# UCLA Department of Electrical Engineering
## EE101A – Engineering Electromagnetics
## Fall 2018
## Midterm, November 13, 2018, (100 minutes)

Name: ______________________________ Student number: ____________________

This is a closed book exam – you are allowed 2 pages (A4 size) of notes (front + back). You are allowed to use a calculator. You are NOT allowed to use other electronic devices such as laptops and cell phones.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focus on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

| | Topic | Max Points | Your points |
|---|---|---|---|
| Problem 1 | Electrostatics | 15 | |
| Problem 2 | Plane Wave | 15 | |
| Problem 3 | Faraday's Law | 30 | |
| Problem 4 | Capacitor | 20 | |
| Problem 5 | Ampere's Law | 20 | |
| | | | |
| Total | | 100 | |

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Constants (SI units):

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$
$$\mu_0 = 4\pi \times 10^{-7} \text{H/m}$$

1. ## Electrostatics (15 points)

Which of the two following expressions does not meet the electrostatic field assumptions? Explain why.
(A): $\vec{\boldsymbol{E}} = 3[(5x + 2z^2)\hat{\boldsymbol{x}} + 2yz\hat{\boldsymbol{y}} + (4xz + y^2)\hat{\boldsymbol{z}}]$
(B): $\vec{\boldsymbol{E}} = 2[(2x + y^2)\hat{\boldsymbol{x}} + (3xy + z^2)\hat{\boldsymbol{y}} + (2yz)\hat{\boldsymbol{z}}]$

Solution:

The electrostatic field should satisfy the condition of $\nabla \times \vec{\boldsymbol{E}} = 0$. (4 points)

(A): $\nabla \times \vec{\boldsymbol{E}} = \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$ (3 points)

$= 3\hat{x}(2y - 2y) + 3\hat{y}(4z - 4z) + 3\hat{z}(0 - 0) = 0$ (4 points)

Which satisfies the electrostatic field assumption.

(B): $\nabla \times \vec{\boldsymbol{E}} = \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$

$= 2\hat{x}(2z - 2z) + 2\hat{y}(0 - 0) + 2\hat{z}(3y - 2y) = 2\hat{z}y \neq 0$ (4 points)

Which does not satisfy the electrostatic field assumption.

2. Plane Wave (15 points)

The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\vec{\boldsymbol{H}} = \hat{\boldsymbol{z}}\,45\cos(2\pi \times 2 \times 10^8 t + 2\pi\ \cdot y)\quad \text{(mA/m)}$$

Find the following:
(a) The direction of wave propagation.
(b) The phase velocity.
(c) The wavelength in the material.
(d) The relative permittivity of the material.
(e) The electric field phasor.

Solution:

(a) The wave is propagating in -$\hat{y}$ direction. (3 points)

(b) The phase velocity is given by: (3 points)

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = 2\pi \cdot \frac{(2 \times 10^8)}{2\pi} = 2 \times 10^8\ \text{m/s}$$

(c) The wavelength in the material is given by: (3 points)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2\pi} = 1\ \text{m}$$

(d) The relative permittivity is given be: (3 points)

$$\frac{\epsilon}{\epsilon_0} = \frac{\omega^2 \epsilon\mu}{\omega^2 \epsilon_0 \mu} = \frac{\beta^2}{k_0^2} = \frac{(2\pi)^2}{(2\pi \cdot 2 \times 10^8)^2 \cdot 8.85 \times 10^{-12} \cdot 4\pi \times 10^{-7}} = 2.25$$

(e) The electric field phasor: (3 points)

$$\vec{\boldsymbol{E}} = \hat{\boldsymbol{x}}\,45 \cdot \eta\cos(2\pi \times 2 \times 10^8 t + 2\pi\ \cdot y)$$

