SOLUTIONS

UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Winter 2015
Final Exam, March 17 2015, (3 hours)

Name ______________________________ Student number ______________________________

This is a closed book exam – you are allowed 2 page of notes (each page front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit.

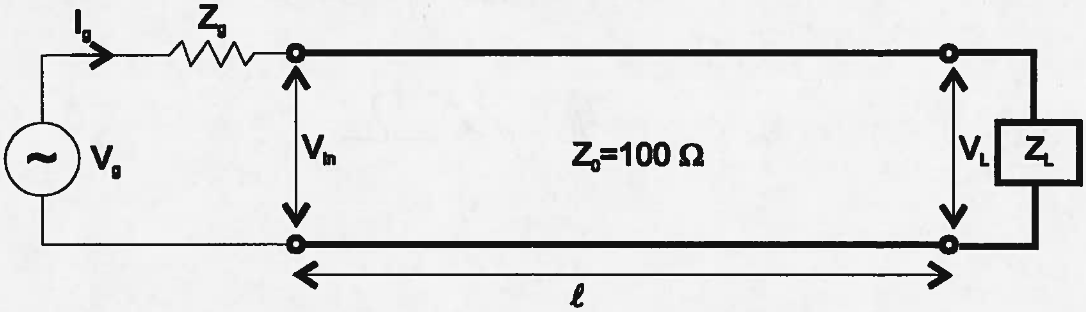
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Smith Chart	15	
Problem 2	Impedance Matching	40	
Problem 3	Impedance of Transmission Line	15	
Problem 4	Phasors and Maxwell's Eq	30	
Total		100	

1. Smith chart basics (15 points)

Consider the generic transmission line problem as shown below. Assume that the transmission line is a coaxial transmission line filled with a material that has $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$.



(a) (5 points) For each of the following loads, mark their position on the Smith chart below (using the letter as a label), and write below the reflection coefficient (magnitude and phase angle).

A: $Z_L = 60\ \Omega$. $\Gamma =$ -0.24 or $0.24 \angle 180^\circ$

$z_L = 0.6$

B: $Z_L = 150 + j300\ \Omega$. $\Gamma =$ $0.78 \angle 30^\circ$

$z_L = 1.5 + j3$

(b) (5 points) For each of the following loads impedances, convert to unnormalized load admittance Y_L and give the value in units Ω^{-1}. Mark the position on the Smith chart below (using the letter as a label).

$y_L = 1.65$

A: $Z_L = 60\ \Omega$. A' $Y_L =$ $0.0165\ \Omega^{-1}$

$0.13 - j\,0.26$

B: $Z_L = 150 + j300\ \Omega$. B' $Y_L =$

$0.0013 - j\,0.0026\ \Omega^{-1}$

(c) (5 points) What is the non-normalized input impedance of the transmission line $Z_{in}(-l)$ for each of the loads if l=2 cm and f=1 GHz? Label each point on the Smith Chart using A'', B''.

A: Z_L= 60 Ω. A'' Z_{in} = $92 + j50\,\Omega$

$0.92 + j0.5$

B: Z_L = 150 + j300 Ω. B'' Z_{in} = $40 - j150\,\Omega$

$0.4 - 1.5j$

$l = 2\text{cm}$ $f = 1\text{GHz}$

$\varepsilon = 4\varepsilon_0, \mu = \mu_0$

$V_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{2} = 1.5\times10^8\,m/s$

