UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2018
Quiz, October 16 2018, (30 minutes)

This is a closed book quiz – you are allowed 1 page of notes (front+back). Calculator is allowed, but other electronic such as cell phones, laptops etc. are not allowed.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Vector Calculus	40	40
Problem 2	Electrostatics	60	60
Total		100	100

EE101A – Engineering Electromagnetics

Quiz

$Q_x - P_y = (2x + 3y - 2y)\hat{z} =$

1. Vector Calculus

For the vector field $\vec{\mathbf{E}} = \hat{\mathbf{x}}\overset{P}{y^2} + \hat{\mathbf{y}}\overset{Q}{(x^2 + 3xy)}$, calculate

(a) $\nabla \times \vec{\mathbf{E}}$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 & x^2+3xy & 0 \end{vmatrix} = \hat{x}(0) + \hat{y}(0) + \hat{z}\left[\frac{\partial}{\partial x}(x^2+3xy) - \frac{\partial}{\partial y}(y^2)\right]$$

$$= \hat{z}\,[2x + 3y - 2y] = \boxed{\hat{z}\,[2x+y]}$$

✓

(b) $\oint_C \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{l}}$ around the triangular contour shown in Fig. 1

$$\vec{E}\cdot d\vec{l} = y^2\,dx + (x^2+3xy)\,dy$$

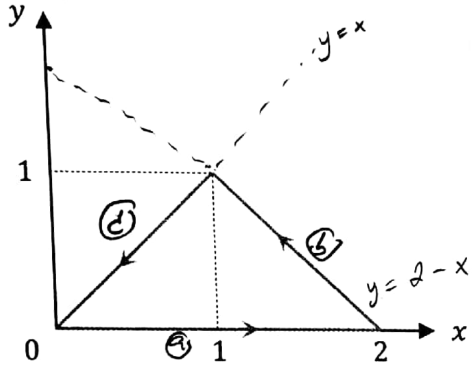


Fig. 1

or $x = 2 - y$

$$\oint_C \vec{E}\cdot d\vec{l} = \int_a \vec{E}\cdot d\vec{l}\,\Big|_{y=0} + \int_b \vec{E}\cdot d\vec{l}\,\Big|_{y=2-x} + \int_c \vec{E}\cdot d\vec{l}\,\Big|_{y=x}$$

$$= 0 + \int_{x=2}^{x=1} (2-x)^2\,dx + \int_{y=0}^{y=1} \left[(2-y)^2 + 3(2-y)y\right] dy + \int_{x=1}^{x=0} x^2\,dx + \int_{y=1}^{y=0} 4y^2\,dy$$

$$= \int_2^1 (4 - 4x + x^2)\,dx + \int_0^1 (4 - 4y + y^2 + 6y - 3y^2)\,dy + \frac{x^3}{3}\Big|_1^0 + \frac{4}{3}y^3\Big|_1^0$$

$$= 4x - 2x^2 + \frac{x^3}{3}\Big|_2^1 + 4y - 2y^2 + \frac{y^3}{3} + 3y^2 - y^3\Big|_0^1 - \frac{1}{3} - \frac{4}{3}$$

$$= (4 - 2 + 1/3) - (8 - 8 + 8/3) + 4 - 2 + 1/3 + 3 - 1 - 1/3 - 4/3 = \boxed{7/3}$$

✓

check Stokes' Theorem:

$$\iint (\nabla\times E)\cdot d\vec{s} = \int_0^1\int_{y=0}^{y=x} (2x+y)\,dy\,dx + \int_1^2\int_0^{2-x} (2x+y)\,dy\,dx = 7/3$$

$$= \int 2xy + y^2/2\,\Big|^x \quad \int^2 \quad 2-x$$

2. Two infinitely conducting hollow spheres ($\sigma = \infty$) with infinitely thin surfaces are positioned at the center of the coordinate system. A cross section of the spheres is shown in the figure. The inner sphere has a total charge of Q and the outer one has a charge of $-Q$. The radii of two spheres are $a = 2.5\ \mu m$, $b = 4.0\ \mu m$. The region between the two hollow spheres (the grey region) is filled with Barium Titanate having a dielectric constant $\epsilon_r = 1200$ ($\epsilon_0 = 8.85 \times 10^{-12} F/m$). Find the capacitance of this two-conductor system. (Hint: use spherical coordinates)

60

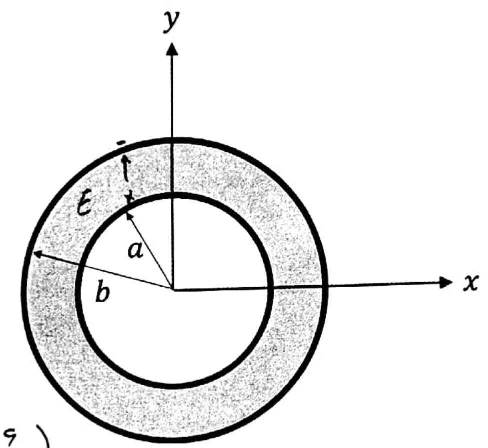
$C = \frac{Q}{V}$ wher $V = -\int \vec{E} \cdot d\vec{l}$

$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon}$

btwn a and b:

$E(4\pi R^2) = \frac{Q}{\epsilon}$

$\Rightarrow E = \frac{Q}{4\pi\epsilon R^2}\ \hat{R}$



$V(a) - V(b) = -\int_b^a \frac{Q}{4\pi\epsilon R^2}\, dR$

$= \frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\epsilon}\left(\frac{b-a}{ba}\right)$

$\epsilon = \epsilon_r \epsilon_0$

$C = 4\pi\epsilon\left(\frac{ba}{b-a}\right) = \boxed{4\pi\epsilon_r\epsilon_0 \frac{ba}{b-a}}$ [F]

$C = 4\pi(1200)(8.85\times10^{-12}) \frac{(4\mu m)(2.5\mu m)}{(4-2.5)\mu m} = \boxed{0.89\ pF}$