EC ENRG 101A Midterm
Winter 2019

Name:_ ____________________

Student ID#: ____________________

A. Please put all the work on the test paper.
B. Show your work! Provide clear explanations and show calculations. You may not get credit for work that is not shown.
C. List units with all answers

Name of person on LEFT even if "far" way. If wall, then write "Wall". If aisle, then write "Aisle".	**ROW NUMBER: (as measured from front)**	**Name of person on RIGHT even if "far" way. If wall, then write "Wall". If aisle, then write "Aisle".**

BE SURE TO ENTER THE FOLLOWING INFORMATION

Allowed:
ʾencil
ılator
compass
formula sheet: ***one side*** *of a 8 ½" by 11" sheet of paper.*

Score

1	10/10	4	25/25
2	5 /5	5	23/25
3	14 /15	6	23/25
		Total	100 /105

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table 2-1 **Transmission-line parameters R', L', G', and C' for three types of lines.**

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{2R_s}{\pi d}$	$\dfrac{2R_s}{w}$	Ω/m
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\dfrac{\mu h}{w}$	H/m
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\sigma w}{h}$	S/m
C'	$\dfrac{2\pi\epsilon}{\ln(b/a)}$	$\dfrac{\pi\epsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\epsilon w}{h}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ϵ, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \approx \ln(2D/d)$.

Problem 1: (10 points): Sphere of charge
Given a sphere with radius a and charge density ρ_V inside the sphere, find the flux density, $\underline{D}$, for all locations. (Note: this should seem familiar!)

$$\oint \vec{D}\cdot d\vec{s} = Q$$

$$\oint D_r \hat{r}\cdot d\vec{s} = D_r(4\pi r^2)$$

$$\vec{D} = D_r \hat{r}$$

if $r<a$

$$Q = \int_V \rho_v \, dV = \rho_v V \Big|_0^r$$

$$= \rho_v \tfrac{4}{3}\pi r^3 \Big|_0^r = \rho_v \tfrac{4}{3}\pi r^3$$

$$D_r(4\pi r^2) = \rho_v \tfrac{4}{3}\pi r^3$$

$$D_r = \frac{\rho_v r}{3}$$

$$\boxed{\vec{D} = \begin{cases} \dfrac{\rho_v r}{3}\hat{r} & r \le a \\ \dfrac{\rho_v a^3}{3r^2}\hat{r} & r > a \end{cases}}$$

if $r>a$

$$Q = \int_V \rho_v \, dV = \rho_v V \Big|_0^a = \rho_v \tfrac{4}{3}\pi r^3 \Big|_0^a = \rho_v \tfrac{4}{3}\pi a^3$$

$$D_r(4\pi r^2) = \rho_v \tfrac{4}{3}\pi a^3 \qquad D_r = \frac{\rho_v a^3}{3r^2}$$

10

Problem 2: (5 points) Units!

Simplify the units for $\sqrt{\frac{1}{\mu\epsilon}}$ as much as you can (you should be able to get it down to fundamental SI units.

$$\sqrt{\frac{1}{\left(\frac{H}{m}\right)\left(\frac{F}{m}\right)}} = \sqrt{\frac{1}{\left(\frac{Vs^2}{Cm}\right)\left(\frac{C}{mV}\right)}} = \sqrt{\frac{1}{\left(\frac{s^2}{m^2}\right)}} = \sqrt{\frac{m^2}{s^2}}$$

$$= \boxed{\frac{m}{s}}$$

$Q = CV$

$\{C\} = \{F\}\{V\}$

$\left\{\frac{C}{V}\right\} = \{F\}$

$V = L\frac{di}{dt}$

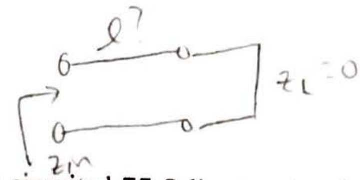
$\{V\} = \{H\}\left\{\frac{A}{s}\right\}$

