SOLUTIONS

UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Winter 2014
Final Exam, March 20 2014, (3 hours)

Name ______________________________ Student number ____________________________

This is a closed book exam – you are allowed 2 page of notes (each page front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit.

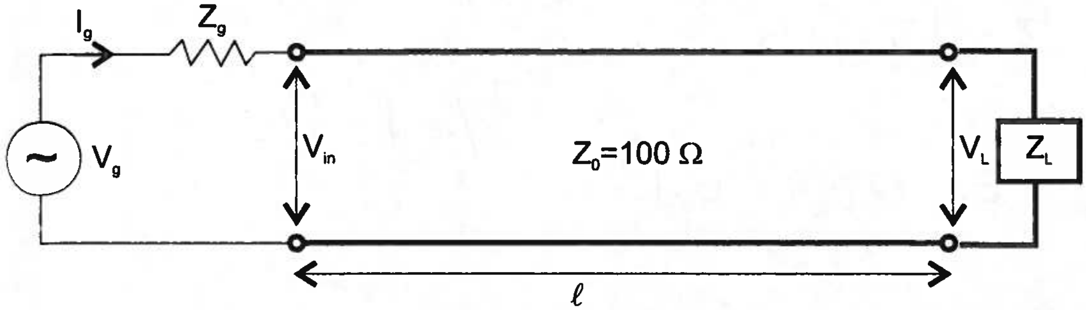
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Smith Chart	20	
Problem 2	Impedance Matching	30	
Problem 3	Plane wave propagation	25	
Problem 4	Capacitor with conductor	25	
Total		100	

1. Smith chart basics (20 points)

Consider the generic transmission line problem as shown below. Assume that the transmission line is a coaxial transmission line filled with a material that has $\varepsilon = 9\varepsilon_0$, $\mu = \mu_0$.



(a) (7 points) For each of the following loads, mark their position on the Smith chart below (using the letter as a label), and write below the reflection coefficient (magnitude and phase angle).

A: Z_L= 30 Ω. Γ = $0.53 \angle 180°$

$z_L = 0.3$

B: Z_L = 50 - j200 Ω. Γ = $0.83 \angle -51°$

$z_L = 0.5 - 2j$

(b) (6 points) For each of the following loads impedances, convert to unnormalized load admittance Y_L and give the value in units Ω^{-1}. Mark the position on the Smith chart below (using the letter as a label).

A: Z_L= 30 Ω. A' Y_L= $0.034\,\Omega^{-1}$

$y_L = 3.4$

B: Z_L = 50 - j200 Ω. B' Y_L= $0.0011 + j\,0.0047\,\Omega^{-1}$

$y_L = 0.11 + j0.47$

(c) (7 points) What is the non-normalized input impedance of the transmission line $Z_{in}(-l)$ for each of the loads if l=2 cm and f=1 GHz? Label each point on the Smith Chart using A'', B''.

