EC ENRG 101A Midterm
Winter 2020

Name: [redacted]

Student I[redacted]

A. Please put all the work on the test paper.
B. Show your work! Provide clear explanations and show calculations. You may not get credit for work that is not shown.
C. List units with all answers

Name of person on LEFT even if "far" way. If wall, then write "Wall". If aisle, then write "Aisle".	ROW NUMBER: (as measured from front)	Name of person on RIGHT even if "far" way. If wall, then write "Wall". If aisle, then write "Aisle".
[redacted]		

BE SURE TO ENTER THE FOLLOWING INFORMATION

Allowed:
pen/pencil
calculator
ruler and/or compass
formula sheet: ***one side*** *of a 8 ½" by 11" sheet of paper.*

Score

1	5/5	5	15 /15
2	4/5	6	20 /20
3	20/20	7	20 /20
4	10/10		
		Total	94/95

Some possibly useful information

$$\mu_0 = 4\pi * 10^{-7}\,\frac{H}{m} \qquad \epsilon_0 = 8.85 * 10^{-12}\,\frac{F}{m}$$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\|\mathbf{A}\| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 2-1 Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\epsilon w}{h}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ϵ, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \approx \ln(2D/d)$.

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos\phi + A_y \sin\phi$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos\phi$ $y = r \sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos\phi - A_\phi \sin\phi$ $A_y = A_r \sin\phi + A_\phi \cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin\theta \cos\phi + \hat{\mathbf{y}} \sin\theta \sin\phi + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta \cos\phi + \hat{\mathbf{y}} \cos\theta \sin\phi - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$	$A_R = A_x \sin\theta \cos\phi + A_y \sin\theta \sin\phi + A_z \cos\theta$ $A_\theta = A_x \cos\theta \cos\phi + A_y \cos\theta \sin\phi - A_z \sin\theta$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$
Spherical to Cartesian	$x = R \sin\theta \cos\phi$ $y = R \sin\theta \sin\phi$ $z = R \cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin\theta \cos\phi + \hat{\boldsymbol{\theta}} \cos\theta \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin\theta \sin\phi + \hat{\boldsymbol{\theta}} \cos\theta \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_x = A_R \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi$ $A_y = A_R \sin\theta \sin\phi + A_\theta \cos\theta \sin\phi + A_\phi \cos\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin\theta + A_z \cos\theta$ $A_\theta = A_r \cos\theta - A_z \sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R \sin\theta$ $\phi = \phi$ $z = R \cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$ $A_\phi = A_\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$

Problem 1: (5 points) UNITS!

Simplify the units for $\sqrt{\frac{\mu}{\epsilon}}$ as much as possible, where μ is permeability and ϵ is permittivity. You must show every step to receive full credit, and you should simplify down to a single unit

$V = L \cdot \frac{dI}{dt}$ $\quad Q = CV$

$\mu = \frac{H}{m}$ $\quad \varepsilon = F/m$ $\quad V = H \cdot \frac{A}{s}$ $\quad Q = F \cdot V$ $\quad V = IR$

$H = \frac{Vs^2}{Q}$ $\quad F = \frac{Q}{V}$ $\quad V = A \cdot \Omega$

5

$$\sqrt{\frac{\mu}{\varepsilon}} \Rightarrow \sqrt{\frac{H}{F}} \Rightarrow \sqrt{\frac{\frac{Vs^2}{Q}}{\frac{Q}{V}}} \Rightarrow \sqrt{\frac{Vs^2}{Q}\frac{V}{Q}} \Rightarrow \frac{Vs}{Q} \Rightarrow \frac{V}{A} \Rightarrow \Omega$$

Problem 2: (5 points) Lumped element model

You are analyzing a system that involves a source connected to the load via a 10 cm line. The circuit operates at 3 GHz (10^9 Hz), and the wave velocity is the speed of light ($3 * 10^8\ m/s$). Do you need to treat the 10 cm line as a transmission line, or can you treat the entire system as a lumped element (where the 10 cm line is all a single node)? Show calculations to justify your answer.

