UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2020
Midterm, November 5, 2020 (120 minutes)

Name: ________________________ Student number: ____________________

This is a closed book exam – you are allowed 2 pages (A4 size) of notes (front + back). You are allowed to use a calculator. You are NOT allowed to use other electronic devices such as laptops and cell phones.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focus on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Image Theory	10	
Problem 2	Gauss's Law	15	
Problem 3	Poisson's Equation	15	
Problem 4	Faraday's Law	20	
Problem 5	Ampere's Law	20	
Problem 6	Electrostatics	20	
Total		100	

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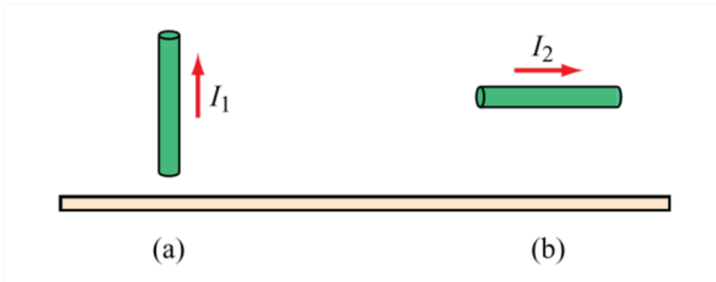
Constants (SI units):

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$$
$$\mu_0 = 4\pi \times 10^{-7}\mathrm{H/m}$$

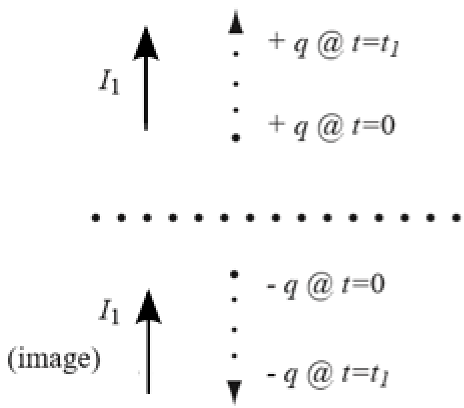
1. Image Theory (10 points)

Conducting wires above a conducting plane carry currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the directions shown in the figure above. Keeping in mind that the direction of a current is defined in terms of the movement of positive charges, what are the directions of the image currents corresponding to $\mathbf{I}_1$ and $\mathbf{I}_2$?

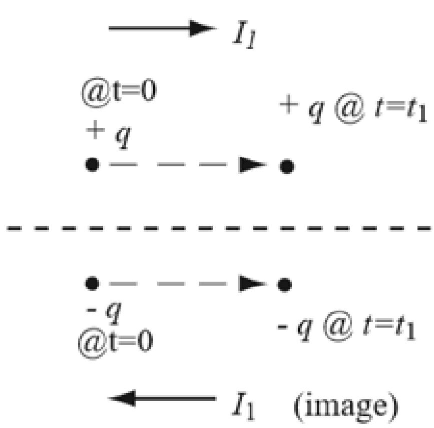


Solution:

(a)



(b)

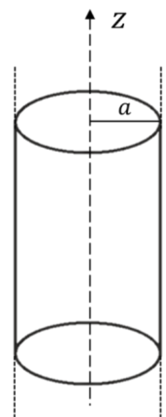


2. Gauss's Law (15 points)

Consider a long cylinder of radius a with its axis along the z-axis. The cylinder is uniformly charged with a constand *volume* charge density of ρ (positive).

(a) Using Gauss's Law find the electric field **E** as a function of r ($0 \le r \le a$ and $r > a$).

(b) Plot **E** as a function of distance r



(a) Symmetry $\longrightarrow E = E(r)\hat{r}$

From Gauss's law, $\oint E \cdot ds = Q/\varepsilon_0$.

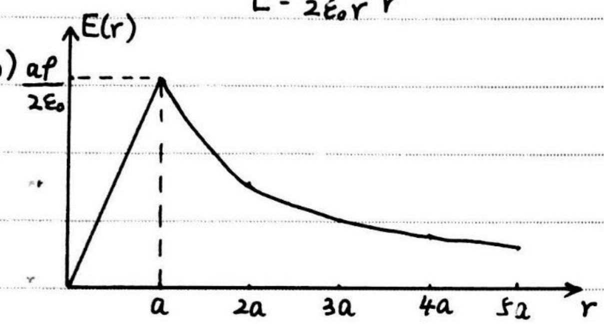
For $0 \le r \le a$, $2\pi r l\, E(r) = \rho \pi r^2 l/\varepsilon_0$, $E(r) = \frac{r\rho}{2\varepsilon_0}$

$$E = \frac{r\rho}{2\varepsilon_0}\hat{r}$$

For $r > a$, $2\pi r l\, E(r) = \rho \pi a^2 l/\varepsilon_0$. $E(r) = \frac{a^2\rho}{2\varepsilon_0 r}$

$$E = \frac{a^2\rho}{2\varepsilon_0 r}\hat{r}$$

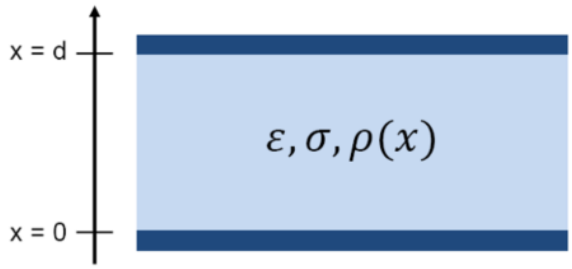
(b)