A: Z_L= 30 Ω. A'' Z_{in} = $170 + j150\,\Omega$

$z_{in} = 1.7 + j1.5$

B: Z_L = 50 - j200 Ω. B'' Z_{in} = $9 + j13\,\Omega$

$z_{in} = 0.009 + j0.13$

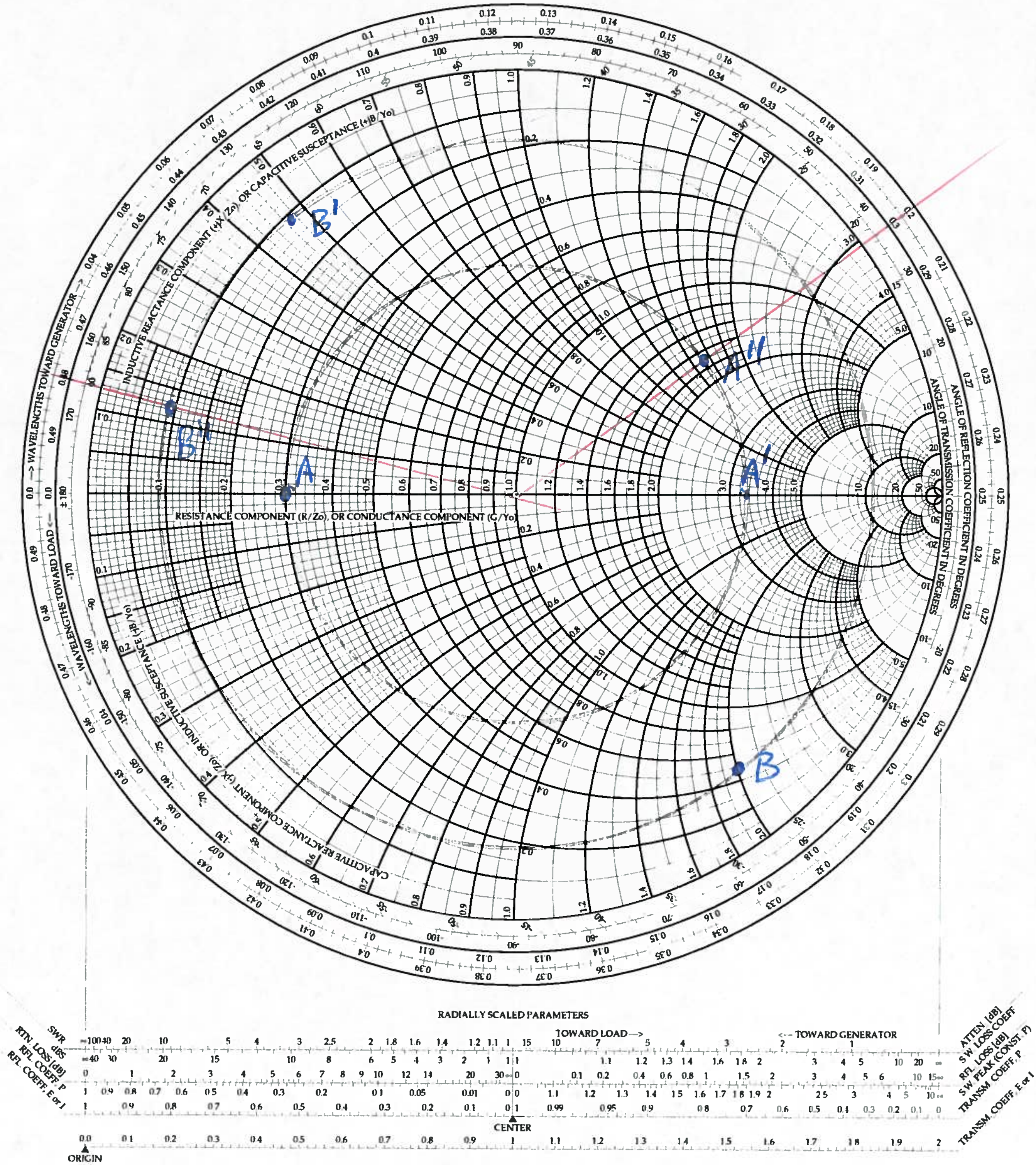
$f = 1\,GHz$

$\varepsilon = 9\varepsilon_0 \quad \mu = \mu_0 \qquad V = \frac{c}{3} = 10^8 m/s$

