frequency when the opposing field has time to build. the perfect conductor needs no time $T_e \to 0$

10

B fields

at high frequencies transient behavior dominates steady state behavior

$T_m \propto \sigma$ The real conductor acts more like a perfect conductor at high frequency.

The transient behavior of currents creating opposing B fields does not have time to die out before the source is inverted. In a perfect conductor they do not die out for any amount of time.

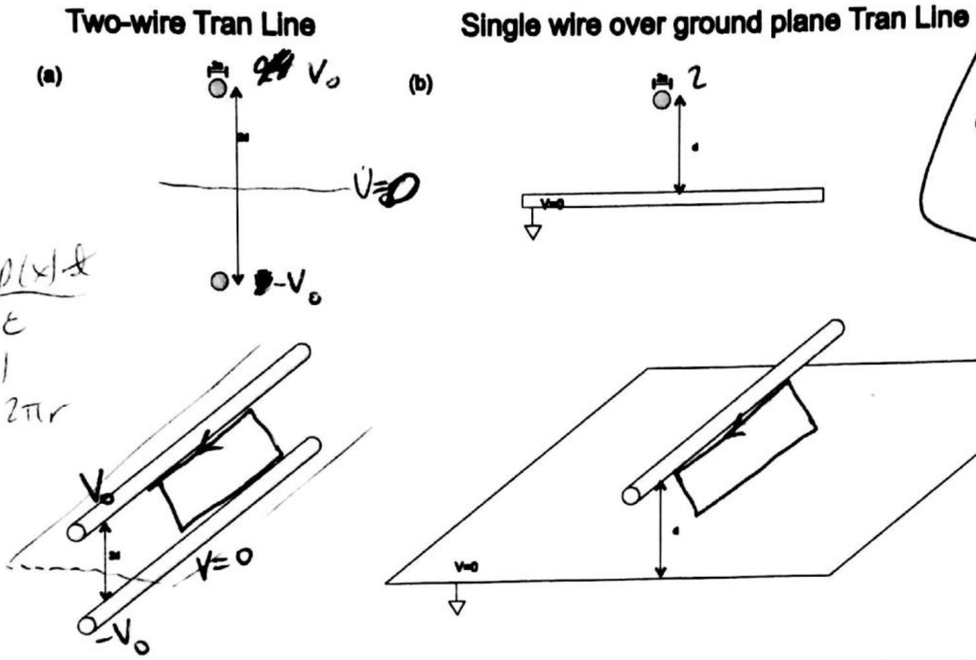
$T_m \to \infty$

the material has no resistivity in a perfect conductor.

3. Transmission line (30 points) 30/

Consider the lossless two-wire transmission line in case (a) and the single wire over a perfectly conducting semi-infinite ground plane shown in case (b).

extra reasoning on scratch paper



$E\, 2\pi r d = \frac{\rho(x) d}{\varepsilon}$

$E = \frac{\rho(x)}{\varepsilon\, 2\pi r}$

$\int_0^{2d}$

(a) (15 points) The capacitance per unit length of the two-wire line with dimensions shown above (wire radius = a, separation = $2d$) is approximately: $C' = \frac{C}{\ell} = \frac{\pi\varepsilon}{\ln(2d/a)}$ (valid when $d \gg a$).

15/

What is the capacitance per unit length of the single wire over ground plane (wire radius = a, distance from ground plane = d)?

To approah the single wire problem with ~~the~~ image ~~theory~~ method gives the same image as the two wire transmission line.

Infinite wire of charge

$\vec{E} = \frac{\rho(x)}{\varepsilon\, 2\pi r}\hat{r}$

Infinite wire current

$\vec{B} = \frac{\mu_0}{2\pi r} I_{enc} (\hat{J} \times \hat{r})$

There is only half of the voltage picture however, but similar charge is present.

prediction: $c' = \frac{C}{l} = \frac{2\pi\varepsilon}{\ln\left(\frac{2d}{a}\right)}$

since $C = \frac{Q}{V}$

$C \to \frac{2Q}{V}$

same E field, but only half the distance

was previously

$c' = \frac{C}{l} = \frac{2\pi\varepsilon}{\ln\left(\frac{2d}{a}\right)}$

(b) (15 points) The inductance per unit length of the same two-wire line is approximately:

5/ $L' = \frac{L}{\ell} = \frac{\mu}{\pi}\ln(2d/a)$. What is the inductance per unit length of the single wire over ground ?

once again the image method shows the same picture as the two wire line.

