UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2019
Midterm, October 30, 2019, (100 minu[redacted]

Name: [redacted] Student number [redacted]

This is a [redacted]lowed 2 pages (A4 size) of notes (front + back). You are allowed to use a calculator. You are NOT allowed to use other electronic devices such as laptops and cell phones.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focus on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Electrostatics	15	15
Problem 2	Gauss's Law	20	20
Problem 3	Faraday's Law	20	17
Problem 4	Ampere's Law	20	17
Problem 5	Electrostatics	25	17
Total		100	86

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Constants (SI units):

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$$
$$\mu_0 = 4\pi \times 10^{-7}\mathrm{H/m}$$

1. Electrostatics (15 points)

Which of the two following expressions does not meet the electrostatic field assumptions? Explain why.

a): $\vec{E}_a = 3[(4x + y^2)\hat{\mathbf{x}} + \left(5xy + \frac{1}{2}z^2\right)\hat{\mathbf{y}} + (yz)\hat{\mathbf{z}}]$

b): $\vec{E}_b = 2[(3x + 2z^2)\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + (4xz + y^2)\hat{\mathbf{z}}]$

$\nabla \times \vec{E} = \vec{0} \rightarrow$ in electrostatics, $\vec{E}$-fields are conservative

$\nabla \cdot \vec{D} = \rho_v$

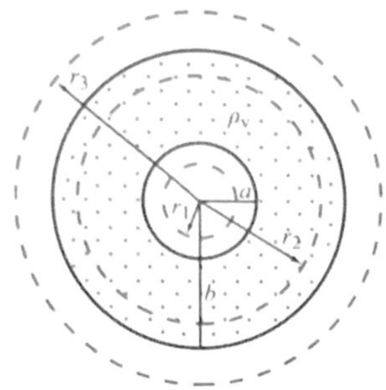
$$\nabla \times \vec{E}_a = 3 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (4x+y^2) & (5xy+\frac{1}{2}z^2) & yz \end{vmatrix} = 3\left(\hat{x}(z - z) - \hat{y}(0-0) + \hat{z}(5y - 2y)\right) = 3(3y\hat{z})$$

$$\nabla \times \vec{E}_b = 2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3x+2z^2) & 2yz & (4xz+y^2) \end{vmatrix} = 2\left(\hat{x}(2y-2y) - \hat{y}(4z-4z) + \hat{z}(0-0)\right) = 0$$

$\boxed{\text{(a) does not meet the electric field assumptions}}$ in electrostatics.

Since $\nabla \times \vec{E}_a = 9y\hat{z} \neq \vec{0}$ and $\nabla \times \vec{E} = \vec{0}$ in electrostatics.

2. Gauss's Law (20 points)



A spherical shell with outer radius b surrounds a charge-free cavity of radius $a < b$ (in the figure above). If the shell contains a charge density given by

$$\rho_v = -\frac{\rho_{v0}}{R^2}, \qquad a \le R \le b,$$

Where ρ_{v0} is a positive constant, determine $\vec{\mathbf{D}}$ in all 3 regions: $\cancel{a < R}$ $R < a$, $a \le R \le b$ and $R \ge b$?

$\vec{E}$?

$\oint_S \vec{D} \cdot d\vec{s} = Q_{enc}$ ✓

assume $\vec{D}$ would point outwards $\hat{R}$; choose surface of sphere

$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D R^2 \sin\theta \, d\theta \, d\phi = 2\pi D R^2 [-\cos\theta]_0^{\pi} = 4\pi D R^2$

$a < R$: $Q_{enc} = 0$, so $\boxed{\vec{D} = \vec{0}}$ ✓

$a \le R \le b$: $Q_{enc} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{r=a}^{R} \left(-\frac{\rho_{v0}}{R^2}\right) R^2 \sin\theta \, dR \, d\phi \, d\theta = -\rho_{v0} (R-a)(2\pi)(2)$
$= -4\pi \rho_{v0} (R-a)$

so, $\vec{D} = \hat{R} \frac{1}{4\pi R^2} (-4\pi \rho_{v0} (R-a)) = \boxed{-\frac{\rho_{v0}(R-a)}{R^2} \hat{R}}$ ✓

