EC ENGR 101A Final

Winter 2020

Name:_______________________________________

Student ID#: ________________________________

A. Please put all the work on the test paper.
B. Show your work! Provide clear explanations and show calculations.

<u>Allowed:</u>
pen/pencil
calculator
formula sheet: ***two sides*** *of an 8 ½" by 11" sheet of paper.*
Open book and notes

<u>Not</u> *<u>Allowed</u>**<u>:</u>***
Internet searches
Communication with anyone

Score

1	/10	6	/20
2	/5	7	/20
3	/5	8	/20
4	/10	9	/20
5	/10	10	/20
		Total	/140

Permittivity for free space: $\varepsilon_0 = 8.85\times10^{-12}$ F/m   Permeability for free space: $\mu_0 = 4\pi\times10^{-7}$ H/m

Possibly useful information

$$\int \frac{1}{u(a+bu)}\,du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

$$\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

1. **Off to a good start (easy points!) (10 points)** Write down your favorite one of Maxwell's equations. Any one of them will be the correct answer.

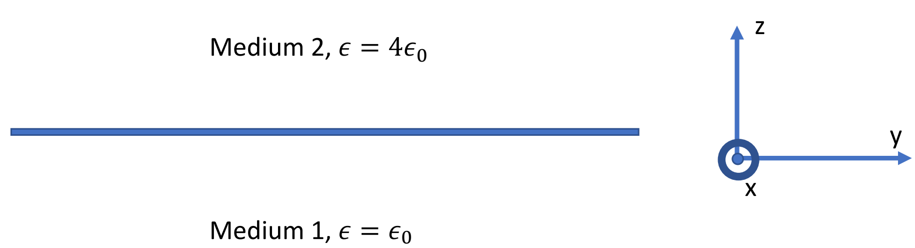
2. **Phasors (5 points)** Write the equivalent time domain expression, $\vec{H}(x, y, z, t)$ for the phasor $\widetilde{H}(x, y, z) = \hat{x} j e^{jkz} + \hat{y}\frac{e^{jkz}}{j}$, assuming frequency of ω.

3. **Units! (5 points)** *Show the units for* $W_m = \frac{1}{2}\vec{H} \cdot \vec{B}$. The answer should simplify to a fraction with one unit in the numerator and one in the denominator. It is possible that one or both units will have an exponent.

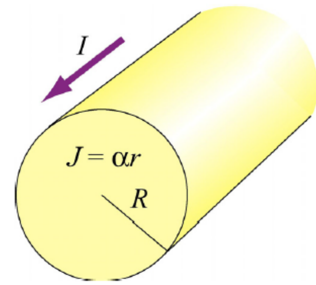
4. **Reflection coefficient (10 points):** Write on your paper all the values that could possibly be a valid value of Γ.

a) $\Gamma = 0$
b) $\Gamma = 0.2 + j0.3$
c) $\Gamma = -50\ \Omega$
d) $\Gamma = j$
e) $\Gamma = 0.8 + j0.7$

5. **Boundary conditions (10 points):** You have an electric field in Medium 1 given by $\overrightarrow{E_1} = = E_0(\hat{x} + \hat{y} + \hat{z})$. The field encounters Medium 2, which has a boundary with Medium 1 that comprises the x-y plane. There is no surface charge. Write an expression for the electric field in Medium 2, $\overrightarrow{E_2} = ?$



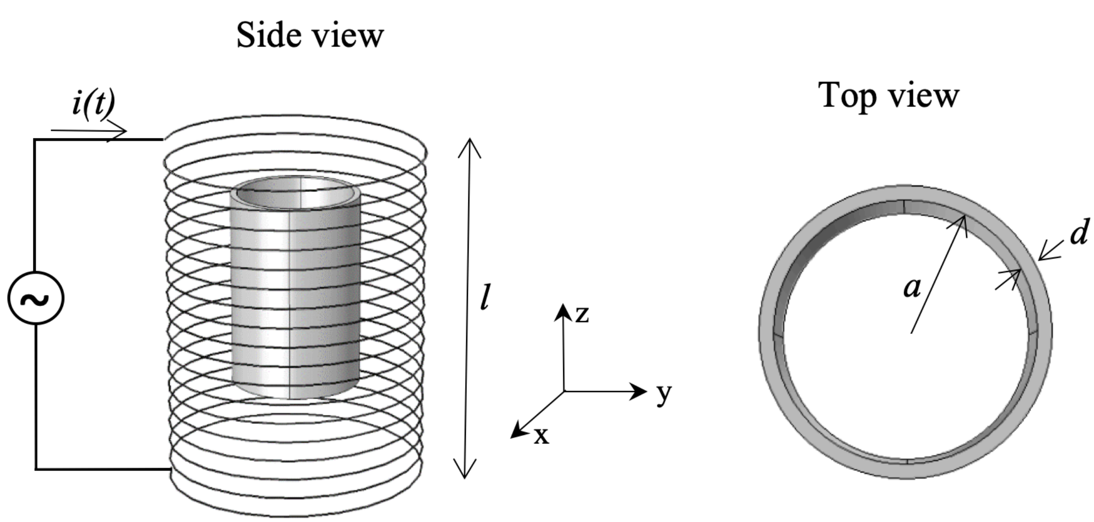
6. **Field from a Wire (20 points)** We have an infinitely long, cylindrical conductor of radius R carrying a current I with a non-uniform current density J = αr, where α is a constant. Find the magnetic field $\vec{H}$ everywhere.