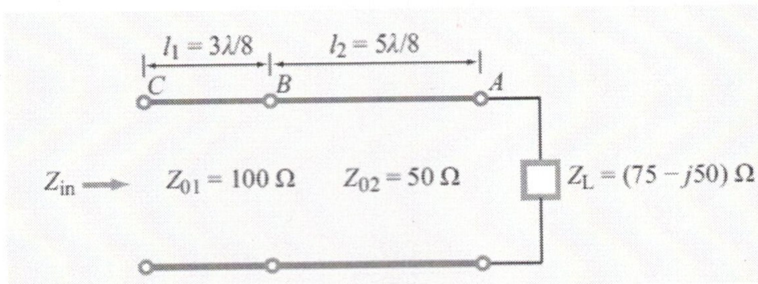
4

$$V_p = f\lambda \Rightarrow \lambda = \frac{V_p}{f} = \frac{3 \cdot 10^8 m/s}{3\times 10^9 Hz} = 0.1 m = 10cm$$

λ 30 The 10cm line is on the same order as the 30cm wavelength

↳ treat as transmission line

Problem 3: (20 points) Smith Chart
Use the Smith chart to determine the input impedance Z_{in} of the two-line configuration shown below. (Additional workspace is provided on the next page)



$\frac{Z_L}{Z_{02}} = 1.5 - j$ (A)