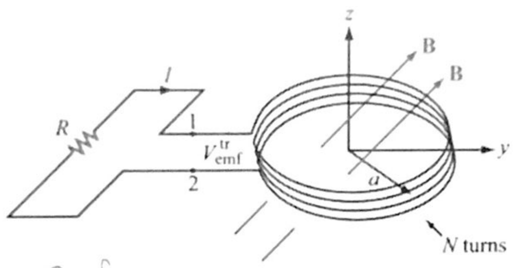
$R \ge b$: $Q_{enc} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{r=a}^{b} \left(-\frac{\rho_{v0}}{R^2}\right) R^2 \sin\theta \, dR \, d\phi \, d\theta = -\rho_{v0}(b-a)(2\pi)(2)$
$= -4\pi \rho_{v0} (b-a)$

$\boxed{\vec{D} = -\frac{\rho_{v0}(b-a)}{R^2} \hat{R}}$ ✓

3. Faraday's Law (20 points)

An inductor is formed by winding N turns of thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor R, as shown in the figure below. In the presence of a magnetic field $\vec{B} = B_0(\hat{y} + 2\hat{z})\sin\omega t$, where ω is the angular frequency. Find the following parameters:

(a) the magnetic flux linking a single turn of the inductor;
(b) the $V_{emf}^{tr} = V_1 - V_2$, given that $N = 10$, $B_0 = 0.2$ T, $a = 15$ cm, and $\omega = 10^3$ rad/s;



a)

$\Phi = \iint_S \vec{B}\cdot d\vec{s}$ surface is in xy plane, so $d\vec{s} = r\,dr\,d\phi\,\hat{z}$

$= \int_{\phi=0}^{2\pi}\int_{r=0}^{a} (B_0(\hat{y}+2\hat{z})\sin\omega t)\cdot(r\,dr\,d\phi\,\hat{z}) = \int_{\phi=0}^{2\pi}\int_{r=0}^{a} 2B_0 \sin\omega t\, r\,dr\,d\phi = \boxed{2\pi a^2 B_0 \sin\omega t}$

b) $V_{emf}^{tr} = V_1 - V_2$

$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(2\pi a^2 B_0 \sin(\omega t)) = \boxed{-2\pi a^2 B_0 \omega \cos(\omega t)}$

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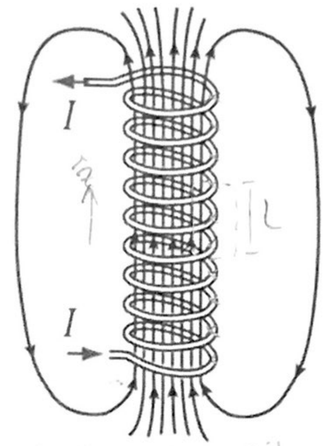
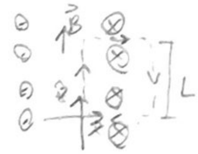
4. Ampere's Law (20 points)

Find the magnetic flux density **B** in the interior region of a tightly wound solenoid. The solenoid is of length l and radius a, and comprises N turns carrying current I.





$\vec{B}_{outside} = 0$

$$\oint_C \vec{B}\cdot d\vec{l} = \int_{z=0}^{L} B\,dz = BL$$

The other sides of the loop would have $\int \vec{B}\cdot d\vec{l} = 0$ since for the sides that go through the sides of the solenoid, $\vec{B} \perp d\vec{l}$ inside and outside $\vec{B} \approx \vec{0}$, so the path outside also has $\int \vec{B}\cdot d\vec{l} = 0$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \checkmark \qquad I_{enc} = L\left(\frac{N}{l}\right)$$

So, $BL = \mu_0 \left(L\frac{N}{l}\right)$, $\boxed{\vec{B} = \mu_0 \frac{N}{L} I \hat{z}}$

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5. Electrostatics (25 points)

A uniform line charge (positively charged, line charge density ρ_l, length l) stands perpendicularly on a grounded perfectly conducting plane (infinite large in x and y direction). Assuming that the free space has a dielectric permittivity of ε_0. Find the following parameters:

(a) Electric field at the ground plane surface $\mathbf{E}$ ($r, z = 0^+$), where r is the cylindrical radial coordinate shown in the figure below.

(b) Surface charge density at the ground plane surface ρ_s ($r, z = 0^+$)

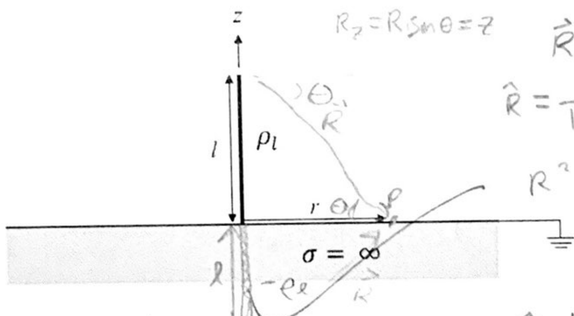
Hint: One or more of the following indefinite integrals may be useful.