$\lambda = \frac{1.5\times10^8}{10^9} = 1.5\times10^{-1} = 0.15m$

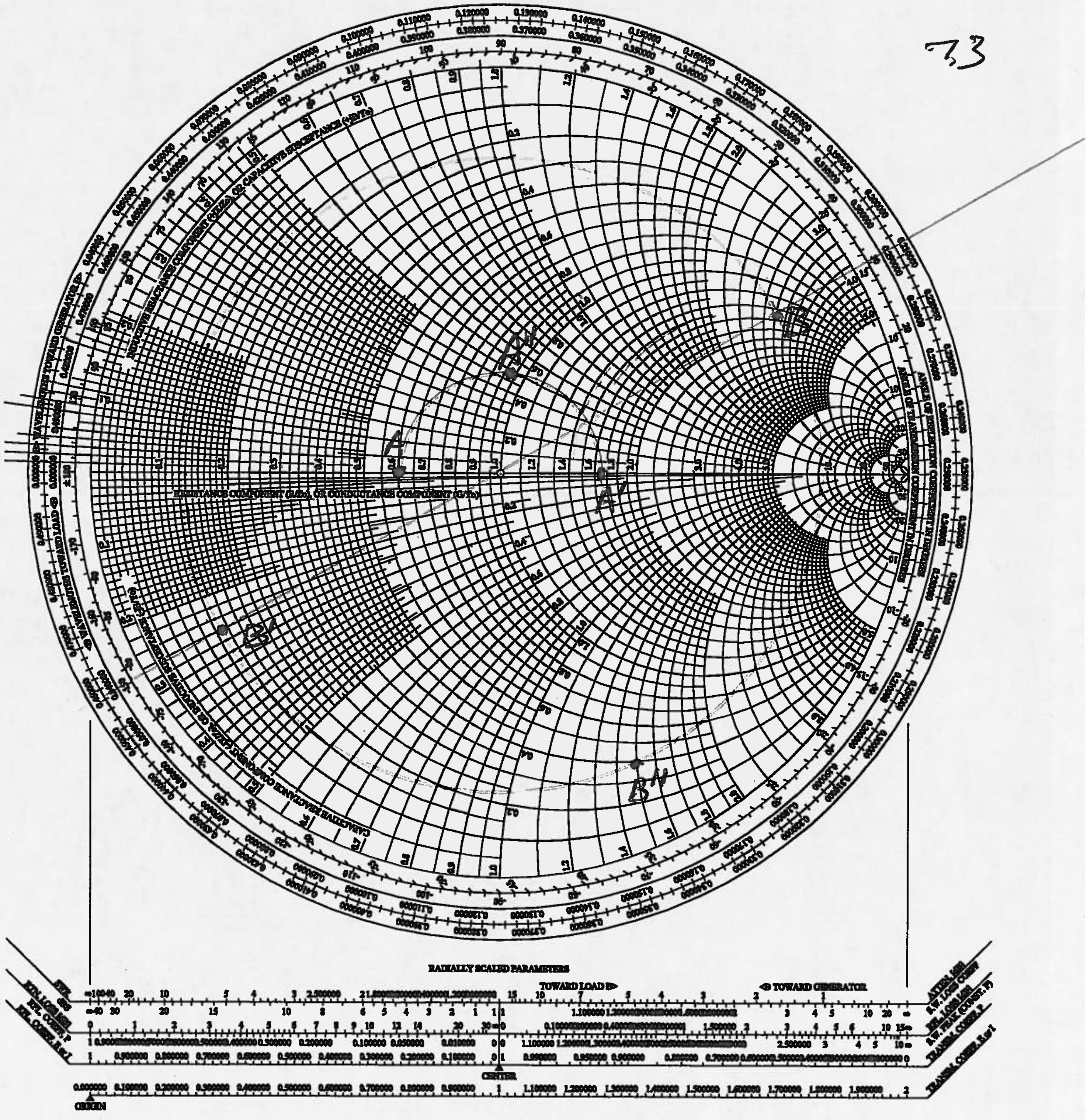
$= 15\,\text{cm}$

$l = \frac{2}{15} = 0.133\lambda$

$0.25 + .133 = 0.383$

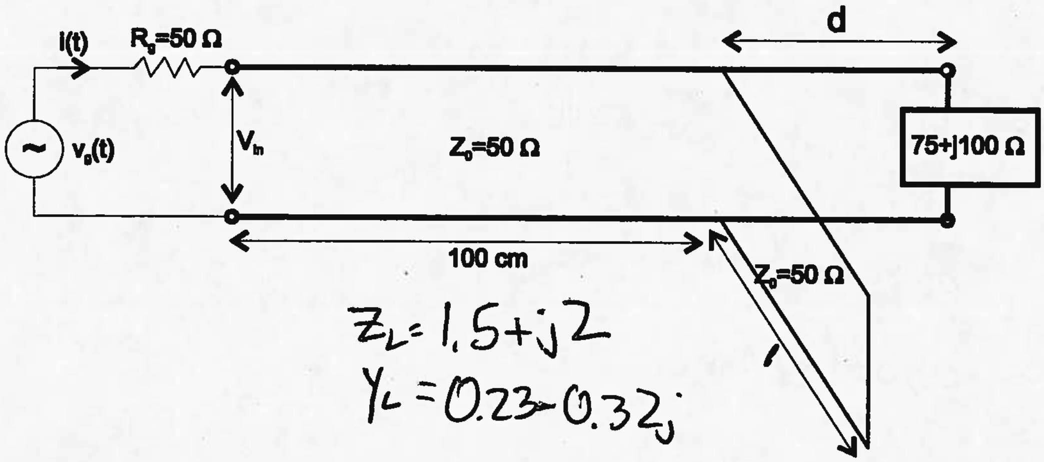
Smith chart for problem 1

7.3



2. **Transmission line – Impedance Matching** **(40 points)**
For this problem, you may use any methods you wish, including the Smith chart. Also, throughout this problem assume that the transmission line is coaxial filled with a dielectric material $\varepsilon=9\varepsilon_0$, $\mu=\mu_0$, and the generator voltage is $v(t)=V_0 \cos(2\pi ft)$, where $f=5$ GHz and $V_0=1$ V throughout the problem.

(a) (20 points) The goal of this problem is to design an impedance matching network that prevents any reflections into the network and maximizes the power delivered to the load, using a shorted stub. All transmission lines have the same characteristic impedance Z_0. Find the lengths d and ℓ in order to impedance match the load the line. Give your answer in terms of wavelengths. (Note, there are multiple solutions – you only need give one).