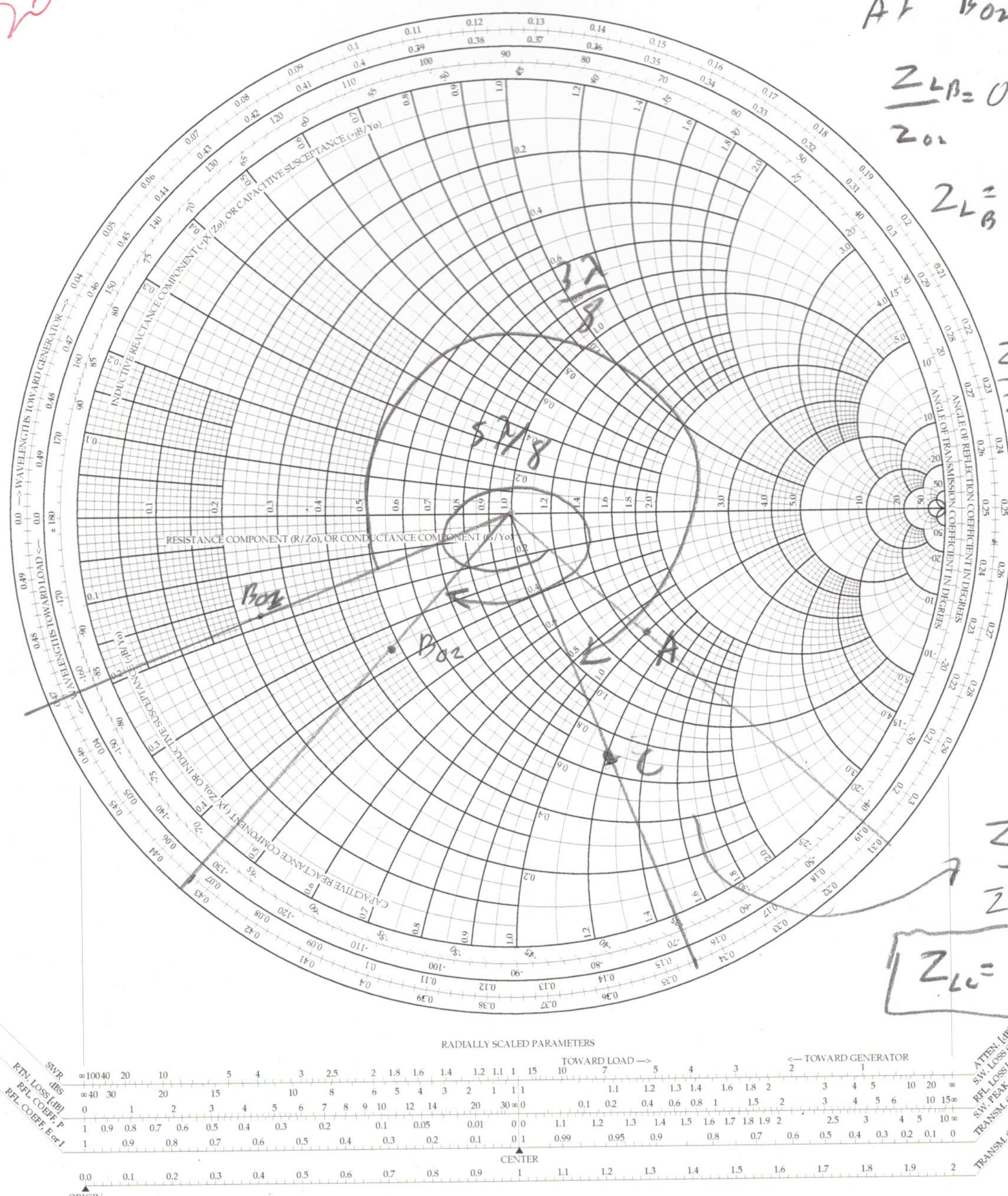
20

At B_{02}:

$\frac{Z_{LB}}{Z_{02}} = 0.48 - 0.36j$

$Z_{LB} = 24 - 18j$

$\frac{Z_{LB}}{Z_{01}} = 0.24 - 0.18j$



$\frac{Z_{LC}}{Z_{01}} = 0.7 - 1.2j$

$\boxed{Z_{LC} = Z_{in} = 70 - 120j}$

Problem 4: (10 points) Power Transmission
You have a transmission line with characteristic impedance $Z_0 = 50\ \Omega$ that ends in a load with $Z_L = 30 + j40\ \Omega$. What percentage of the power in the line is transmitted to the load?

$$P = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2) \rightarrow \text{power to load}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j40 - 50}{30 + j40 + 50} = \frac{-20 + j40}{80 + j40}$$

10

$$= \frac{-1 + j2}{4 + j2}, \quad |\Gamma| = \frac{\sqrt{1^2 + 2^2}}{\sqrt{4^2 + 2^2}} = \frac{\sqrt{5}}{\sqrt{20}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\frac{P_{\text{to load}}}{P_{\text{forward wave}}} = \frac{\frac{|V_0^+|^2}{2Z_0}(1 - (\frac{1}{2})^2)}{\frac{|V_0^+|^2}{2Z_0}} = 1 - \frac{1}{4} = \frac{3}{4}$$

75% of power is ~~transfered~~ transmitted to load

$Z_0 = 50\,\Omega$, $Z_L = 30 + j40\,\Omega$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-20 + j40}{80 + j40} = \frac{-1 + j2}{4 + j2}$$

$$\frac{\sqrt{1^2 + 4}}{\sqrt{16 + 4}} = \sqrt{\frac{5}{20}} = \frac{1}{2}$$

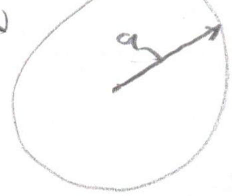
$$P_{av} = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2)$$

$$= \frac{|V_0^+|^2}{2Z_0}(1 - \frac{1}{4})$$

75%

Problem 5: (15 points) Gauss' Law
An electric charge +Q is uniformly distributed across the surface of a non-conducting *hollow sphere* of radius *a*. Determine the electric field flux $\vec{D}$ everywhere inside and outside the sphere. You can assume the sphere is infinitesimally thin.

