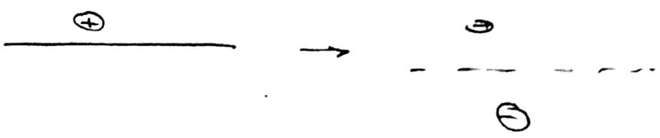
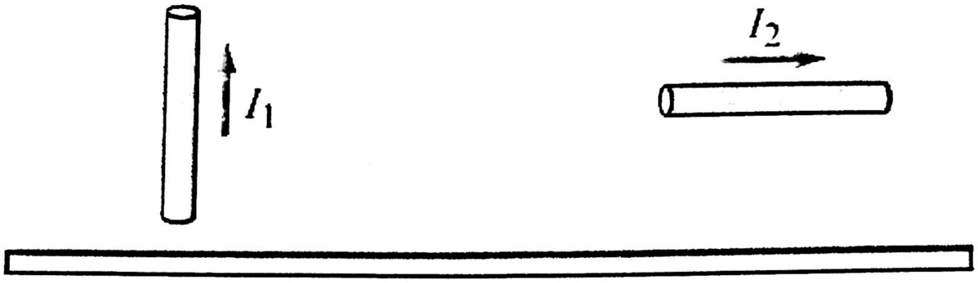
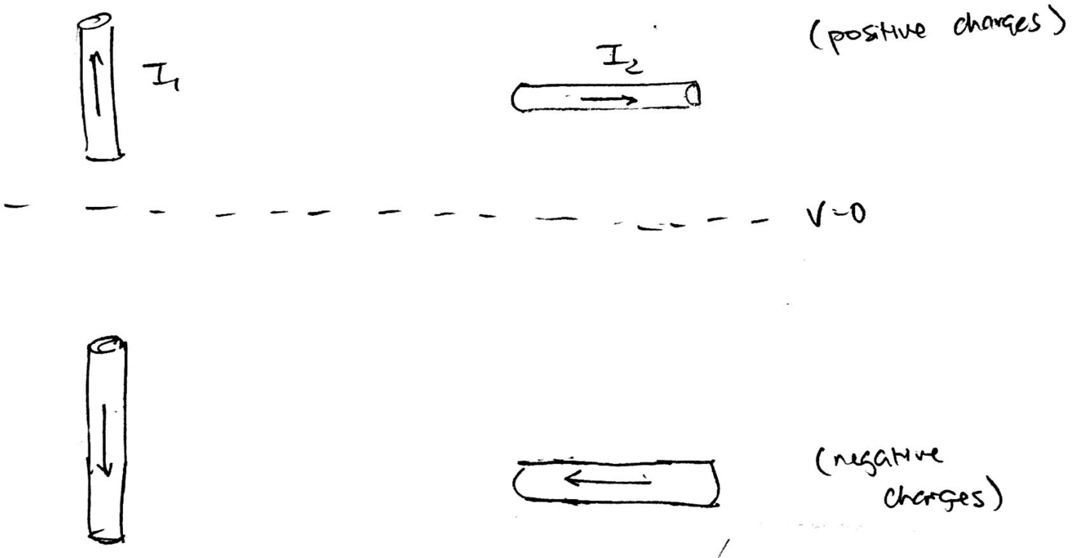
1. Image Theory (10 points)





(a) (b)

Conducting wires above a conducting plane carry currents I_1 and I_2 in the directions shown in the figure above. Keeping in mind that the direction of a current is defined in terms of the movement of positive charges, what are the directions of the image currents corresponding to I_1 and I_2?



X

✓

-5

EE101A – Engineering Electromagnetics

2. Electrostatics (20 points)

Which of the two following cases does not meet the electrostatic field assumptions? Explain.