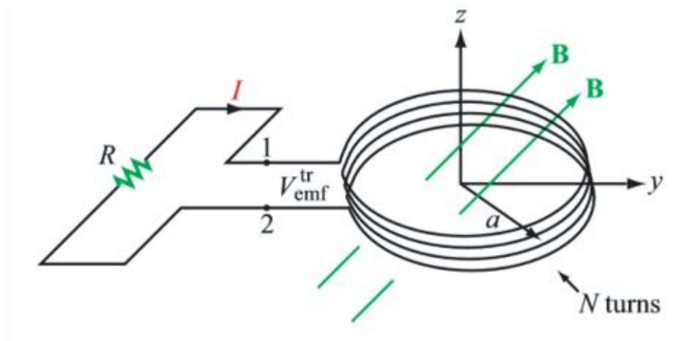
$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\mu_0/\epsilon} = \sqrt{4\pi \times 10^{-7}/(2.25 \cdot 8.85 \times 10^{-12})} = 251\Omega$$

$$\widetilde{\boldsymbol{E}} = \hat{\boldsymbol{x}}\,45 \cdot 251\mathrm{e}^{j2\pi y} = \hat{\boldsymbol{x}}\,11295\mathrm{e}^{j2\pi y}\text{mV/m} = \hat{\boldsymbol{x}}\,11.295\mathrm{e}^{j2\pi y}\ \text{V/m}$$

## 3. Faraday's Law (30 points)

An inductor is formed by winding $N$ turns of thin conducting wire into a circular loop of radius $a$. The inductor loop is in the $x$-$y$ plane with its center at the origin, and connected to a resistor $R$, as shown in the figure below. In the presence of a magnetic field $\vec{\boldsymbol{B}} = B_0(\hat{\boldsymbol{y}} + 2\hat{\boldsymbol{z}})\cos\omega t$, where $\omega$ is the angular frequency. Find the following parameters:

(a) the magnetic flux linking a single turn of the inductor;
(b) the $V_{emf}^{tr} = V_1 - V_2$, given that $N = 20$, $B_0 = 0.2$ T, $a = 15$ cm, and $\omega = 10^3$ rad/s;
(c) the polarity of $V_{emf}^{tr}$ at $\omega t = \pi/2$;




Solution:

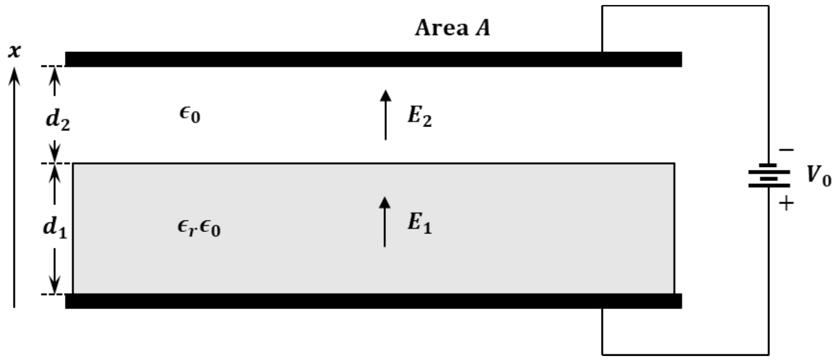
(a) $\Phi = \int \vec{B} \cdot d\vec{s} = \int_0^a \int_0^{2\pi} B_0(\hat{\boldsymbol{y}} + 2\hat{\boldsymbol{z}})\cos\omega t \cdot \rho d\rho d\phi\, \hat{\boldsymbol{z}} = 2B_0 \cos\omega t\, (\pi a^2)$ (10 points)

(b) $V_{emf}^{tr} = V_1 - V_2 = -N \cdot \frac{d\Phi}{dt} = \mathrm{N} \cdot 2\mathrm{B}_0 \omega(\pi a^2)\sin\omega t = 180\pi \sin(1 \times 10^3 t)$ (10 points)

(c) At $\omega t = \pi/2$, $V_{emf}^{tr} = 180\pi \sin(\pi/2) = 180\pi > 0$. The magnetic flux is decreasing, resulting in a induced potential with $V_1 - V_2 > 0$. (10 points)

## 4. Capacitor (20 points)

A parallel plate capacitor with electrodes of area $A$ has its upper electrode in a free space region of thickness $d_2$ in series with a solid dielectric of thickness $d_1$ and dielectric relative permittivity $\epsilon_r$. The $x = d_1$ interface has no free surface charge.