$H = \left\{\frac{VS}{A}\right\}$

$A = \frac{C}{S}$

$H = \left\{\frac{VS^2}{C}\right\}$

5



Problem 3: (15 points) Smith chart

Use the Smith chart to find the shortest lengths of a short-circuited 75 Ω line to give the following input impedance. You can give your answer in terms of wavelength:

a) $Z_{in} = 0$ → $l_{min} = 0.5\lambda$

b) $Z_{in} = \infty$ → l

c) $Z_{in} = j75$ → $Z_{in} = j$

d) $Z_{in} = j10$ → $Z_{in} = j0.13$

more towards load

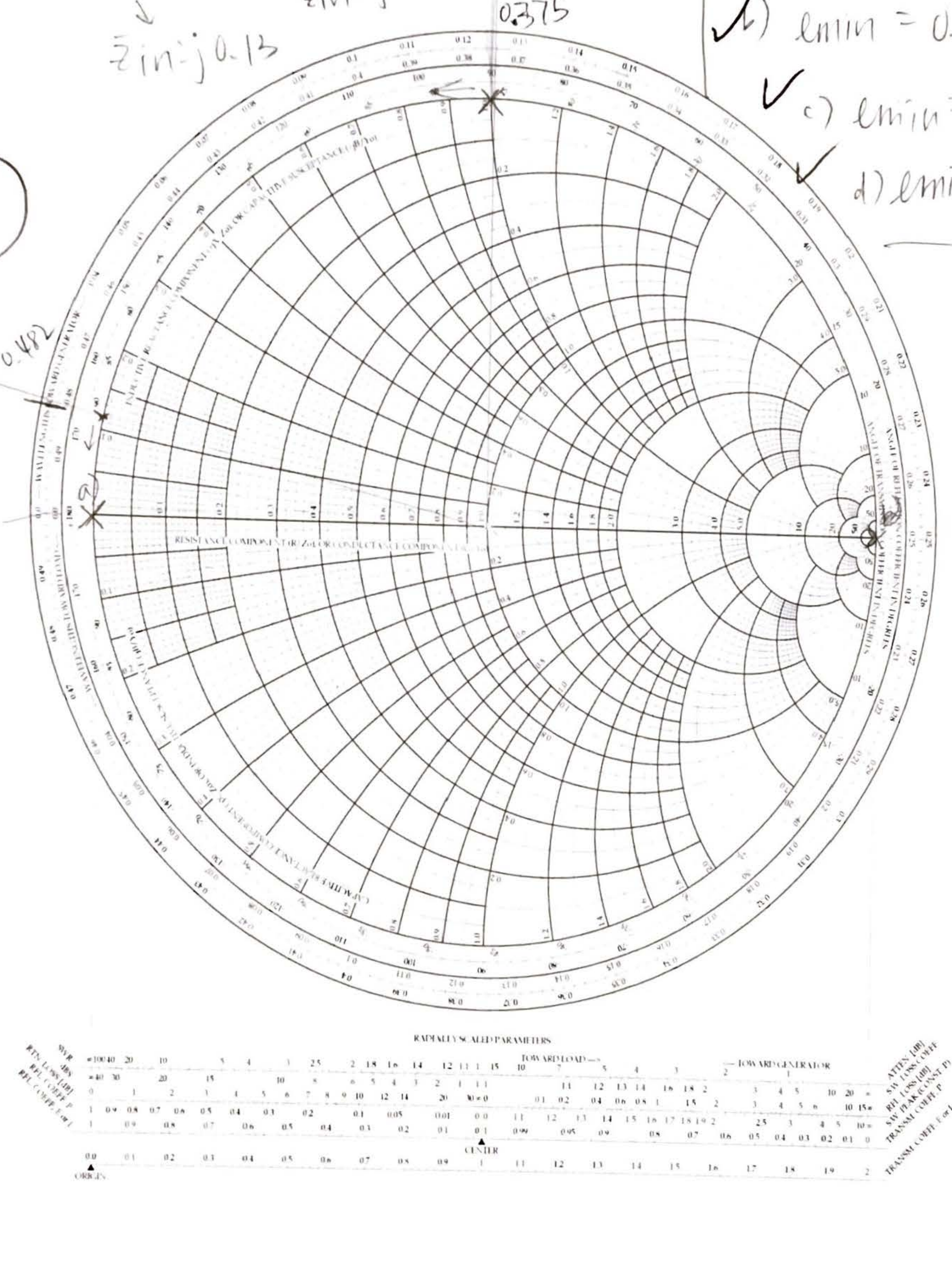
a) $l_{min} = 0.5\lambda$ or 0λ

b) $l_{min} = 0.25\lambda$ ✓

c) $l_{min} = 0.125\lambda$ ✓

d) $l_{min} = 0.018\lambda$ ✓

14



Problem 4: (25 points) Transmission line basics

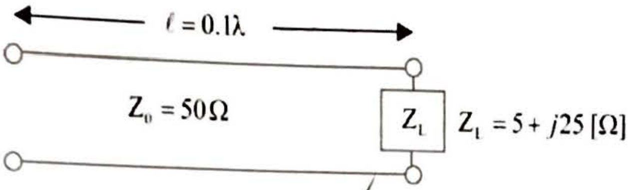
For the transmission line below:

a) Mark the location of z_L on the Smith chart
b) What is the impedance at $l = 0.1\,\lambda$
c) What is the voltage standing wave ratio on the line?
d) What is Γ_L?
e) What is Γ at $l = 0.1\,\lambda$ from the load?

25

NOTE: It is your choice if you use the Smith chart or just calculate the answers. Either was is fine

$\bar{z}_L$ = normalized z_L



a) $\bar{z}_L = 0.1 + j0.5$ ✓

b) $l = 0.1\lambda$ z_{in}?

$$z_{in} = z_0 \frac{z_L + jz_0 \tan(\beta l)}{z_0 + jz_L \tan(\beta l)} = 50 \frac{5+j25+j50\tan\left(\frac{2\pi}{\lambda}0.1\lambda\right)}{50+j(5+j25)\tan\left(\frac{2\pi}{\lambda}0.1\lambda\right)}$$

$$= 50\frac{5+j25+j36}{50+3.6j-18} = 50\frac{5+j61.3}{31.8+j3.6} = 50\left(1.92e^{j(85-6.5)}\right)$$

$$\boxed{z_{in} = 96e^{j78.8}\,\Omega = 18.6+j94.2\,\Omega}$$ ✓

d) $$\Gamma_L = \frac{z_L - z_0}{z_L + z_0} = \frac{5+j25-50}{5+j25+50} = \frac{-45+j25}{55+j25} = 0.85e^{j(151-24)}$$

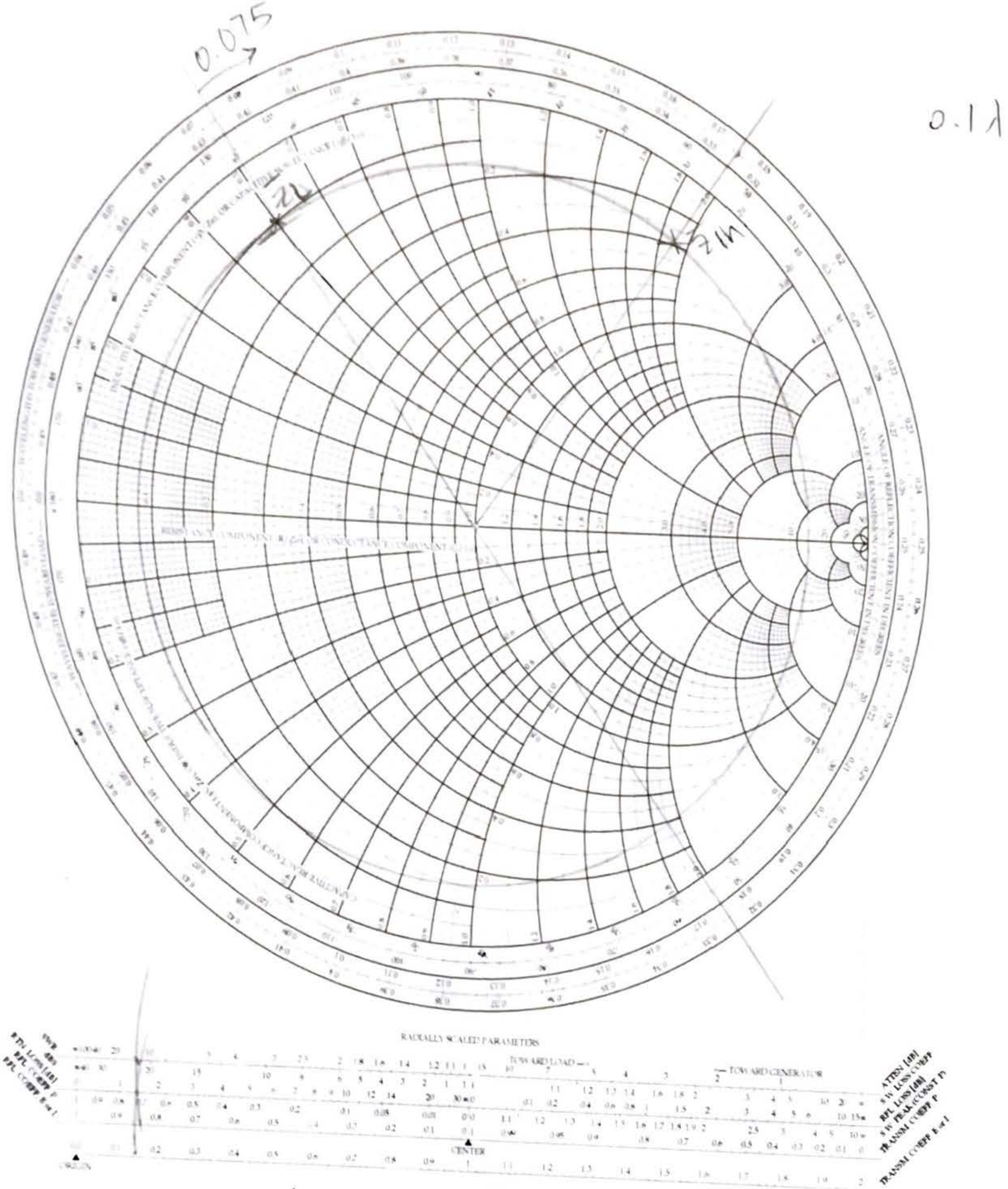
✓ $\boxed{\Gamma_L = 0.85e^{j126.5^\circ}}$