$L = \frac{\Phi N}{I}$

in this case $N=1$
and I does not change between the image and the two wire.
Φ changes to half the area

if $\Phi = \int \vec{B} \cdot d\vec{s}$
$= \vec{B}A$

area $= \frac{A_0}{2}$ where $A_0 \leftarrow$ 2 wire problem

prediction: $L' = \frac{L}{\ell} = \frac{\mu}{2\pi} \ln\left(\frac{2d}{a}\right)$

→ is half the area half the flux? yes by symmetry

vs

was $L' = \frac{L}{\ell} = \frac{\mu}{2\pi} \ln\left(\frac{2d}{a}\right)$ ✓

→ changing area bounds changes the integral $\Phi = \int \vec{B} \cdot d\vec{s}$ $B \propto \frac{1}{r}$ of a wire

Maxwell's Equations in media:

$$\nabla\cdot\mathbf{D} = \rho_f$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}$$

Auxillary Fields: $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ $\quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$

In linear media: $\mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$ $\quad \mathbf{D} = \varepsilon\mathbf{E}$

$\mathbf{M} = \chi_m\mathbf{H}$ $\quad \mathbf{B} = \mu\mathbf{H}$

Ohm's law: $\mathbf{J}_f = \sigma\mathbf{E}$

Electrostatic Scalar Potential: $\mathbf{E} = -\nabla V$ $\quad$ Vector potential: $\mathbf{B} = \nabla\times\mathbf{A}$

Electrodynamic Potential: $\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t}$

Gradient Theorem: $\int_a^b (\nabla f)\cdot d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\int_V (\nabla\cdot\mathbf{A})\,dV = \oint_S \mathbf{A}\cdot d\mathbf{S}$

Stokes's Theorem: $\int_S (\nabla\times\mathbf{A})\cdot d\mathbf{S} = \oint_C \mathbf{A}\cdot d\mathbf{l}$

Electric energy density: $W_e = \frac{1}{2}\mathbf{E}\cdot\mathbf{D}$ or $W_e = \frac{1}{2}\varepsilon E^2$ (in linear media)

Magnetic energy density: $W_m = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$ or $W_m = \frac{1}{2}\mu H^2$ (in linear media)

Joule power dissipation density: $W_p = \mathbf{E}\cdot\mathbf{J}$ or $W_m = \sigma E^2$ (in Ohm's law media)

Poynting Vector: $\mathbf{S} = \mathbf{E}\times\mathbf{H}$

Time averaged Poynting vector: $\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*\}$

Capacitance: $C = \frac{Q}{V}$

Inductance: $L = \frac{\Lambda}{I} = N\frac{\Phi}{I}$

Boundary conditions: $E_{t,2} - E_{t,1} = 0$ $\quad H_{t,1} - H_{t,2} = J_s$

$D_{n,2} - D_{n,1} = \rho_s$ $\quad B_{n,2} - B_{n,1} = 0$

Bound charge: $\rho_{b,v} = -\nabla\cdot\mathbf{P}$ $\quad \rho_{b,s} = \mathbf{P}\cdot\hat{\mathbf{n}}$

Bound current: $\mathbf{J}_{b,v} = \nabla\times\mathbf{M}$ $\quad \mathbf{J}_{b,s} = \mathbf{M}\times\hat{\mathbf{n}}$

Definition of phasor $\tilde{F}$ for time harmonic function $f(t)$:
$$\begin{cases} f(t) = \mathrm{Re}\{\tilde{F}e^{j\omega t}\} = |F|\cos(\omega t + \phi) \\ \tan^{-1}(\phi) = \mathrm{Im}\{\tilde{F}\}/\mathrm{Re}\{\tilde{F}\} \end{cases}$$

Constants (SI units): $\varepsilon_0 = 8.85\times10^{-12}$ F/m (or $C^2\,N^{-1}\,m^{-2}$) $\quad \mu_0 = 4\pi\times10^{-7}$ H/m (or $N\,A^{-2}$)