$\lambda = \frac{V}{f} = 0.1 m = 10 cm$

$l = 2 cm = 0.2\lambda$

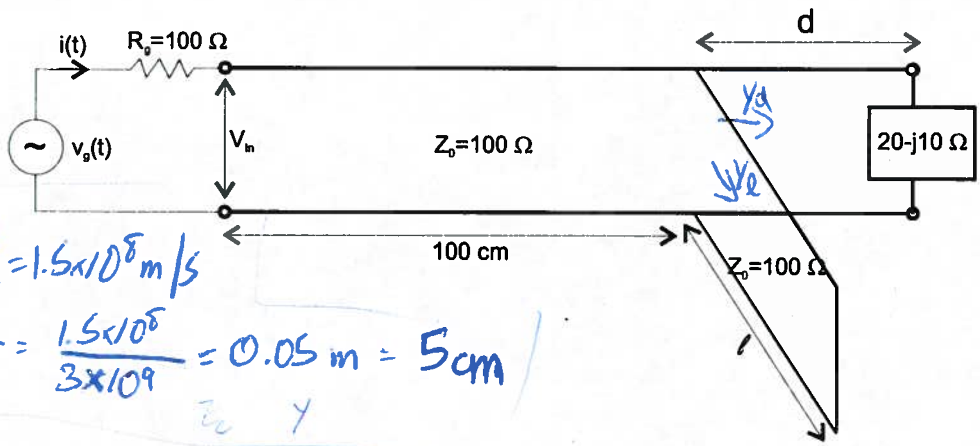
Smith chart for problem 1



2. Transmission line – Impedance Matching **(~~35~~ 30 points)**

For this problem, you may use any methods you wish, including the Smith chart. Also, assume that the transmission line is coaxial filled with a dielectric material $\varepsilon=4\varepsilon_0$, $\mu=\mu_0$, and the generator voltage is $v(t)=V_0 \cos(2\pi f t)$, where $f=3$ GHz and $V_0=1$ V throughout the problem.

(a) (~~15~~ 20 points) The goal of this problem is to design an impedance matching network that prevents any reflections into the network and maximizes the power delivered to the loads. All transmission lines have the same characteristic impedance Z_0. Find the lengths d and ℓ in order to impedance match the load the line. Give your answer both in terms of wavelengths and meters.



$z_L = 0.2 - j0.1$

$V = \frac{c}{2} = 1.5\times10^8$ m/s

$\lambda = \frac{V}{f} = \frac{1.5\times10^8}{3\times10^9} = 0.05$ m $= 5$ cm

Choose d such that $g_d = 1$, choose ℓ such that $x_L = +1.8$

$(y_d = 1 - j1.8)$

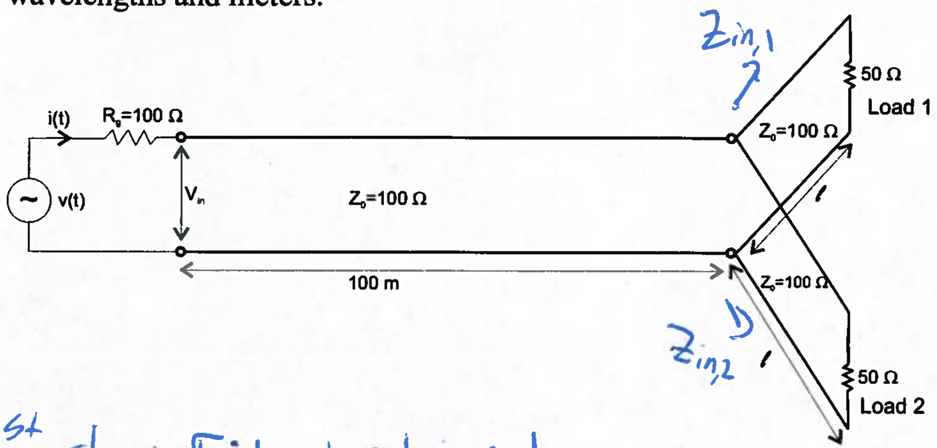
Solution 1: $d = 0.084\lambda = 0.42$ cm

$\ell = 0.42\lambda = 2.1$ cm

Solution 2: $d = 0.45\lambda = 2.25$ cm

$\ell = 0.08\lambda = 0.4$ cm

(b) (10 points) Consider the following case, where a transmission line is split into two parallel branches each terminated with a load of 50 Ω. What choice of length ℓ will result in no reflections back to the generator? You should give your answer in terms of both numbers of wavelengths and meters.

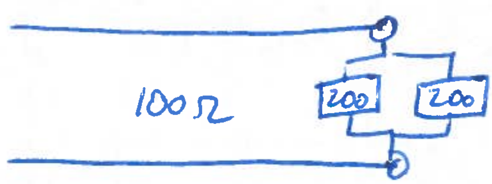


1st step: Find input impedance at entrance to each sub branch $Z_{in,1}$, $Z_{in,2}$.

Since there are no stubs available, we must choose ℓ such that $Z_{in,1}$ and $Z_{in,2}$ are real. There are only 2 options: $\ell = \frac{\lambda}{4}$ or $\ell = \frac{\lambda}{2}$.