7. **Current from a coil (20 points):** Consider a cylindrical metal shell with conductivity σ placed inside a long solenoid of length l with N turns driven to produce a magnetic field in free space. The shell is continuous (i.e., no gap like in the homework problem). The current in the solenoid is increasing linearly in time for all time: $i(t) = kt$, where k is a positive constant (units of A/s), with a direction as indicated in the figure. The shell has dimensions shown, where $d \ll a$.

   What is the current density $\vec{J}$ flowing in the metal shell?

   Note: This is different from the homework problem, where there was a small gap in the cylinder. This cylinder is continuous all the way around.



8. **Capacitance (20 points)**

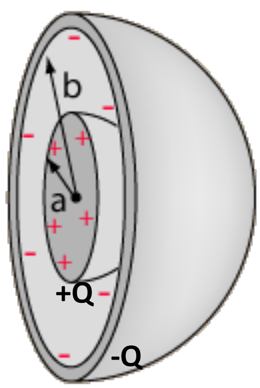
Consider a spherical capacitor of two concentric conducting spheres with inner radius = *a*, outer radius = *b*. The inner sphere carries +Q charges and outer sphere carries -Q charges.

(a) Find the potential in region $a < r < b$ and the capacitance of this spherical capacitor. Use ε_0 for the primitivity between the spheres.

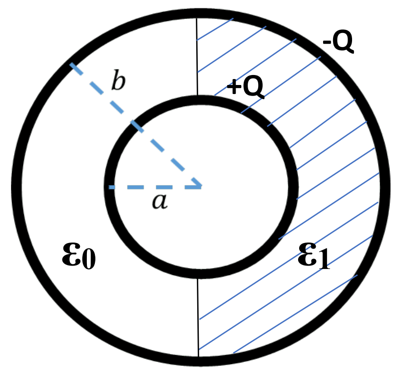
Now we fill half of the empty space between the spheres by a hemispherical shell of dielectric $\varepsilon = \varepsilon_1$.

(b) Calculate the electric fields $\vec{E}$ and electric flux density $\vec{D}$ in each half.

(c) Calculate the surface-charge distribution on the inner sphere as well as the capacitance of the entire capacitor.



(a)



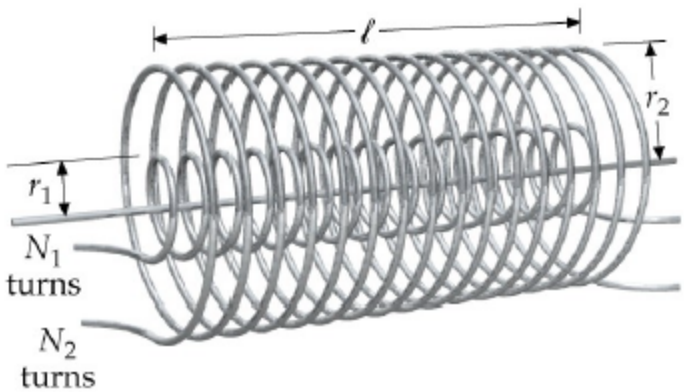
(b) (c)

9. **Inductance (20 points)**

Consider a tightly wound concentric solenoids. Assume that the inner solenoid carries current I_1 and the magnetic flux on the outer solenoid Φ_{B2} is created due to this current.

(a) What is the mutual inductance L_{21}?

(b) These coils can be used as car ignition coil. Spark jumps across gap in a spark plug and ignites a gasoline-air mixture.
Given $N_1 = 16{,}000$ turns, $N_2 = 400$ turns, $l = 10$ cm, $r_1 = 3$ cm, $r_2 = 8$ cm. A current through the primary coil $I_1 = 3$ A is broken in 10^{-4} sec. What is the induced emf?



10. **Plane waves (20 points)**

a. A uniform plane wave with $\vec{E} = E_x \cdot \hat{x}$ propagates in a lossless medium ($\varepsilon_r=4$, $\mu_r=1$, $\sigma=0$) in the z-direction. If E_x has a frequency of 100 MHz and has a maximum value of 10^{-4} (V/m) at $t = 0$ and $z = 1/8$ (m),

a) Write the instantaneous expression for $\vec{E}$ and $\vec{H}$,

b) Determine the locations where E_x is a positive maximum when $t=10^{-8}$ s.