(A): $\vec{E} = 4[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}]$

(B): $\vec{E} = 2[y^2\hat{x} + (2xy + z^2)\hat{y} + (2yz)\hat{z}]$

$\nabla \times E = 0$

(A) $\hat{x}(0 - 2y) + \hat{y}(0 - 3z) + \hat{z}(0 - x)$

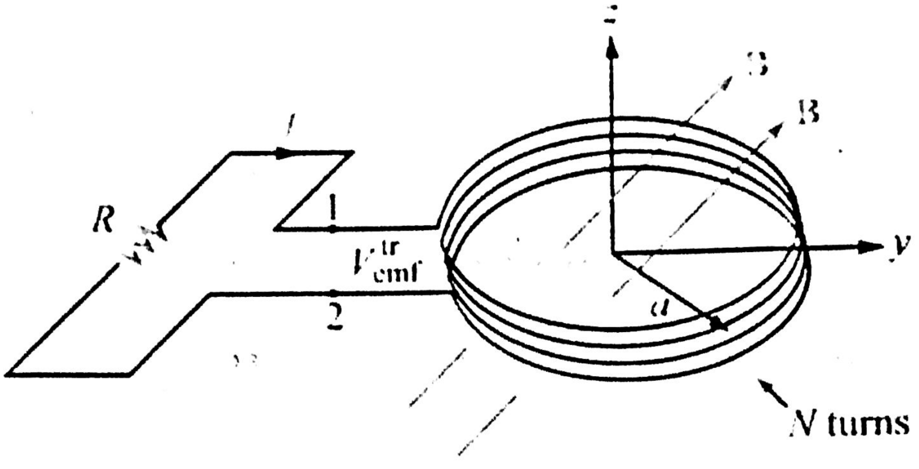
→ Does not meet electrostatic field assumption bc

$\nabla \times E \neq 0$. ✓

(B) $\hat{x}(2z - 2z) + \hat{y}(0 - 0) + \hat{z}(2y - 2y)$

$\nabla \times E = 0$ ✓

3. Inductor in a changing magnetic field (30 points)



An inductor is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the $\boxed{\text{x-y plane}}$ with its center at the origin, and connected to a resistor R, as shown in the figure above. In the presence of a magnetic field $\vec{B} = B_0(\hat{\mathbf{y}}3 + \hat{\mathbf{z}})sin\omega t$, where ω is the angular frequency. Note: $\frac{dsinx}{dx} = cosx; \frac{dcosx}{dx} = -sinx$

Find the following parameters:

(a) the magnetic flux linking a <u>single turn</u> of the inductor

$N = 1$

$$\Phi = \int B \cdot ds = B \cdot A = B_0(\hat{y}3 + \hat{z})\sin(\omega t) \cdot (\pi a^2)$$

$$= B_0 \sin(\omega t) \cdot (\pi a^2)$$

✓

(b) the $V_{emf}^{tr} = V_1 - V_2$, given that N=10, B₀=0.2T, a=10cm, and $\omega = 10^3$ rad/s.

$$-N\frac{d\Phi}{dt} = V_{emf} \qquad \Phi = B_0(\hat{y}3 + \hat{z})\sin(\omega t)\cdot\pi a^2$$

$$= -10\left[0.2\,(\hat{y}3+\hat{z})\cos(\omega t)\cdot\omega\cdot\pi a^2\right]$$

$$V_{emf} = -10\left[0.2\cos(10^3 t)\cdot 10^3(\pi\cdot 0.1^2)\right]$$

$$V_{emf} = -20\pi\cos(10^3 t) \ \ \mathrm{Tm^2 rad/sec}$$

✓

(c) the polarity of V_{emf}^{tr} at t=0

$$V_{emf} = V_1 - V_2 = -10\left[0.2\overset{1}{\cos}(0)\cdot 10^3(\pi\cdot 0.1^2)\right] = \boxed{\text{"}-\text{"}}$$

✓

(d) the induced current in the circuit for R=1kΩ (assume the wire resistance to be much smaller than R)

$I = \int J\cdot ds \cdots$???

$$I = \frac{V_{emf}}{R} \qquad I = \frac{-20\pi\cos(10^3 t)}{1000}$$

R=1k

✓