UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2018
Midterm, November 13, 2018, (100 minutes)

This is a closed book exam – you are allowed 2 pages (A4 size) of notes (front + back). You are allowed to use a calculator. You are NOT allowed to use other electronic devices such as laptops and cell phones.

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Electrostatics	15	15
Problem 2	Plane Wave	15	15
Problem 3	Faraday's Law	30	30
Problem 4	Capacitor	20	20
Problem 5	Ampere's Law	20	20
Total		100	100

1. Electrostatics (15 points)

assume $\nabla \times \vec{E} = 0$

15

Which of the two following expressions does not meet the electrostatic field assumptions? Explain why.

(A): $\vec{E} = 3[(5x + 2z^2)\hat{x} + 2yz\hat{y} + (4xz + y^2)\hat{z}]$

(B): $\vec{E} = 2[(2x + y^2)\hat{x} + (3xy + z^2)\hat{y} + (2yz)\hat{z}]$

(A) $\frac{1}{3}\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 5x+2z^2 & 2yz & 4xz+y^2 \end{vmatrix} = \hat{x}\left[\frac{\partial}{\partial y}[4xz+y^2] - \frac{\partial}{\partial z}(2yz)\right] - \hat{y}\left[\frac{\partial}{\partial x}[4xz+y^2] - \frac{\partial}{\partial z}[5x+2z^2]\right] + \hat{z}[0 - 0]$

$= \hat{x}(2y - 2y) - \hat{y}(4z - 4z) + \hat{z}(0) = \boxed{0}$ ✓ meets assumption.

(B) $\frac{1}{2}\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+y^2 & 3xy+z^2 & 2yz \end{vmatrix}$ (similar)

$= \hat{x}[2z - 2z] - \hat{y}[0 - 0] + \hat{z}[3y - 2y]$

$= \boxed{\hat{z}[y] \neq 0}$

Since $\nabla \times \vec{E} \neq 0$, (B) is not an ES field

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2. Plane Wave (15 points)

The magnetic field of a wave propagating through a certain nonmagnetic material is given by (μ_0)

$$\vec{H} = \hat{z}\underbrace{45}_{H_0}\cos(\underbrace{2\pi \times 2 \times 10^8}_{\omega} t + \underbrace{2\pi}_{K} \cdot y) \quad (\mathrm{mA/m})$$

Find the following:

(a) The direction of wave propagation.
(b) The phase velocity.
(c) The wavelength in the material.
(d) The relative permittivity of the material.
(e) The electric field phasor.

$\tilde{H} = \hat{z}\ 45 e^{+2\pi y}\ \left[\frac{mA}{m}\right]$

(a) $\hat{K} = \boxed{-\hat{y}}$ direction

(b) $u_p = \frac{\omega}{K} = \frac{2\pi \times 2\times10^8}{2\pi} = \boxed{2\times10^8\ m/s}$

(c) $\lambda = \frac{2\pi}{K} = \frac{2\pi}{2\pi} = \boxed{1\ m}$

(d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\epsilon_r}}\frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\epsilon_r}} \times c$

$\Rightarrow \frac{u_p}{c} = \frac{1}{\sqrt{\epsilon_r}} = \frac{2\times10^8}{3\times10^8} = \frac{1}{\sqrt{\epsilon_r}}$

$\Rightarrow \sqrt{\epsilon_r} = 3/2 \Rightarrow \boxed{\epsilon_r = 9/4}$

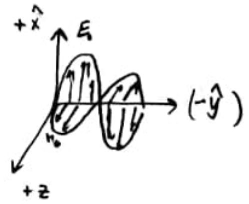
(e) $\tilde{E} = -\eta(\hat{k}\times\tilde{H})$ where $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu}{\epsilon_r\epsilon_0}} = \frac{1}{\sqrt{\epsilon_r}}\times\eta_0 = \frac{1}{3/2}\times 120\pi = 80\pi$

$= -80\pi\left[(-\hat{y})\times(\hat{z})\right]45 e^{2\pi y}\ \left[\frac{mV}{m}\right]$

$= 11309.7 e^{2\pi y}(\hat{x})\left[\frac{mV}{m}\right]$

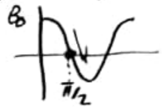
$\boxed{\tilde{E} = (+\hat{x})\ 11.31 e^{2\pi y}\ \left[\frac{V}{m}\right]}$