(a) What are the electric field $E_1$ and $E_2$ in the dielectric and free space regions?
(b) What is the free surface charge density on the lower electrode?
(c) What is the capacitance $C$ of the capacitor?

Solution:

(a) Based on the boundary condition at $x = d_1$ interface:

$$\epsilon_r\epsilon_0 E_1 = \epsilon_0 E_2 \quad \Rightarrow \quad E_2 = \epsilon_r E_1 \quad \text{(3 points)}$$

$$E_1 d_1 + E_2 d_2 = V_0 \quad \text{(2 points)}$$

$$E_1(d_1 + \epsilon_r d_2) = V_0 \quad \Rightarrow \quad E_1 = \underset{\text{(3 points)}}{\frac{V_0}{d_1 + \epsilon_r d_2}}, \quad E_2 = \underset{\text{(3 points)}}{\frac{\epsilon_r V_0}{d_1 + \epsilon_r d_2}}$$

(b) Based on the boundary condition at $x = 0$: (2 points)

$$\rho_s(x = 0) = \epsilon_r\epsilon_0 E_1 = \underset{\text{(3 points)}}{\frac{\epsilon_r\epsilon_0 V_0}{d_1 + \epsilon_r d_2}}$$

(c) Capacitance can be found by:

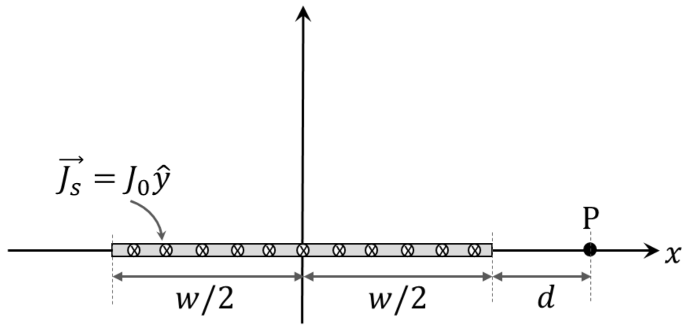
$$C = \underset{\text{(2 points)}}{\frac{Q}{V_0} = \frac{\rho_s A}{V_0}} = \underset{\text{(2 points)}}{\frac{\epsilon_r\epsilon_0 A}{d_1 + \epsilon_r d_2}}$$

Or by using the serial connection equivalent:

$$C = \frac{C_1 C_2}{C_1 + C_2}, \quad C_1 = \frac{\epsilon_r\epsilon_0 A}{d_1}, C_2 = \frac{\epsilon_0 A}{d_2} \quad \text{(2 points)}$$

$$C = \underset{\text{(2 points)}}{\frac{\epsilon_r\epsilon_0 A}{d_1 + \epsilon_r d_2}}$$

## 5. Ampere's Law (20 points)




An infinitely long thin conducting sheet of width $w$ positioned along the *x* direction and lying in the *x*-*y* plane as shown in the figure. It carries a surface current density of $\vec{\boldsymbol{J}}_s = J_0\hat{\boldsymbol{y}}$. Find the magnetic field $\vec{\boldsymbol{H}}$ at point P.

Solution:

We can think of this current sheet as composed of infinite number of current filaments, and $d\vec{H}$ is the H field contribution due to a single current filament (5 points). And based on Ampere's Law (5 points):

$$d\vec{H} = -\hat{z}J_0 dx \cdot \frac{1}{2\pi(\frac{w}{2} + d - x)}$$

(5 points)

Where $\frac{w}{2} + d - x$ is the distance between point P and the current filament at *x*.

$$\vec{H} = -\hat{z}\frac{J_0}{2\pi}\int \frac{1}{\frac{w}{2} + d - x}dx = \hat{z}\frac{J_0}{2\pi}\ln\left(\frac{w}{2} + d - x\right)|_{-w/2}^{w/2}$$

$$= \hat{z}\frac{J_0}{2\pi}\ln(\frac{d}{w+d})$$

(5 points)