If we choose $\ell = \frac{\lambda}{4}$, $Z_{in,1} = Z_{in,2} = 200\,\Omega$

We then have the circuit:



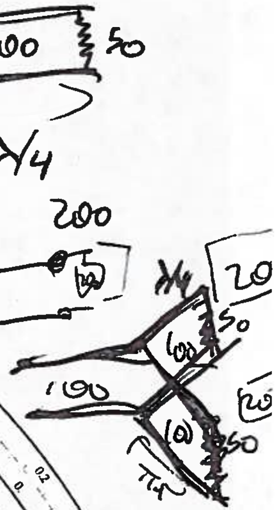
The two 200Ω input impedances in parallel give 100Ω, which is a match to the line.

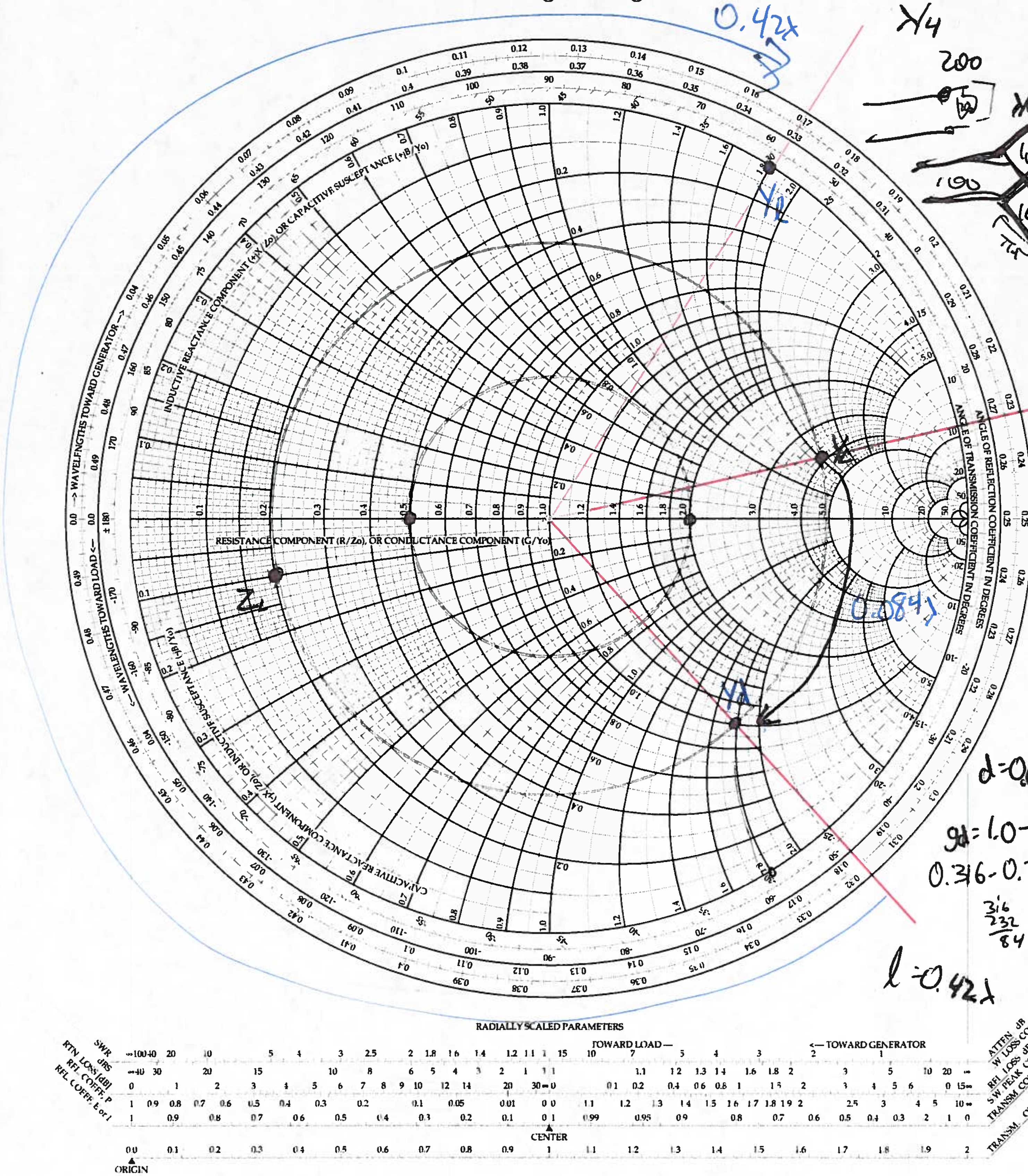
$$\boxed{\ell = \frac{\lambda}{4} = 0.125\ \text{cm}}$$