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of A, $\|A\|$ =	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1}$ =	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$, for $P(x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{R}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$ $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$ $\hat{x}\times\hat{y} = \hat{z}$ $\hat{y}\times\hat{z} = \hat{x}$ $\hat{z}\times\hat{x} = \hat{y}$	$\hat{r}\cdot\hat{r} = \hat{\phi}\cdot\hat{\phi} = \hat{z}\cdot\hat{z} = 1$ $\hat{r}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{z} = \hat{z}\cdot\hat{r} = 0$ $\hat{r}\times\hat{\phi} = \hat{z}$ $\hat{\phi}\times\hat{z} = \hat{r}$ $\hat{z}\times\hat{r} = \hat{\phi}$	$\hat{R}\cdot\hat{R} = \hat{\theta}\cdot\hat{\theta} = \hat{\phi}\cdot\hat{\phi} = 1$ $\hat{R}\cdot\hat{\theta} = \hat{\theta}\cdot\hat{\phi} = \hat{\phi}\cdot\hat{R} = 0$ $\hat{R}\times\hat{\theta} = \hat{\phi}$ $\hat{\theta}\times\hat{\phi} = \hat{R}$ $\hat{\phi}\times\hat{R} = \hat{\theta}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l}$ =	$\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$	$\hat{r}\,dr + \hat{\phi}r\,d\phi + \hat{z}\,dz$	$\hat{R}\,dR + \hat{\theta}R\,d\theta + \hat{\phi}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{x}\,dy\,dz$ $d\mathbf{s}_y = \hat{y}\,dx\,dz$ $d\mathbf{s}_z = \hat{z}\,dx\,dy$	$d\mathbf{s}_r = \hat{r}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\phi}\,dr\,dz$ $d\mathbf{s}_z = \hat{z}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{R}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\theta}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\phi}R\,dR\,d\theta$
Differential volume, dV =	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

GRADIENT, DIVERGENCE, CURL & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

SOME USEFUL VECTOR IDENTITIES

$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB}$ Scalar (or dot) product

$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}}AB\sin\theta_{AB}$ Vector (or cross) product, $\hat{\mathbf{n}}$ normal to plane containing **A** and **B**

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$

$\nabla(U + V) = \nabla U + \nabla V$

$\nabla(UV) = U\nabla V + V\nabla U$

$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$

$\nabla \cdot (U\mathbf{A}) = U\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla U$

$\nabla \times (U\mathbf{A}) = U\nabla \times \mathbf{A} + \nabla U \times \mathbf{A}$

$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times \nabla V = 0$

$\nabla \cdot \nabla V = \nabla^2 V$

$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$\int_v (\nabla \cdot \mathbf{A})dv = \oint_S \mathbf{A} \cdot d\mathbf{s}$ Divergence theorem (S encloses v)

$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$ Stokes's theorem (S bounded by C)

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$\vec{B} = \frac{\mu_0 I}{2\pi r}$

$\ln$

$\int_0^{\cancel{d}} \frac{1}{r}\, dr$

$L = \frac{\Phi}{I}$

$\Phi = \frac{\mu_0}{2\pi} \int$

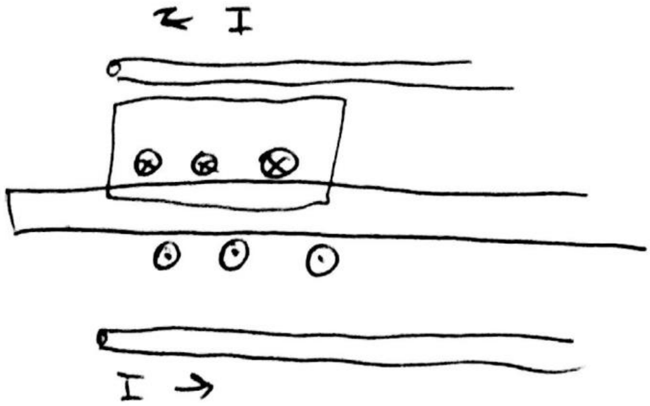
$\ln\left(\frac{2d}{a}\right)$

$= \int_s \frac{\mu_0}{2\pi r_1} + \frac{\mu_0}{2\pi r_2}\, ds$

$= \frac{\mu_0}{2\pi} \ln \frac{x}{d-x}$

we use the image of two currents, ~~but~~ the B field only exists above the ground plane, where do we take flux? not the whole space between the wires, but only above the plane.

I is the same, N is the same, B is the same above the plane,



$\Phi = \frac{\mu_0}{2\pi} \int_{\cancel{a}\,0}^{2d} \frac{1}{x\,dx}$