$\vec{D}$ only has $\hat{r}$ component due to symmetry



$\vec{D} = D_r \hat{r}$

15

for $r < a$:

$\nabla \cdot D = \rho_v$

$\Rightarrow \oint_s \vec{D} \cdot d\vec{s} = Q_{enc} = 0$

$\boxed{\hookrightarrow \vec{D} = 0}$

For $r > a$:

$\oint_s \vec{D} \cdot d\vec{s} = Q_{enc} = Q$

$\int_0^{2\pi} \int_0^{\pi} D_r \hat{r} \cdot \hat{r} \, R^2 \sin\theta \, d\theta \, d\phi \Big|_{R=r}$

$D_r \cdot 2\pi \cdot r^2 \int_0^{\pi} \sin\theta \, d\theta$

$D_r \cdot 2\pi \cdot r^2 (-\cos\theta \big|_0^{\pi})$

$D_r \cdot 2\pi \cdot r^2 (-(-1) - (-1))$

$D_r \cdot 4\pi r^2 = Q$

$\boxed{D_r = \frac{Q}{4\pi r^2}}$

$\boxed{\vec{D} = \frac{Q \hat{r}}{4\pi r^2}} \; \frac{C}{m^2}$

Problem 6: (20 points) Stub matching

Use an open-circuited stub to match the transmission line below. Determine the length of *d* and *l*. You may use the Smith chart on the next page or calculate your answer. Either way, make sure to show your work.

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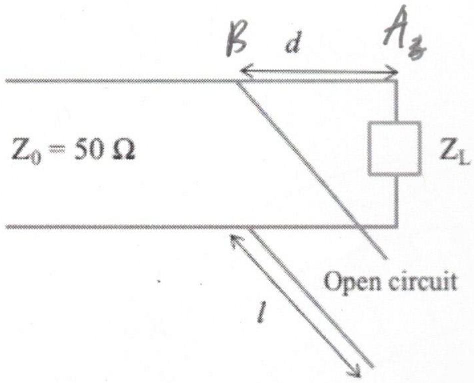