3. Poisson's Equation (15 points)

For a parallel plate capacitor with dielectric permittivity ε, conductivity σ, and thickness d, assume an initial condition of $\rho\,(x) = \rho_0$. The two plates were shorted at time t = 0, so you can assume $V(x = 0,\ t = 0^+) = V(x = d,\ t = 0^+) = 0$.

Find $V(x, t = 0^+)$, $\boldsymbol{E}(x, t = 0^+)$, and surface charge at two plates at $t = 0^+$



2. Posson's equations

$\nabla^2 V = -\frac{\rho_0}{\varepsilon}$

1D problem, $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} = -\frac{\rho_0}{\varepsilon}$

Form of the solution: $V(x) = -\frac{\rho_0 x^2}{2\varepsilon} + Cx + D$

Apply boundary condition: $\begin{cases} V(0) = 0. & D = 0. \\ V(d) = 0, & -\frac{\rho_0 d^2}{2\varepsilon} + Cd = 0. \quad C = \frac{\rho_0 d}{2\varepsilon} \end{cases}$

$V(x) = -\frac{\rho_0 x^2}{2\varepsilon} + \frac{\rho_0 d}{2\varepsilon} x$, $\vec{E}(x) = -\hat{x}\frac{dV}{dx} = \hat{x}\left(\frac{\rho_0 x}{\varepsilon} - \frac{\rho_0 d}{2\varepsilon}\right)$

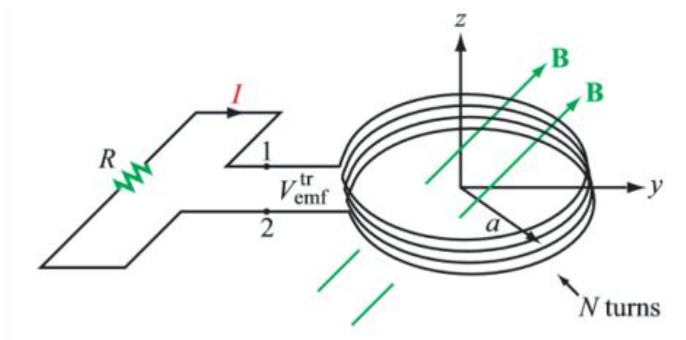
Surface charge $\begin{cases} \text{bottom: } E_n = \hat{n}\cdot\vec{E}(x=0) = \hat{x}\cdot\left(-\hat{x}\frac{\rho_0 d}{2\varepsilon}\right), \ E_n = \rho_s/\varepsilon. \ \rho_s = -\frac{\rho_0 d}{2} \\ \text{top: } E_n = \hat{n}\cdot\vec{E}(x=d) = -\hat{x}\cdot\left(\hat{x}\frac{\rho_0 d}{2\varepsilon}\right), \ E_n = \rho_s/\varepsilon, \ \rho_s = -\frac{\rho_0 d}{2} \end{cases}$

4. Faraday's Law (20 points)

An inductor is formed by winding *N* turns of thin conducting wire into a circular loop of radius a. The inductor loop is in the *x*-*y* plane with its center at the origin, and connected to a resistor *R*, as shown in the figure below. In the presence of a magnetic field $\vec{\boldsymbol{B}} = B_0(\hat{\boldsymbol{y}} + 3\hat{\boldsymbol{z}})\sin\omega t$, where ω is the angular frequency. Find the following parameters:

(a) the magnetic flux linking a single turn of the inductor

(b) the $V_{emf}^{tr} = V_1 - V_2$, given that *N* = 50, B_0 = 0.3 T, a = 20 cm, and $\omega = 10^2$ rad/s



3. (a) $\Phi = \int \vec{B}\cdot d\vec{s} = \int_0^a\int_0^{2\pi} B_0(\hat{y}+3\hat{z})\sin(\omega t)\cdot r\,dr\,d\phi\,\hat{z} = 3\pi a^2 B_0 \sin(\omega t)$

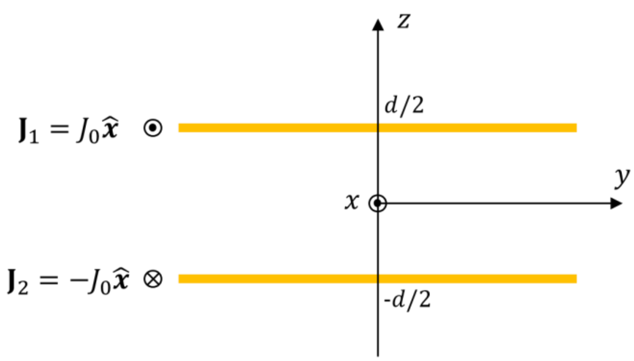
(b) $V_{emf}^{tr} = -N\frac{\partial\Phi}{\partial t} = -N\cdot 3\pi a^2 B_0 \omega\cos(\omega t) = -180\pi\cos(10^2 t)$