$Z_L = 1.5 + j2$
$Y_L = 0.23 - 0.32j$

Solution 1

$d = 0.233\lambda$
$\ell = 0.084\lambda$

Stub: $Y_{in} = -j1.7$

Solution 2

$d = 0.370\lambda$
$\ell = 0.416\lambda$

Stub $Y_{in} = +j1.7$

(b) (10 points) For the same problem, what should the length d and ℓ be in meters?

$f = 5\text{GHz}$, $V_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{3} = 10^8 \text{m/s}$

$\lambda = \frac{V_p}{f} = \frac{10^8}{5\times10^9} = 0.2\times10^{-1} = 2\text{cm} = 0.02\text{m}$

Solution 1

$d = 0.00466\text{m}$

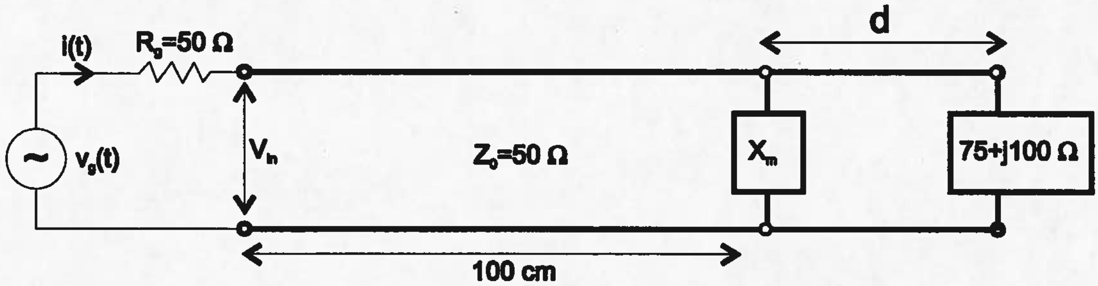
$\ell = 0.00168\text{m}$

Solution 2

$d = 0.0074\text{m}$

$\ell = 0.00832\text{m}$

(c) (10 points) Now consider using a lumped circuit element to match instead of a shorted stub (shown as reactance X_m in the figure). Should you use an inductor or a capacitor? What value should you use (in either units Farads or Henries)?



If you choose $d = 0.233\lambda$, you must choose

$$Y_m = -j0.034\ \Omega^{-1} \qquad X_m = +j29.4$$

choose inductor

$$X_m = \omega L \qquad L = \frac{29.4}{2\pi \times 5\times10^9} = 9.36\times10^{-10}\ H$$