c) $$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.85}{1-0.85} = 12.5$$

$\boxed{SWR = 12.5}$ ✓

e) $$\Gamma_{in} = \frac{96e^{j78.8}-50}{96e^{j78.8}+50} = \frac{18.6+j94.2-50}{18.6+j94.2+50} = \frac{-31.4+j94.2}{68.6+j94.2}$$

$$= 0.85e^{j(108.4-53.9)} = \boxed{0.85e^{j(54.5^\circ)} = \Gamma_{in}}$$ ✓



$\Gamma_L = 0.86 e^{j(126)}$ ✓

$SWR = 12$ ✓

$\Gamma_{in} = 0.86 e^{j(54)}$

$\bar{z}_{in} = 0.4 + j1.9$

$z_{in} = 20 + j95$ ✓

results confirmed using smith charts

Problem 5 (25 points): Transmission line (this should be familiar from discussion section!)

The following two-step procedure has been carried out with a 50 Ω coaxial slotted line to determine an unknown load impedance

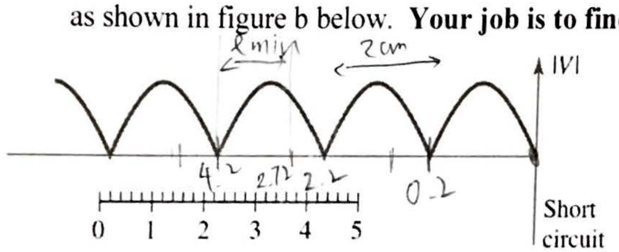
1. A short circuit is placed as the load, resulting in a standing wave on the line with infinite standing wave ratio and sharply defined voltage minima, as shown in the figure below. On the arbitrarily positioned scale on the slotted line, voltage minima are recorded at

$$z = 0.2\ cm, 2.2\ cm, 4.2\ cm$$

2. The short circuit is removed and replaced with the unknown load. The standing wave ratio (SWR) is measured as $SWR = 1.3$, and voltage minima, which are not as sharp as those defined as the case with a short circuit load, are recorded at

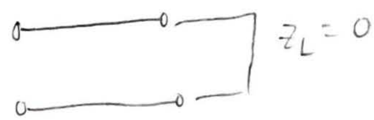
$$z = 0.72\ cm, 2.72\ cm, 4.72\ cm$$

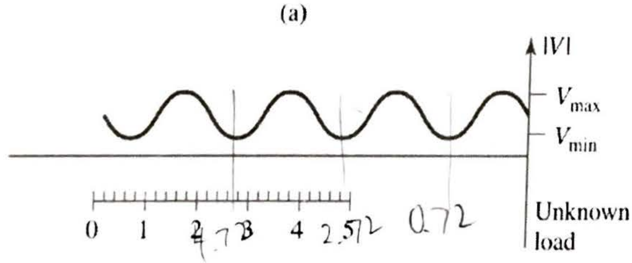
as shown in figure b below. **Your job is to find the load impedance.**



