UCLA Department of Electrical Engineering
ECE101A – Engineering Electromagnetics
Fall 2020
Quiz, October 20, 2020 (60 minutes)

Name ______________________________ Student number_____________________________

This is an open book quiz. Calculator is allowed, but no wireless device communication other than for downloading/viewing/uploading the exam/solution materials.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Vector Calculus	15	
Problem 2	Electrostatics	20	
Problem 3	Capacitance	15	
Total		50	

1. Vector Calculus

For the vector field $\vec{\boldsymbol{E}} = \hat{\boldsymbol{x}}y^2 + \hat{\boldsymbol{y}}(x^2 + 3xy)$, calculate

(a) $\nabla \times \vec{\boldsymbol{E}}$

(b) $\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$ around the triangular contour shown in Fig. 1

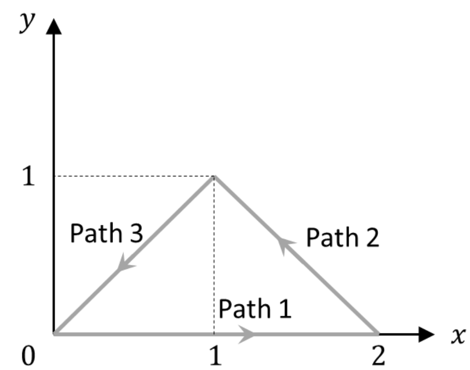

Fig. 1

(a) $\nabla \times \vec{\boldsymbol{E}} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & x^2 + 3xy & 0 \end{vmatrix} = \hat{\boldsymbol{z}}(2x + 3y - 2y) = \hat{\boldsymbol{z}}(2x + y)$

(5 points)

(b) Path 1, 2, 3 are denoted in Fig. 1.

Path 1: $y = 0$
Path 2: $x + y = 2, dx = -dy$
Path 3: $x = y, dx = dy$

$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = \int_{path1} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} + \int_{path2} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} + \int_{path3} \overrightarrow{\boldsymbol{E} \cdot} d\vec{\boldsymbol{l}}$$

$$= \int_0^1 \hat{\boldsymbol{y}}x^2 \cdot \hat{\boldsymbol{x}}dx + \int_0^1 [\hat{\boldsymbol{x}}y^2 + \hat{\boldsymbol{y}}(4 + 2y - 2y^2)] \cdot (-\hat{\boldsymbol{x}}dy + \hat{\boldsymbol{y}}dy)$$

$$+ \int_1^0 [\hat{\boldsymbol{x}}y^2 + \hat{\boldsymbol{y}}(y^2 + 3y^2)] \cdot (\hat{\boldsymbol{x}}dy + \hat{\boldsymbol{y}}dy)$$

$$= 0 + \int_0^1 (4 + 2y - 3y^2)dy + \int_1^0 (5y^2)dy$$

$$= 4 - \frac{5}{3} = \frac{7}{3}$$

Also works if use Stokes's theorem (10 points)

2. The circular disk of radius *a* shown in Fig. 2 has uniform charge density ρ_s across its surface.

(a) Obtain an expression for the electric potential *V* at a point *P*(0, 0, *z*) on the *z*-axis.

(b) Use your result to find **E** and then evaluate it for $z = h$.

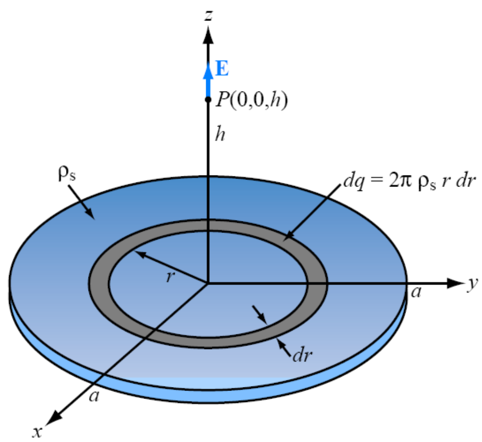

Fig. 2

Solution:

(a) Consider a ring of charge at a radial distance r. The charge contained in width dr is

$$dq = \rho_s(2\pi r\,dr) = 2\pi\rho_s r\,dr.$$

The potential at P is

$$dV = \frac{dq}{4\pi\varepsilon_0 R} = \frac{2\pi\rho_s r\,dr}{4\pi\varepsilon_0(r^2+z^2)^{1/2}}.$$

The potential due to the entire disk is

$$V = \int_0^a dV = \frac{\rho_s}{2\varepsilon_0}\int_0^a \frac{r\,dr}{(r^2+z^2)^{1/2}} = \frac{\rho_s}{2\varepsilon_0}(r^2+z^2)^{1/2}\Big|_0^a = \frac{\rho_s}{2\varepsilon_0}\left[(a^2+z^2)^{1/2} - z\right].$$

(10 points)

(b)

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{x}}\frac{\partial V}{\partial x} - \hat{\mathbf{y}}\frac{\partial V}{\partial y} - \hat{\mathbf{z}}\frac{\partial V}{\partial z} = \hat{\mathbf{z}}\frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right].$$

The expression for **E** reduces to Eq. (4.24) when $z = h$.

(10 points)

3. Calculate the capacitance of the capacitor in Fig. 3 (using the definition of capacitance): a parallel plate capacitor with two dielectric layers. Show that the overall capacitance C is equal to the series combination of the capacitances of the individual layers, C_1 and C_2, namely

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

where

$$C_1 = \varepsilon_1 \frac{A}{d_1}, \qquad C_2 = \varepsilon_2 \frac{A}{d_2}.$$

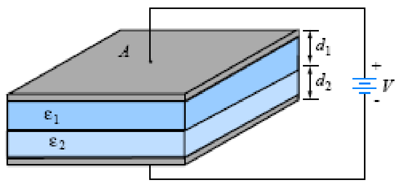

Fig. 3

3. Assume the top plate carry charge $+Q$
bottom plate carry charge $-Q$.

Top $+Q$, E_1, ε_1

$\varepsilon_1 E_1 A = Q \qquad E_1 = \frac{Q}{A\varepsilon_1}$

Bottom E_2, $-Q$

$-\varepsilon_2 E_2 A = -Q \qquad E_2 = \frac{Q}{A\varepsilon_2}$. (5 points)

$$V = \int_0^{d_2} E_2 \, dl + \int_{d_2}^{d_1+d_2} E_1 \, dl$$

$$= \frac{Q}{A\varepsilon_2} d_2 + \frac{Q}{A\varepsilon_1} d_1 \qquad \text{(3 points)}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)} \qquad \text{(5 points)}$$

$$= \frac{\frac{\varepsilon_2 A}{d_2} \cdot \frac{\varepsilon_1 A}{d_1}}{\frac{\varepsilon_1 A}{d_1} + \frac{\varepsilon_2 A}{d_2}} = \frac{C_1 C_2}{C_1 + C_2} \qquad \text{(3 points)}$$