If you choose $d = 0.370\lambda$ you must choose

$$Y_m = +j0.034\Omega^{-1} \qquad X_m = -j29.4$$

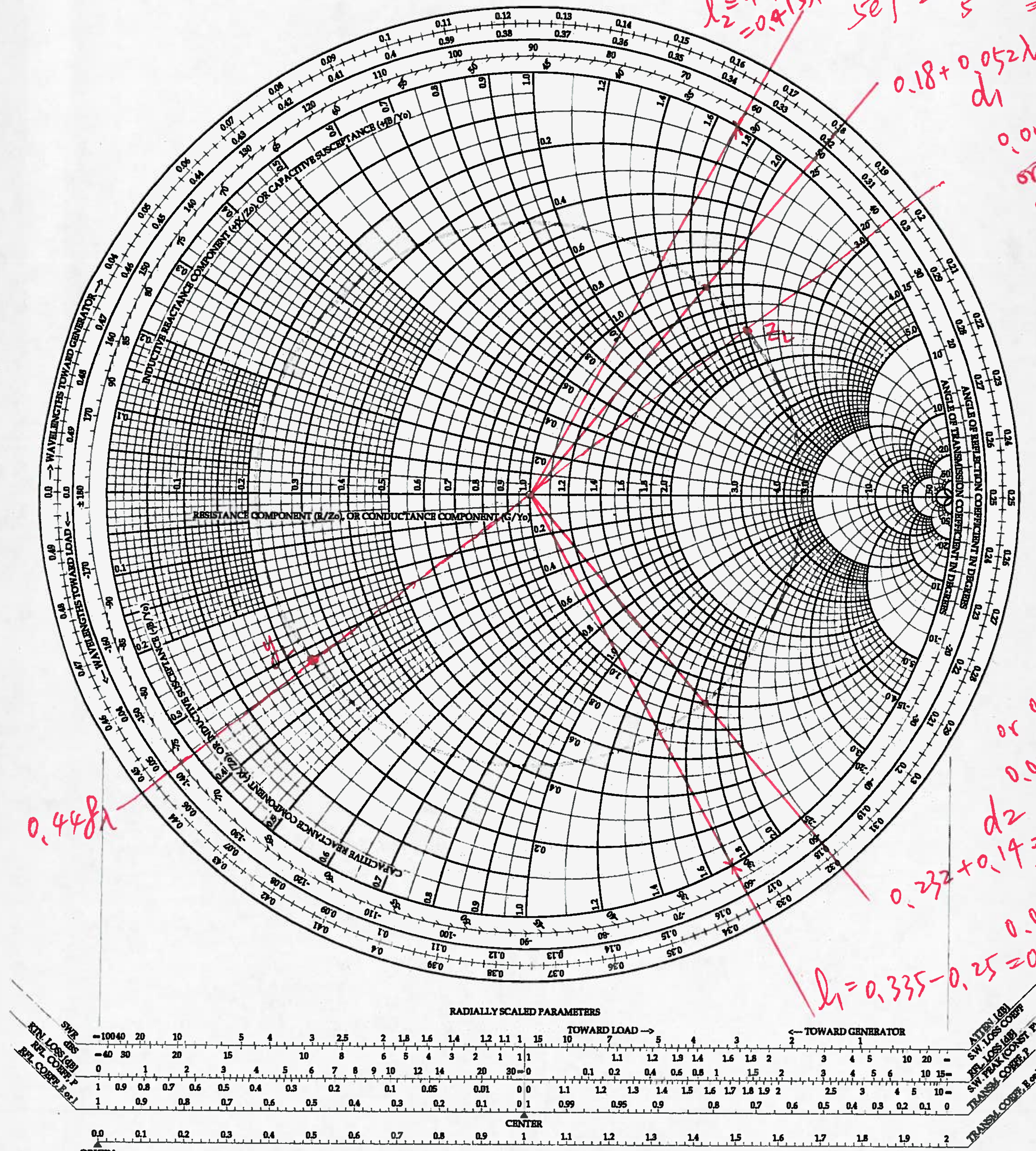
choose Capacitor

$$X_m = -\frac{j}{\omega C} = -j29.4$$

$$C = \frac{1}{2\pi\times5\times10^9\times29.4} = 1.08\times10^{-12}\ F$$

The Complete Smith Chart

Black Magic Design



or 0.00832m

$v = \frac{c}{\sqrt{9}} = 1e8\ m/s$

$\frac{1e8\ m/s}{5e9} = \frac{1}{5} \cdot 10^{-1} = 0.02$

$1e8\ m/s = 5e9 \cdot \lambda$

$= 2c$

0.0083m

0.165

$l_2 = +0.25$

$= 0.415\lambda$

$0.18 + 0.052\lambda = 0.23$

d_1

0.0046

or

0.004

0.448λ

or 0.0074

or 0.0074

d_2

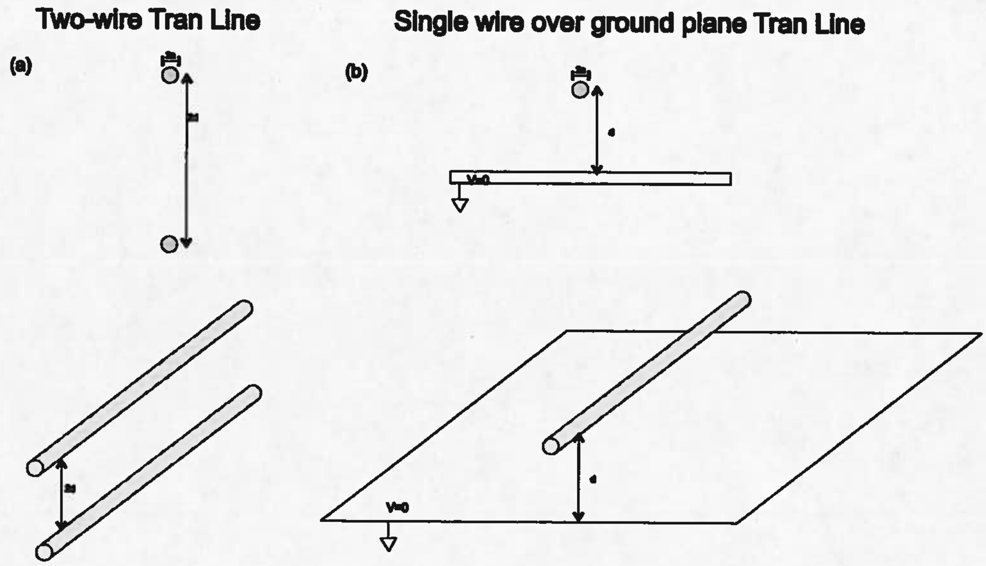
$0.232 + 0.14 = 0.37$

0.00

$l_1 = 0.335 - 0.25 = 0.085$

or 0.00

3. **(15 points)** **Impedance of transmission line.** Consider a two wire transmission line, and a single wire transmission line over a ground plane with dimensions as shown (assume that *d* and *a* have the same values in each case). If the characteristic impedance of the two-wire line is Z_0=40 Ω, what is Z_0 for the wire over the ground plane? Explain the reasoning behind your answer in 1-3 sentences.



The single wire TL is the image theory equivalent to the two wire line. However the voltage drop is $\frac{1}{2}$ of the two-wire case for the same charge so $C_{single} = \frac{Q}{V} \Rightarrow 2C_{two}$

The B-field flux is $\frac{1}{2}$ of the two wire case so $L_{one} \to \frac{L_{two\text{-}w}}{2}$