(a)

$Z_0 = 50\Omega$





(b)

$\frac{\lambda}{2} = 2cm \quad \lambda = 4cm$

$\Gamma = |\Gamma| e^{j\theta_r}$

$SWR = 1.3 = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}}$

$1.3(1-|\Gamma|) = 1+|\Gamma|$

$1.3 - 1.3|\Gamma| = 1+|\Gamma|$

$0.3 = 2.3|\Gamma|$

$|\Gamma| = 0.13$ ✓

ℓ_{min} : distance between minima points for both loads

$\ell_{min} = 4.2 - 2.72 = 1.48\ cm$ ✓

$|\tilde{V}| = V_0^+ \sqrt{1+|\Gamma| 2\cos(\theta_r + 2\beta z) + |\Gamma|^2}$

$|\tilde{V}|_{min}$ occurs when $\cos(\theta_r + 2\beta z) = -1$

$\beta = \frac{2\pi}{\lambda}$

Additional work space for Problem 5

$$\cos\left(\theta_r + 2\left(\frac{2\pi}{\lambda}\right)(\ell_{min})\right) = -1$$

$\cos\theta = -1$ when $\theta = \pi + 2\pi n$

$$\theta_r + 2\left(\frac{2\pi}{4}\right)(1.48) = \pi \qquad \theta = \pi + 2\beta\ell_{min}$$

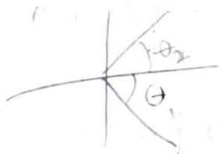
$$\theta_r = -1.5 \text{ rad or } -86.4^\circ \quad \times -2$$

$|\Gamma| = 0.13 \quad \theta_r = -86.4^\circ \qquad Z_0 = 50\Omega$

$$\Gamma = 0.13\, e^{-j86.4^\circ} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma(Z_L + Z_0) = Z_L - Z_0$$

$$\Gamma Z_L + \Gamma Z_0 = Z_L - Z_0$$



$$Z_L(1-\Gamma) = Z_0(1+\Gamma)$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

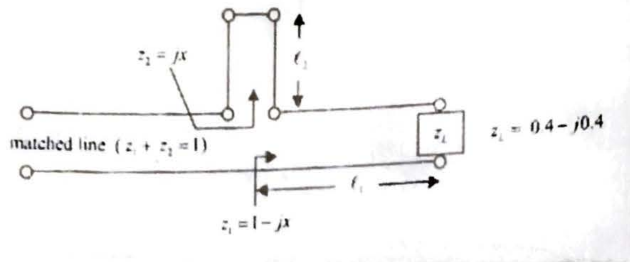
$$Z_L = 50\,\frac{1 + 0.13\, e^{-j86.4}}{1 - 0.13\, e^{-j86.4}} = 50\,\frac{1 + 0.0082 - j0.13}{1 - 0.0082 + j0.13}$$

$$Z_L = 50\,\frac{1.008 - j0.13}{0.992 + j0.13} = 50\left(1.02\, e^{j(-7.4-7.5)}\right)$$