Solution 1

The Complete Smith Chart

Black Magic Design





$d = 0.084\lambda$

$y_d = 1.0 - j1.8$

0.316 - 0.232

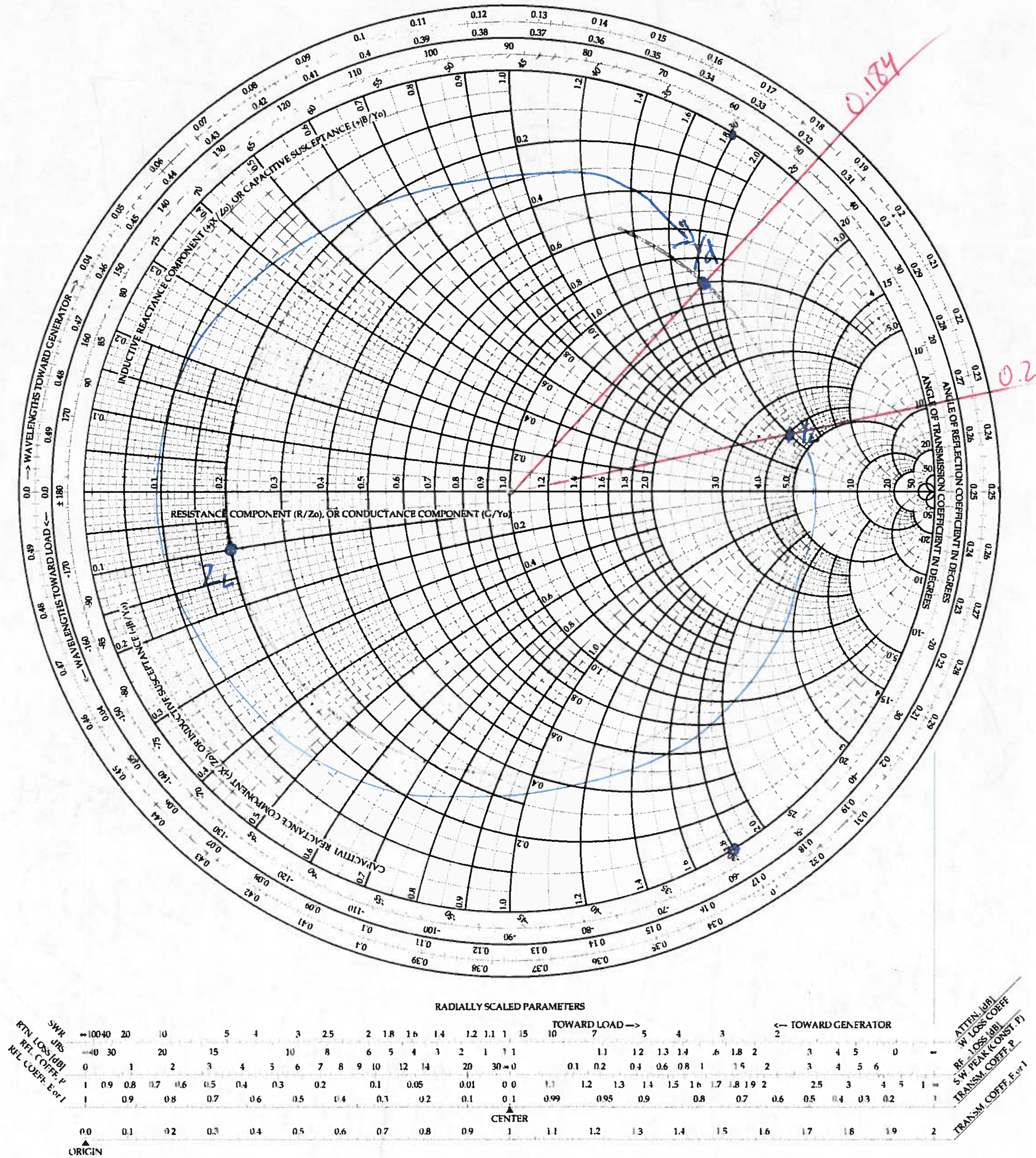
$$\begin{array}{r} 316 \\ -232 \\ \hline 84 \end{array}$$