$$Z_{single} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{L'_{two}/2}{C'_{two}2}} = \frac{Z_{two}}{2} = 20\Omega.$$

4. Phasors and Maxwell's Equations (30 points)

(a) (8 points) Write the following phasor quantities in the time domain assuming an angular frequency ω. (Do not include the expression "Re{}" in your answer). Assume E_0, H_0, V_0, and A are real numbers.

i. $\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}E_0 e^{-jkz}$ $\quad$ $\mathbf{E}(z,t) = \hat{x} E_0 \cos(\omega t - kz)$

$Re\{\hat{x} E_0 e^{-jkz} e^{j\omega t}\} = Re\{\hat{x} E_0 (\cos(\omega t - kz) + j\sin(\omega t - kz))$

ii. $\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}} jH_0 e^{-jkz}$ $\quad$ $\mathbf{H}(z,t) = \hat{y} H_0 \sin(\omega t - kz)$

iii. $\tilde{F} = 3A(1-j)$ $\quad$ $F(t) = 3A(\cos \omega t + \sin \omega t)$

$3\sqrt{2}A \cos(\omega t - \pi/4)$

iv. $\tilde{V}(z) = V_0 \sin(\beta z)$ $\quad$ $V(z,t) = V_0 \sin(\beta z) \cos(\omega t)$

(b) (4 points) Consider a plane wave propagating through a particular medium with the phasor relations for the field:

$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}E_0 e^{jkz}$

$\tilde{\mathbf{H}}(z) = -\hat{\mathbf{y}} \dfrac{E_0}{100} e^{jkz}$

Assuming that $\mu = \mu_0$, what is the value of σ and ϵ? What direction is this wave propagating?

Propagation in $-\hat{z}$ direction.