$-\hat{y}\times\hat{x} = +\hat{z}$

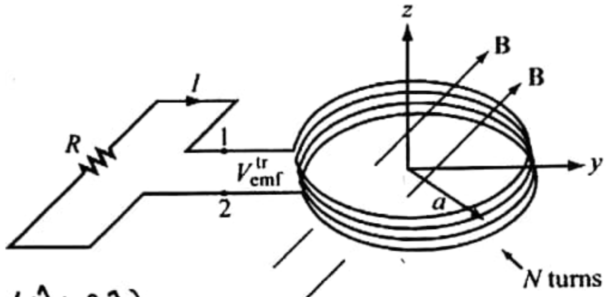


3. Faraday's Law (30 points)

An inductor is formed by winding N turns of thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor R, as shown in the figure below. In the presence of a magnetic field $\vec{B} = B_0(\hat{y} + 2\hat{z})\cos\omega t$, where ω is the angular frequency. Find the following parameters:



(a) the magnetic flux linking a single turn of the inductor;
(b) the $V_{emf}^{tr} = V_1 - V_2$, given that $N = 20$, $B_0 = 0.2$ T, $a = 15$ cm, and $\omega = 10^3$ rad/s;
(c) the polarity of V_{emf}^{tr} at $\omega t = \pi/2$;



(a) $\Phi = \int \vec{B}\cdot d\vec{A} = B_0(\hat{y} + 2\hat{z})\cos\omega t \cdot \{\hat{z}\,\pi a^2\}$

$\boxed{\Phi = 2B_0\pi a^2\cos(\omega t)}$ [Wb]

(b) $V_{emf}^{tr} = -N\frac{d}{dt}\Phi$

$= +N\,2B_0\,\omega\,\pi a^2\sin(\omega t)$

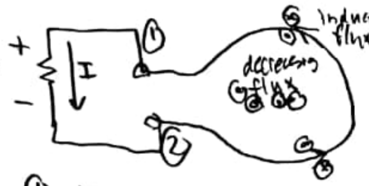
$= 20\times2\times0.2\times1000\times\pi(.15)^2\sin(1000t) = 180\pi\sin(1000t)$

$= \boxed{565.487\sin(1000t)\ [V]}$

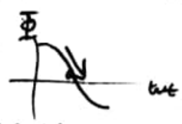
(c) $V_{emf}^{tr}(\omega t = \pi/2) = 565.487\ (\sin 90°)$

$= 565.487$ [V] $\boxed{\text{Positive}}$

sign of Vemf is to generate a current opposing the change in flux



at $\omega t = \pi/2$ flux decreases so induced flux is produced in same direction to maintain field.



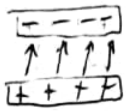
So ① is at higher potential than ②

$E_{cap} = \frac{\rho_s}{\epsilon} (\hat{x}) = \frac{Q}{\epsilon A} (\hat{x})$

$C = \frac{\epsilon A}{d}$ in general for parallel plate cap

4. Capacitor (20 points)

$C = \frac{Q}{V}$



A parallel plate capacitor with electrodes of area A has its upper electrode in a free space region of thickness d_2 in series with a solid dielectric of thickness d_1 and dielectric relative permittivity ϵ_r. The $x = d_1$ interface has no free surface charge.

Boundary

$D_{1n} - D_{2n} = \rho_s \to 0$

$D_{1n} = D_{2n}$

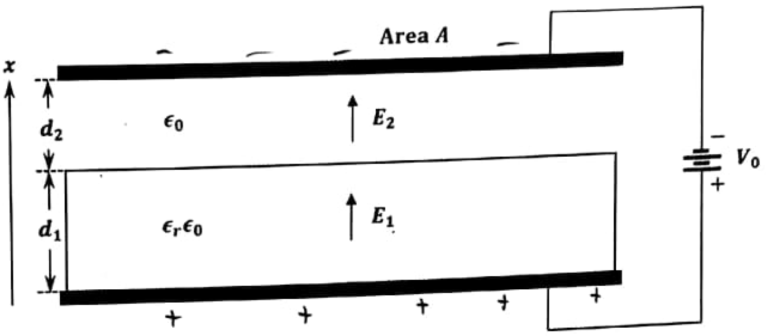
$\epsilon_r \epsilon_0 E_{1n} = \epsilon_0 E_{2n}$

$\epsilon_r E_{1n} = E_{2n}$

$V = -\int \vec{E} \cdot d\vec{l}$

$V_0 = E_1 d_1 + E_2 d_2$

$V_0 = E_1 d_1 + \epsilon_r E_1 d_2$



(a) What are the electric field E_1 and E_2 in the dielectric and free space regions?
(b) What is the free surface charge density on the lower electrode?
(c) What is the capacitance C of the capacitor?