$l = 0.42\lambda$

Solution 2

The Complete Smith Chart

Black Magic Design



3. **Plane waves and interfaces** 25 points ~~(20 points)~~

A microwave oven cooks your food by irradiating it with electromagnetic waves at a frequency of 2.45 GHz. Let's model this process by considering the radiation in the form of a plane wave that is incident upon your "food". Since food is mostly water, we will model the food as a pure water "medium". The permittivity of pure water is very different at different frequencies. We will take the permittivity of water to be $\varepsilon = \varepsilon_0(78\text{-}j7.6)$. You may make any approximations provided they are justified.

6 Points

(a) At 2.45 GHz, is it better to describe water as a good conductor or a lossy dielectric (i.e. poor conductor)?

Approx a ~~conductor~~ lossy dielectric. $\frac{\varepsilon'}{\varepsilon''} \approx 10$

Although it is close to the cross over point.

6 Points

(b) At 2.45 GHz, what is the wavelength in water at this frequency?

$\lambda = \frac{2\pi}{\beta}$ $\beta = \omega\sqrt{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]}$

~~Since~~ Use approx $\sqrt{1+x} \approx 1+\frac{x}{2}$ when $x \ll 1$.

$\beta = \omega\sqrt{\frac{\mu\varepsilon'}{2}\left((1.005)+1\right)} \approx \omega\sqrt{\mu\varepsilon'} = \frac{\omega\sqrt{\varepsilon_r'}}{c}$

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi c}{2\pi f\sqrt{\varepsilon_r'}} = \frac{3\times10^8}{2.45\times10^9\times\sqrt{78}} = 0.014\text{ m}}$$

$\beta = 453\text{ m}^{-1}$

7 points

(c) At 2.45 GHz, over what distance will the power carried by a plane wave drop to 1% of its original value in water?

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]}$$

Use $\sqrt{1-x} \approx 1-\frac{x}{2}$ if $x \ll 1$

$$\alpha \approx \omega\sqrt{\frac{\mu\varepsilon'}{2}\left[1+\frac{1}{2}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2 - 1\right]} = \frac{\omega}{2}\sqrt{\mu\varepsilon'}\,\frac{\varepsilon''}{\varepsilon'} = \frac{\beta}{2}\frac{\varepsilon''}{\varepsilon'}$$

$\beta = 453\ m^{-1}$ $\quad \alpha = \frac{453}{2} \times 0.97 = 22\ m^{-1}$

Power attenuation: $S \propto e^{-2\alpha z}$

$$0.01 = e^{-2\alpha z}$$

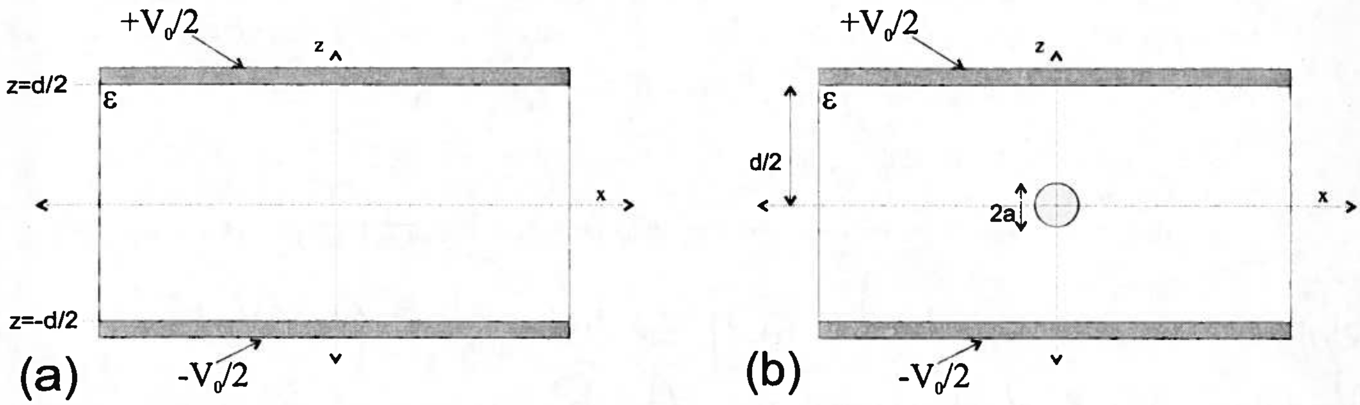
$$-\frac{\ln 0.01}{2\alpha} = z = \boxed{0.10\ m = 10\,cm}$$