$\sigma = 0$

$\eta = 100 = \sqrt{\dfrac{\mu_0}{\varepsilon}}$ $\qquad$ $\varepsilon = \dfrac{\mu_0}{10^4} = \dfrac{4\pi \times 10^{-7}}{10^4} = 4\pi \times 10^{-11}$

$\boxed{\begin{array}{l} \varepsilon = 1.25 \times 10^{-10} \text{ F/m} \\ \varepsilon = 14.2\, \varepsilon_0 \end{array}}$

(c) (4 points)

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}E_0 e^{-\gamma z}$$

$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}(1-j)\frac{E_0}{8}e^{-\gamma z}$$

Assuming that $\mu=\mu_0$, is this wave propagating through good conductor or a poor conductor (i.e lossy dielectric)? Explain how you can tell the difference.

It is propagating through a good conductor with $\frac{\sigma}{\omega} >> \varepsilon$.

You can tell because $\eta \approx \frac{8}{1-j} \approx \frac{(1+j)8}{2}$ Equal parts imag & real.

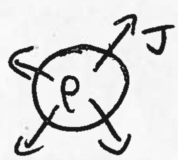
Since $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}}$

When $\varepsilon'' >> \varepsilon'$ $\eta_c \approx \sqrt{\frac{\mu\omega}{j\sigma}} = \sqrt{j}\sqrt{\frac{\mu\omega}{\sigma}} = \frac{(1+j)}{\sqrt{2}}\sqrt{\frac{\mu\omega}{\sigma}}$

$\varepsilon'' = \frac{\sigma}{\omega} \approx 4400\,\varepsilon_0$

When $\varepsilon' >> \varepsilon''$ $\eta_c \approx \sqrt{\frac{\mu}{\varepsilon'}}$ mostly real.

(d) (4 points) Consider the equation: $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$. Apply this equation to a volume V with a surface defined by differential elements $d\mathbf{S}$, and rewrite this equation in integral form. Give a physical explanation of what conservation law that this describes.

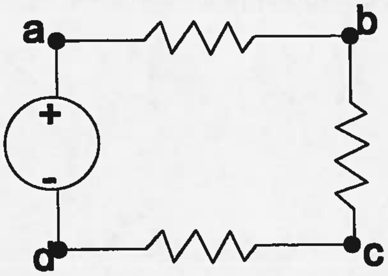


$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\int \frac{\partial \rho}{\partial t} dV = -\frac{\partial}{\partial t}\int \rho\, dV = -\frac{\partial Q_{total}}{\partial t}$$

This describes conservation of charge. If the charge density at a location changes, it must be accompanied by a net current flux inward or outward.

(e) (5 points) In circuit theory, Kirchoff's voltage law says that the sum of voltages in a closed circuit must add up to zero. Qualitatively explain and/or derive how this rule can be derived from one of Maxwell's equations.

$V_{ad}+V_{ba}+V_{cb}+V_{dc}=0$



This originates from Faraday's Law taken in the electrostatic limit

$$\nabla\times\vec{E}=0.$$

Integral form is $\oint_c \vec{E}\cdot d\vec{l}=0$.

~~Th~~ If this is taken around the loop of the circuit, the sum of the voltage drops must equal zero.

(f) (5 points)

Here is a "proof" that there is no such thing as magnetism. Magnetic Gauss's law states that: $\nabla \bullet \mathbf{B} = 0$, When we apply the divergence theorem, we find:

$$\int_V (\nabla \bullet \mathbf{B}) dV = \int_S \mathbf{B} \bullet d\mathbf{S} = 0$$

Because **B** has zero divergence, we are able to define **B** as the curl of the vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$ If we combine the last two equations, we obtain:

$$\int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S} = 0$$

Next we apply Stokes's theorem to the above result to obtain:

$$\int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S} = \oint_C \mathbf{A} \bullet d\mathbf{l} = 0$$

Thus we have shown that the circulation of **A** is path independent. It follows that we can write $\mathbf{A} = \nabla \psi$ where ψ is some scalar function. Since the curl of a gradient is zero, we arrive at the remarkable conclusion that:

$$\mathbf{B} = \nabla \times (\nabla \psi) = 0$$

That is, the magnetic field is zero everywhere!

Obviously I made a mistake somewhere in this proof. Explain where I went wrong.
(Hint: pay careful attention to the definitions of the various laws and theorems – it may be helpful to make sketches).

The problem here originates from a misstatement of the divergence theorem, which should read:

$$\int_V (\nabla \cdot B) dV = \oint_S B \cdot dS = 0$$

It must be applied over a closed surface.

For this reason it can't be "plugged-in" to the Stokes theorem, which is applied over an open surface:

$$\int_S (\nabla \times A) \cdot dS = \oint A \cdot dl \neq 0.$$