$z_L = \frac{Z_L}{z_0} = \frac{Z_L}{50\Omega} = 1.2 - j1.6$

$\boxed{d = 0.109\lambda}$

$\Rightarrow y_B = 1 + 1.4j$

$\boxed{l = 0.348\lambda}$ to have $y_{oc\,B} = -1.4j$

$Z_L = 60 - j80\Omega$

$Z_0 = 50\Omega$

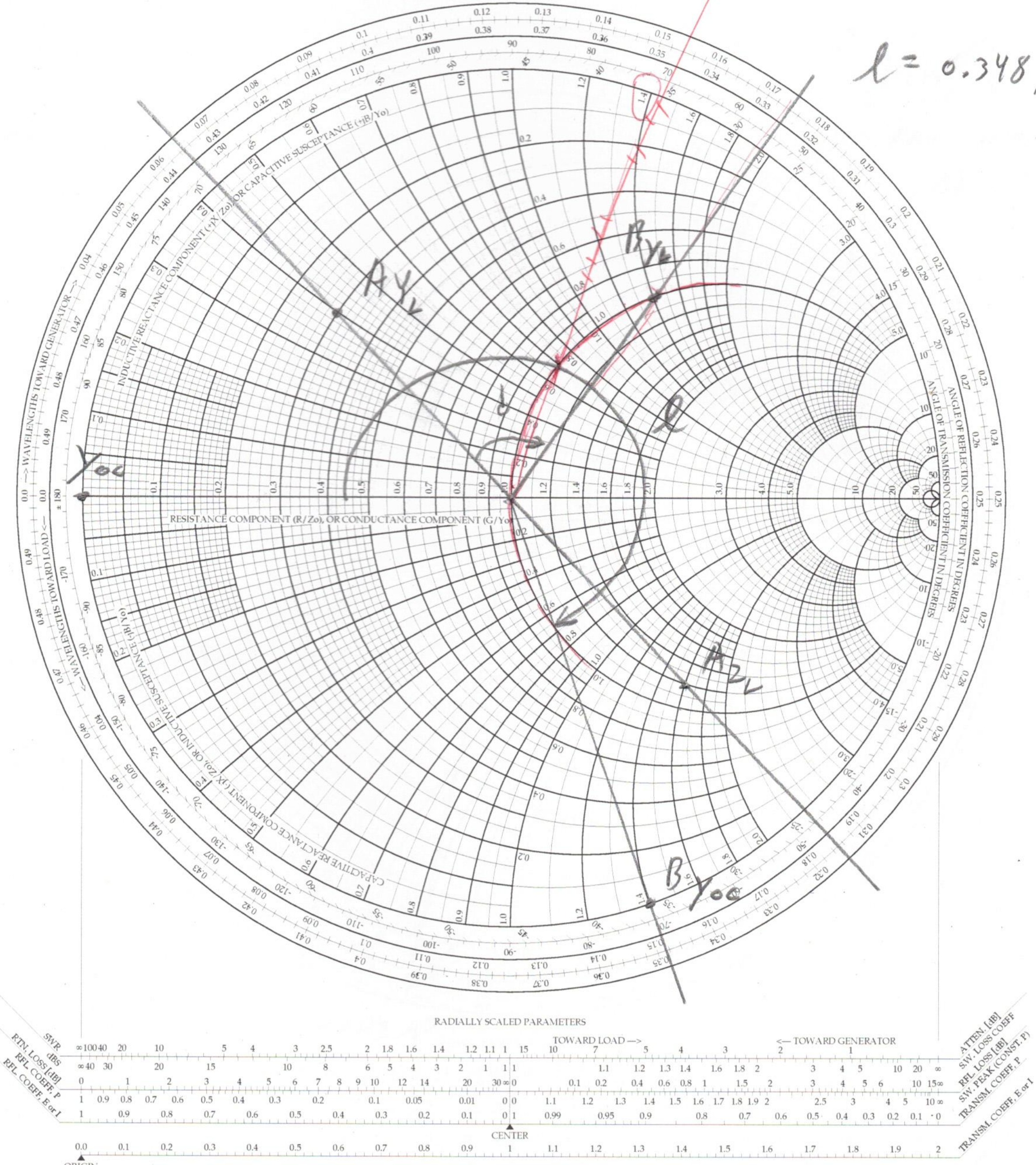
$Z_{in}(z = -d) = Z_0\left(\frac{Z_L + jZ_0 \tan(\frac{2\pi}{\lambda}d)}{Z_0 + jZ_L \tan(\frac{2\pi}{\lambda}\cdot d)}\right)$