6 points

(d) Ice has a dielectric constant of $\varepsilon = \varepsilon_0(6.8\text{-}j0.03)$. Given this, which would you expect to heat up faster in a microwave oven? Liquid water or ice? Why?

Water will heat up faster. Its larger ε'' will result in a larger absorption (α is larger) an thus faster heating. The absorbed energy is dissipated as Joule heating (i.e. Ohmic power dissipation) which heats the sample.

4. Capacitor with conducting sphere (25 points)



(a) (7 points) Consider a capacitor of thickness d, and permittivity ε, where the voltage difference between the upper and lower (perfectly conducting) plates is V_0 (see figure (a)). You may assume that the capacitor width is much greater than d, and neglect any fringing fields (i.e. parallel plate approximation). There is no free charge in the dielectric. Find an expression for the electrostatic potential $V(z)$ inside the dielectric medium ε. Give your answer both in Cartesian coordinates (i.e. $V(z)$), and spherical coordinates (i.e. $V(R,\phi,\theta)$).

$V(z) = \frac{V_0}{d} z$ $\qquad$ $V(R,\theta) = \frac{V_0}{d} R \cos\theta$

(b) (8 points) Consider the case if we place a small perfectly conducting metal sphere of radius a in the center of the capacitor, as shown in part (b) of the figure. The sphere has no net charge. Find the solution for the potential $V(R,\theta)$ for this case by analyzing Poisson's equation in the dielectric medium. You should look for solutions of the form $V(R,\theta) = AR\cos\theta + \frac{B}{R^2}\cos\theta + C$, which is a solution to $\nabla^2 V = 0$. Find the appropriate values of the constants A, B, C that satisfy the boundary conditions. You can assume that $a << d$, so that the field at the capacitor plates (i.e. far away from the sphere) is unperturbed by the presence of the metal sphere (i.e. at the plates it is the same as you found in part (a)).

Symmetry ensures that $\boxed{C=0}$ so that at $z=0$ $V=0$.

Now we must find unknowns A, B.

Boundary condition: For $R >> a$ (i.e. as $R \to \infty$) we should recover the potential for the unperturbed capacitor $V \to \frac{V_0}{d} R\cos\theta$

so: $V(R,\theta) = AR\cos\theta + \cancel{\frac{B}{R^2}\cos\theta}^{\to 0}$

so $AR\cos\theta = \frac{V_0}{d} R\cos\theta \Rightarrow \boxed{A = \frac{V_0}{d}}$

Now we have $V = \frac{V_0}{d} R\cos\theta + \frac{B}{R^2}\cos\theta$

At surface of sphere $(R=a)$ we must have equipotential.

Symmetry determines $V(R=a) = 0$ independent of θ.