$$\boxed{Z_L = 50.8\, e^{-j14.8}\,\Omega}$$ ✓

Problem 6: (25 points): Series stub

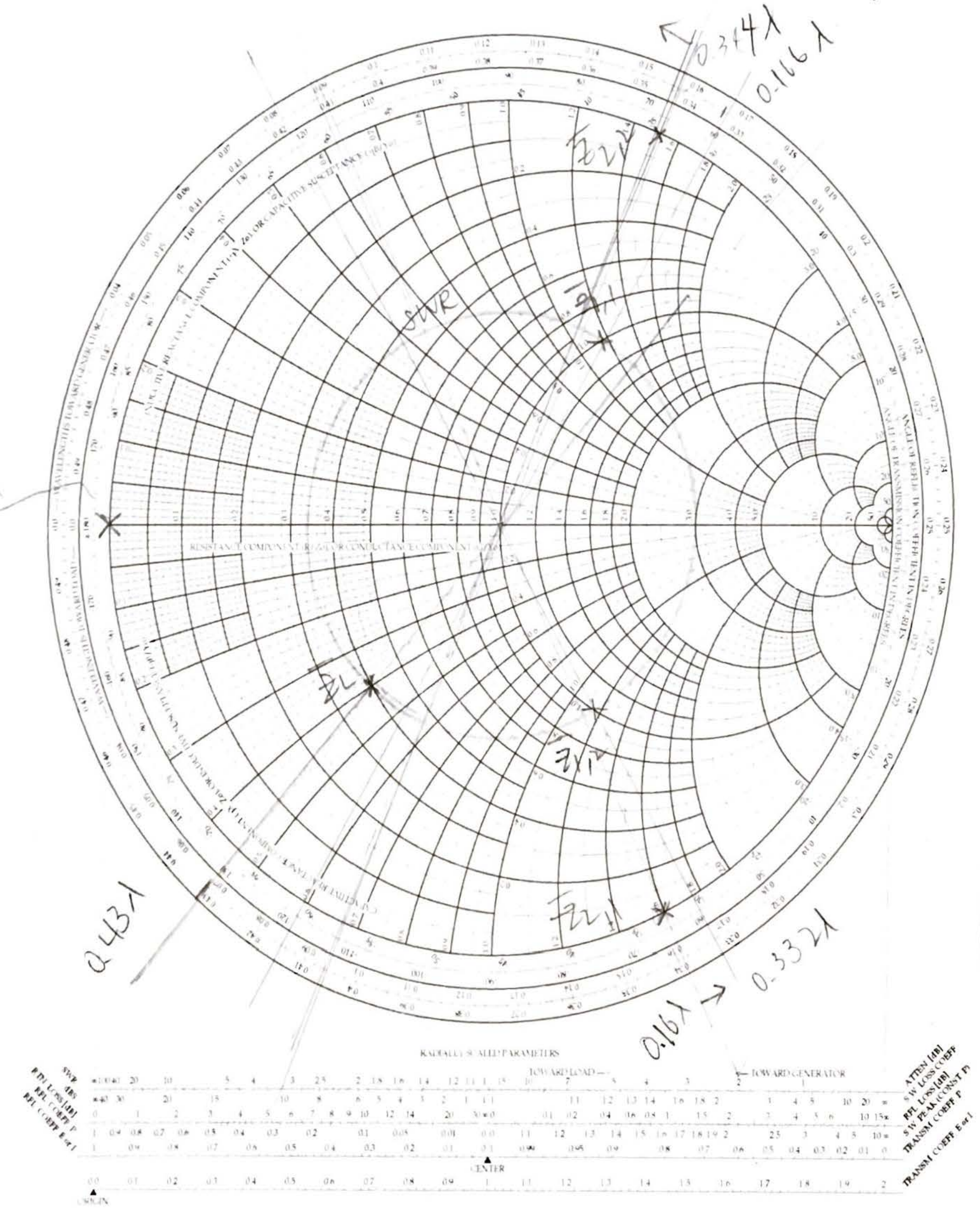
Use a single *series* stub tuner to match the below line to its normalized load. Use a shorted stub and find its distance from the load, l_1, and the length of the stub, l_2.



$\bar{z}_1 + \bar{z}_2 = 1$

$\bar{z}_1 = 1.0 - jX$

$\bar{z}_2 = jX$



$\bar{z}_{1,1} = 1.0 + j1.6$ x −2 $1.13j$

$l_{1,1} = 0.166\lambda + (0.5 - 0.43)\lambda$

$\boxed{l_{1,1} = 0.236\lambda}$ ✓

$\bar{z}_{2,1} = -j1.6$

move towards load ($Z_L = 0$)

$l_{2,1} = (0.5 - 0.16)\lambda$

$\boxed{l_{2,1} = 0.34\lambda}$ ✓

$\bar{z}_{1,2} = 1.0 - j1.5$ $-1.13j$

$l_{1,2} = 0.332 + (0.5 - 0.43)\lambda$

$\boxed{l_{1,2} = 0.402\lambda}$ ✓

$\bar{z}_{2,2} = j1.5$

move towards load ($Z_L = 0$)

$l_{2,2} = 0.5\lambda - 0.344\lambda$

$\boxed{l_{2,2} = 0.156\lambda}$ ✓