(c)

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ where $C_1 = \frac{\epsilon_r \epsilon_0 A}{d_1}$ and $C_2 = \frac{\epsilon_0 A}{d_2}$

$$C_{eq} = \frac{\left(\frac{\epsilon_r \epsilon_0 A}{d_1}\right)\left(\frac{\epsilon_0 A}{d_2}\right)}{\frac{\epsilon_r \epsilon_0 A}{d_1} + \frac{\epsilon_0 A}{d_2}} = \frac{(\epsilon_0 A)^2 \left[\frac{\epsilon_r}{d_1 d_2}\right]}{(\epsilon_0 A)\left[\frac{\epsilon_r}{d_1} + \frac{1}{d_2}\right]} = \epsilon_0 A \left(\frac{\frac{\epsilon_r}{d_1 d_2}}{\frac{d_2 \epsilon_r + d_1}{d_1 d_2}}\right) = \boxed{\frac{\epsilon_0 \epsilon_r A}{d_2 \epsilon_r + d_1}} \quad [\text{Farads}]$$

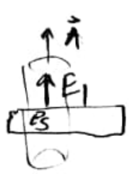
(a)

$$Q = C_{eq} V_0 = \left[\frac{\epsilon_0 \epsilon_r A}{d_2 \epsilon_r + d_1}\right] \times V_0$$

$$\vec{E}_1 = \frac{Q}{(\epsilon_0 \epsilon_r) A} (\hat{x}) = \frac{\left[\frac{\epsilon_0 \epsilon_r A V_0}{d_2 \epsilon_r + d_1}\right]}{\epsilon_0 \epsilon_r A} (\hat{x}) = \boxed{\frac{V_0}{d_2 \epsilon_r + d_1} (+\hat{x})} = \vec{E}_1 \quad \left[\frac{V}{m}\right]$$

$$\vec{E}_2 = \frac{Q}{\epsilon_0 A} \hat{x} = \frac{\left[\frac{\epsilon_0 \epsilon_r A V_0}{d_2 \epsilon_r + d_1}\right]}{\epsilon_0 A} \hat{x} = \boxed{\frac{\epsilon_r V_0}{d_2 \epsilon_r + d_1} (+\hat{x})} = \vec{E}_2$$

(b) Dielectric – Conductor Boundary condition,



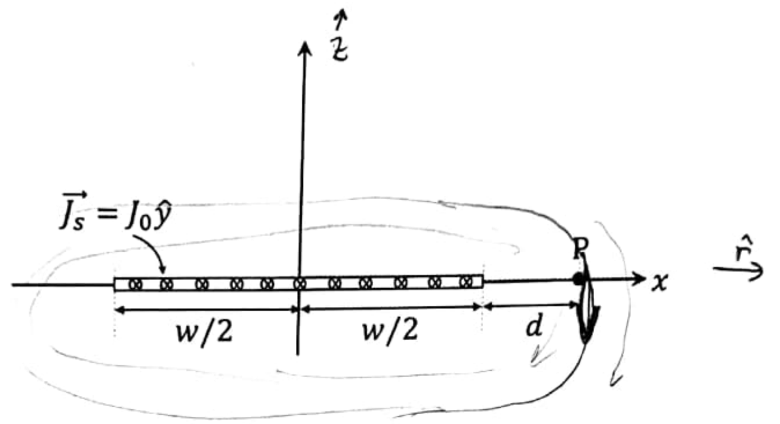
$\hat{n} = \hat{x}$

$\epsilon_r \epsilon_0 \vec{E}_1 \cdot \hat{n} = \rho_s$

$\epsilon_r \epsilon_0 E_{1n} = \rho_s$

$$\boxed{\rho_s = \epsilon_r \epsilon_0 \left[\frac{V_0}{d_1 + d_2 \epsilon_r}\right]} \quad \left[\frac{C}{m^2}\right]$$

5. Ampere's Law (20 points)



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An infinitely long thin conducting sheet of width w positioned along the x direction and lying in the x-y plane as shown in the figure. It carries a surface current density of $\vec{J}_s = J_0\hat{y}$. Find the magnetic field $\vec{H}$ at point P. [A/m]

mpere's Law

$\oint H \cdot dl = I_{enclosed}$ for a long wire $\Rightarrow \vec{H} = \frac{I_{enclosed}}{2\pi r}\hat{\phi}$ but at point P, direction is $(-\hat{z})$

um up the contribution from all the wires.

first one is at d.
last is at d+w away.

$$d\vec{H} = \frac{dI}{2\pi r}(-\hat{z})$$

$$\vec{H} = \int_{d}^{d+w} \frac{J_0\,dr}{2\pi r}(-\hat{z})$$

$$\boxed{\vec{H} = \frac{J_0}{2\pi}\ln\frac{d+w}{d}(-\hat{z})} \quad [A/m]$$