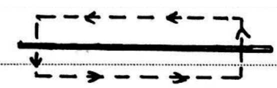
5. Ampere's Law (20 points)

Consider two infinitely large sheets lying in the *xy*-plane, separated by a distance *d*. The two sheets carry surface current densities of $\mathbf{J}_1 = J_0\hat{\boldsymbol{x}}$ and $\mathbf{J}_2 = -J_0\hat{\boldsymbol{x}}$, respectively (as shown in the figure below). The extent of the sheets in the *y* direction is infinity. Note that J_0 is the current per unit width perpendicular to the flow.

(a) Find the magnetic field everywhere due to both current sheets.

(b) How would your answer in (a) change if both current were running in the same direction, with $\mathbf{J}_1 = \mathbf{J}_2 = J_0\hat{\boldsymbol{x}}$.



(a) J_1 ⊙ 

Consider the Loop above. $\oint_c H\cdot dl = I$

$\oint_c H\cdot dl = 2Hl$, $I = J_0 l$

Therefore, the magnetic field from sheet 1:

$$H_1 = \begin{cases} -\frac{J_0}{2}\hat{y}, & z > \frac{d}{2} \\ \frac{J_0}{2}\hat{y} & z < \frac{d}{2} \end{cases}$$

Similarly, the magnetic field from sheet 2

$$H_2 = \begin{cases} \frac{J_0}{2}\hat{y} & z > -\frac{d}{2} \\ -\frac{J_0}{2}\hat{y} & z < -\frac{d}{2} \end{cases}$$

$$H = H_1 + H_2 = \begin{cases} J_0\hat{y} & |z| < \frac{d}{2} \\ 0 & |z| > \frac{d}{2} \end{cases}$$

(b) $H_2 = \begin{cases} -\frac{J_0}{2}\hat{y}, & z > \frac{-d}{2} \\ \frac{J_0}{2}\hat{y} & z < -\frac{d}{2} \end{cases}$ $H = H_1 + H_2 = \begin{cases} -J_0\hat{y}, & z > \frac{d}{2} \\ 0 & -\frac{d}{2} < z < \frac{d}{2} \\ J_0\hat{y} & z < -\frac{d}{2} \end{cases}$

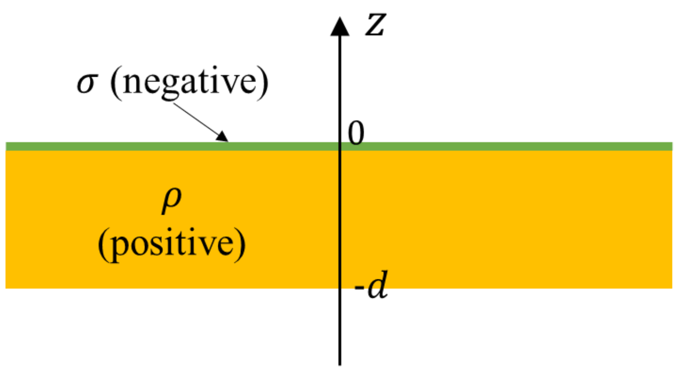
6. Electrostatics (20 points)

As shown in the figure below, an infinite, *non-conducting* sheet (of negligible thickness) carries a negative uniform *surface* charge density σ ($\sigma < 0$). Next to it, an infinite parallel slab of thickness d carries positive uniform *volume* charge density ρ. All charges are fixed.

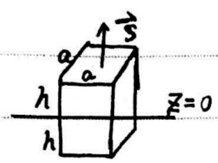
(a) Calculate the magnitude of the electric field **E** in the region above the negatively charged sheet and point out its direction.

(b) Plot **E** as a function of distance z (for $z = 0$ to $+\infty$).

Hint: Gauss's Law and superposition principle could be your best friends when solving this problem! You may want to calculate the electric field due to the sheet of charge alone (let's call it $\mathbf{E}_{sheet}$), and then calculate the electric field due to the slab of charge alone (let's call it $\mathbf{E}_{slab}$).



(a) For $\vec{E}_{sheet}$, since the plate is infinite, $\vec{E}_{sheet}$ is in $\hat{z}$ or $-\hat{z}$ direction



From Gauss's law, $\oint \vec{E}\, d\vec{s} = Q/\varepsilon_0$

$E \cdot a^2 \cdot 2 = \sigma \cdot a^2/\varepsilon_0$, $\vec{E}_{sheet} = \frac{\sigma}{2\varepsilon_0}\hat{z}$ $(z>0)$

Similarly, $\vec{E}_{slab} = \frac{\rho d}{2\varepsilon_0}\hat{z}$ $(z>0)$

$E = \vec{E}_{sheet} + \vec{E}_{slab} = \frac{\rho d+\sigma}{2\varepsilon_0}\hat{z}$ $z>0$

(b)