$$0 = \frac{V_0}{d} a\cos\theta + \frac{B}{a^2}\cos\theta$$

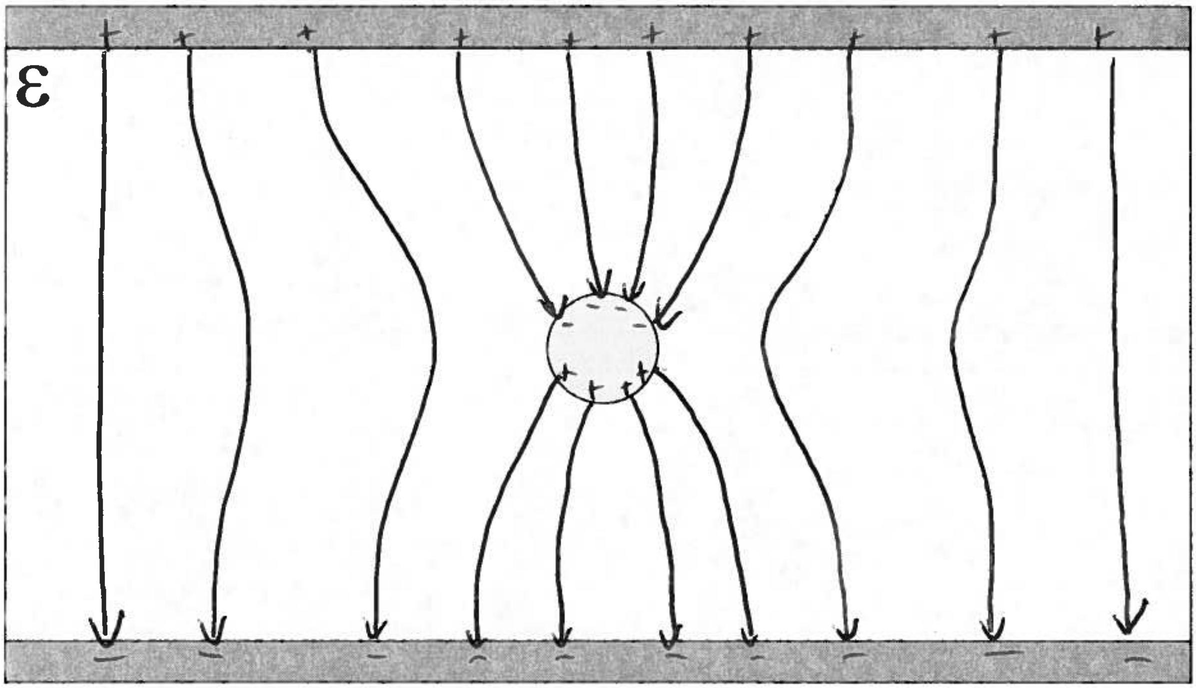
$$\boxed{B = -\frac{V_0}{d} a^3}$$

$$\boxed{V(R,\theta) = \frac{V_0}{d} R\cos\theta - \frac{V_0 a^3}{d R^2}\cos\theta}$$

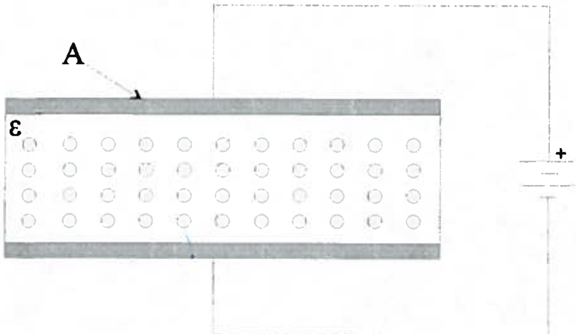
linear in z term from capacitor

dipole term

(c) (5 points) Sketch the electric field lines in the figure below. Indicate the position and polarity of any surface charge. **(PLEASE BE NEAT!)**



(d) (~~7~~ 5 points) Imagine we modify a conventional capacitor by filling the dielectric with very tiny metal spheres. However, they don't touch each other, so the material remains insulating. Will the inclusion of these spheres increase or decrease the capacitance C, or will it remain unchanged? Explain why.



This is known as an "artificial dielectric" or "metamaterial".

C will increase. Each sphere acts as a small dipole. This increases the dipole density per unit volume $\vec{P}$ (the Polarization field). Larger $\vec{P}$ for given $\vec{E}$ corresponds to a larger dielectric susceptibility χ_e.

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$C = \frac{\varepsilon A}{d}$$

As χ_e goes up, ε goes up, and C goes up.

Or: the presence of the metal reduces the E-field (average) within the capacitor. Because V_0 is held constant, more charge will have to be moved to plates to keep $V_0 = -\int_{bottom}^{top} \vec{E} \cdot d\vec{l}$ = constant.