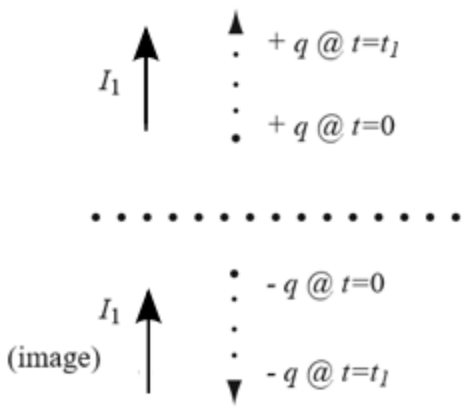
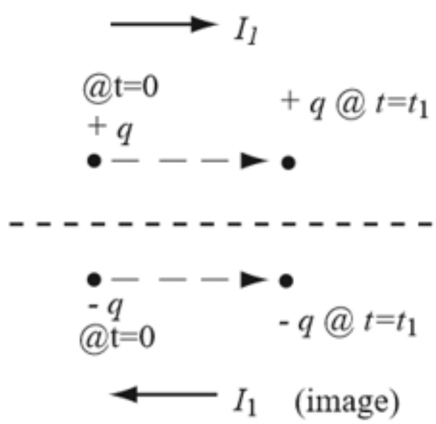
Problem 1
Solution:

(a)



(b)



Problem 2.
Solution:

When the field is electrostatic, $\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial\vec{B}}{\partial t} = 0$

For (A),

$$\nabla\times\vec{E} = 4[\hat{x}(0-2y) + \hat{y}(3z-0) + \hat{z}(0-x)] \neq 0$$

For (B),

$$\nabla\times\vec{E} = 2[\hat{x}(2z-2z) + \hat{y}(0-0) + \hat{z}(2y-2y)] = 0$$

So the E field in (A) is impossible for electrostatic situation.

Problem 3.
Solution:
(a)

The magnetic flux linking single turn of the inductor is

$$\phi = \int_s \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \int_s [B_0(\hat{\boldsymbol{y}}3 + \hat{\boldsymbol{z}})sin\omega t] \cdot \hat{\boldsymbol{z}}\, ds = \int_s B_0 sin\omega t\, ds$$

$$= B_0 sin\omega t \int_0^{2\pi} \int_0^a r dr d\phi = B_0 sin\omega t \cdot \pi a^2$$

(b)

$$V_1 - V_2 = V_{emf}^{tr} = -\frac{Nd\phi}{dt} = -N\frac{d(B_0 sin\omega t \cdot \pi a^2)}{dt}$$

$$= -NB_0 \cdot \pi a^2 \cdot \omega \cdot cos\omega t = -\pi a^2 \omega N B_0 cos\omega t$$

$$= -\pi \times 0.1^2 \times 10^3 \times 10 \times 0.2 \cos(10^3 t) = -20\pi \cos(10^3 t)$$

$$= -62.8 \cos(10^3 t)\ V$$

(c)

At t=0, $V_{emf}^{tr} = -62.8\,V$. The flux is increasing, so the emf voltage is pushing current going from 1 through the inductor to 2. In this situation, (V_1-V_2) is smaller than 0, meaning that $V_1<V_2$.

(d)

$$I = \frac{V_2 - V_1}{R} = \frac{-62.8\cos(10^3 t)}{1000} = -0.0628\cos(10^3 t)\quad A$$

Problem 4.

Solution:

$$\vec{\boldsymbol{D}} = \hat{\boldsymbol{R}} D_R$$

When R<a,

$$D_R = 0$$

When $a \le R \le b$,

$$D_R \cdot 4\pi R^2 = \int \rho dv = \int_a^R -\frac{\rho_{v0}}{R^2} \cdot 4\pi R^2 dR = -4\pi\rho_{v0}(R-a)$$

So we have:

$$\vec{\boldsymbol{D}} = \hat{\boldsymbol{R}} D_R = \frac{\hat{\boldsymbol{R}}[-4\pi\rho_{v0}(R-a)]}{4\pi R^2} = \hat{\boldsymbol{R}}\frac{-\rho_{v0}(R-a)}{R^2}$$

When R ≥ b

$$\vec{\boldsymbol{D}} = \hat{\boldsymbol{R}} D_R = \hat{\boldsymbol{R}}\frac{-\rho_{v0}(b-a)}{R^2}$$

Problem 5.

Solution:

$$\vec{\boldsymbol{H}} = \frac{I}{4\pi}\int_l \frac{d\vec{\boldsymbol{l}} \times \hat{\boldsymbol{R}}}{R^2}$$

We apply the super position of H field by semi-circle a and b.

For semi-circle a:

$$\overrightarrow{\boldsymbol{H_a}} = \frac{I}{4\pi}\int_l \frac{d\vec{\boldsymbol{l}}\times\widehat{\boldsymbol{R}}}{R^2} = \frac{I}{4\pi}\int_0^{\pi} \frac{\widehat{\boldsymbol{\phi}} a d\phi \times(-\widehat{\boldsymbol{R}})}{a^2} = \frac{I}{4\pi}\int_0^{\pi} \hat{\boldsymbol{z}}\cdot\frac{d\phi}{a} = \hat{\boldsymbol{z}}\frac{I}{4a}$$

Similarly, for semi-circle b:

$$\overrightarrow{\boldsymbol{H_b}} = \hat{\boldsymbol{z}}\frac{I}{4b}$$

$$\overrightarrow{\boldsymbol{H}} = \overrightarrow{\boldsymbol{H_a}} + \overrightarrow{\boldsymbol{H_b}} = \hat{\boldsymbol{z}}\frac{I}{4}\left(\frac{1}{b}+\frac{1}{a}\right)$$