i) $\int \frac{x\,dx}{\sqrt{x^2+L^2}} = \sqrt{x^2+L^2}$

ii) $\int \frac{dx}{\sqrt{x^2+L^2}} = \ln(x + \sqrt{x^2+L^2})$

iii) $\int \frac{dx}{(x^2+L^2)^{3/2}} = \frac{x}{L^2\sqrt{x^2+L^2}}$

iv) $\int \frac{x\,dx}{(x^2+L^2)^{3/2}} = -\frac{1}{\sqrt{x^2+L^2}}$



$\vec{R} = -R\sin\theta\,\hat{z} + R\cos\theta\,\hat{r}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{R\sin\theta\hat{z} + R\cos\theta\hat{r}}{R}$

$R^2 = z^2 + r^2$

$R_z = \sin\theta = \frac{z}{R}$

$E_z = \frac{1}{4\pi\varepsilon_0}\int_V \rho \frac{1}{R^2}\sin\theta\, dv = \frac{\rho}{4\pi\varepsilon_0}\int \frac{z}{R^3}$

net $\vec{E}$ will point in $-\hat{z}$ direction on plane by conductor's properties + symmetry

Choose point $P(r, z=0^+)$

So we only care about E_z

$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\int_{z=0}^{l} \rho_l \frac{z}{(r^2+z^2)^{3/2}}dz + \frac{1}{4\pi\varepsilon_0}\int_{z=-l}^{0} -\rho_l \frac{z}{(r^2+z^2)^{3/2}}dz$

$= \left[\frac{\rho_l}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{z^2+r^2}}\right)\right]_{z=0}^{l} + \frac{-\rho_l}{4\pi\varepsilon_0}\left[-\frac{1}{\sqrt{z^2+r^2}}\right]_{z=-l}^{0}$

$= \frac{\rho_l}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{l^2+r^2}} + \frac{1}{r} + \frac{1}{r} - \frac{1}{\sqrt{l^2+r^2}}\right) = \frac{\rho_l}{4\pi\varepsilon_0}\left(\frac{2}{r} - \frac{2}{\sqrt{l^2+r^2}}\right)$

$\ln\left(\frac{l+\sqrt{l^2+r^2}}{r}\right) - \ln\left(\frac{r}{-l+\sqrt{r^2+l^2}}\right) = \ln\left(\frac{r^2+l^2-l^2}{r^2}\right)$

so $\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0}\left(\frac{2}{r} - \frac{2}{\sqrt{l^2+r^2}}\right)(-\hat{z})$ since net $\vec{E}$ is in $+\hat{z}$ direction on plane

$\rho_s(r, z=0^+)$, $V=0$ on surface of plane, ρ_s would be induced so it's strongest around the base of the line and decreasing as you go further away, and $Q_{tot} = -l\rho_l$ across entire plane

so, induced charge cancels out voltage from line

$V_{\text{on the plane, from line}} = \frac{1}{4\pi\varepsilon_0}\int_{z=0}^{l}\frac{\rho_l}{R}dz = \frac{\rho_l}{4\pi\varepsilon_0}\int_0^l \frac{1}{\sqrt{z^2+r^2}}dz = \frac{\rho_l}{4\pi\varepsilon_0}\left[\ln(z+\sqrt{z^2+r^2})\right]_0^l = \frac{\rho_l}{4\pi\varepsilon_0}\ln\left(\frac{l+\sqrt{l^2+r^2}}{r}\right)$

and since on the plane $V=0$, $\frac{\rho_l}{4\pi\varepsilon_0}\ln\left(\frac{l+\sqrt{l^2+r^2}}{r}\right) = V_{\text{from plane}}$, take $\nabla^2 V = -\frac{\rho_s}{\varepsilon_0}$ (cont.)

$$\nabla^2 V = \frac{\partial^2 V}{\partial r^2} = \frac{\rho_l}{4\pi\varepsilon_0} \frac{\partial}{\partial r} \left(\frac{l + \sqrt{l^2+r^2}}{r} \left(\frac{\partial}{\partial r} \left(\frac{l+\sqrt{l^2+r^2}}{r} \right)\right)\right) = \frac{-\rho_s}{\varepsilon_0}$$

$$\rho_s = -\frac{\rho_l}{4\pi\varepsilon_0} \frac{\partial}{\partial r} \left(\frac{l + \sqrt{l^2+r^2}}{r} \left(\frac{\partial}{\partial r} \left(\frac{l+\sqrt{l^2+r^2}}{r} \right)\right)\right)$$

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