$= 50\,($

A: $Z_L = 1.2 - j1.6$

$d = (0.174 - 0.065)\lambda$
$= 0.109\lambda$

$l = 0.348\lambda$

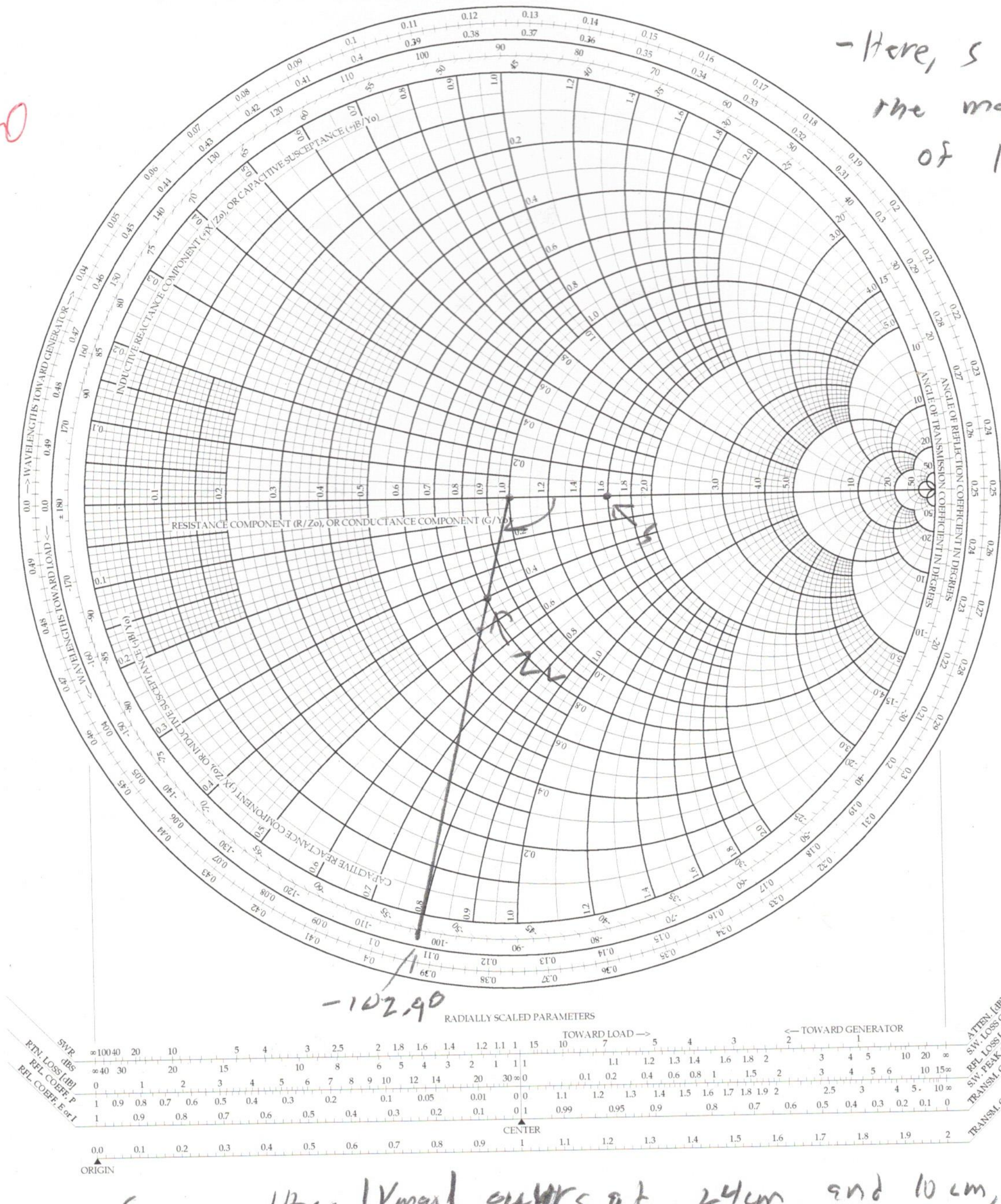


Problem 7: (20 points) Smith Chart
Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: S = 1.6, and $|\tilde{V}|_{max}$ occurred at 10 cm and 24 cm from the load. Use the Smith chart to find Z_L. You can find additional workspace on the next page.

Hint: An *air-spaced* lossless line tells you that the permittivity and permeability of the dielectric in the transmission line are the values for air.

20

- Here, S implies the magnitude of Γ.



Since ~~the~~ |Vmax| occurs at 24cm and 10 cm,

$\frac{\lambda}{2} = 24_{cm} - 10cm = \cancel{10cm}\ 14cm$, $\lambda = 28cm$

The formula: $|\tilde{V}(d)| = |V_0^+|\,(1 + |\Gamma|^2 + 2|\Gamma|\cdot\cos(2\beta l - \theta_r))^{1/2}$

max occurs at $l = 10cm \Rightarrow 2\beta(10cm) - \theta_r = 2\pi n$

$\Rightarrow 2\cdot\frac{2\pi}{28cm}\cdot 10cm - 2\pi n = \theta_r \Rightarrow \theta_r = 4.488 - 2\pi n$

Additional workspace for problem 7

$\theta_r = 4.488 - 2\pi n$

$-\pi < \theta_r < \pi$ so $n = 1$

$\theta_r = -1.795 = -102.9°$

From chart : $\Gamma = 1.6\, e^{-j\,102.9°}$

$\frac{Z_L}{Z_0} = 0.8 - 0.4j$

$\boxed{Z_L = 40 - 20j\ \Omega}$