UCLA Department of Electrical Engineering
EE101A – Engineering Electromagnetics
Fall 2015
Midterm, November 3 2015, (1:45 minutes)

Name ______________________ Student number ______________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Capacitor	40	
Problem 2	Conductors and fields	30	
Problem 3	Transmission Line	30	
Total		100	

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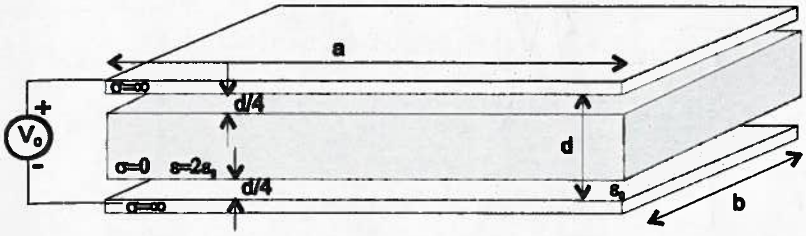
1. Capacitor (40 points)



(a) (10 points) Consider a parallel plate capacitor with the metal plates separated by *d*, that is partially filled with a dielectric with permittivity $\varepsilon=2\varepsilon_0$ and thickness $d/2$. The rest of the gap (above and below) is vacuum. What is the capacitance of this capacitor?

Two ways to solve.

① → C_1, C_2, C_1 in series

$C_1 = \frac{4A\varepsilon_0}{d}$ $\quad C_2 = \frac{A2\varepsilon_0}{d/2} = \frac{4A\varepsilon_0}{d}$

$\frac{1}{C} = \frac{d}{4A\varepsilon_0} + \frac{d}{4A\varepsilon_0} + \frac{d}{4A\varepsilon_0} = \frac{3}{4A\varepsilon_0}$ $\quad \boxed{C = \frac{4A\varepsilon_0}{3d}} = \boxed{\frac{4ab\varepsilon_0}{3d}}$

② → Field in vacuum = $\vec{E}_1 = -\hat{z}E_1$ (Same above + below since each plate has Charge $\pm Q$, $\rho_s = \pm \frac{Q}{ab}$)

Field in dielectric = $\vec{E}_2 = -\hat{z}E_2$

Bnd cond $\varepsilon_0 E_1 = 2\varepsilon_0 E_2$ $\quad E_1 = 2E_2$ $\quad \rho_s = \pm E_1 \varepsilon_0$

$V_0 = \frac{d}{4}E_1 + \frac{d}{4}E_1 + \frac{d}{2}E_2 = \frac{d}{2}E_1 + \frac{d}{2}E_2 = \cancel{\left(d + \frac{d}{2}\right)E_2}$

$= \frac{d}{2}E_1 + \frac{d}{4}E_1 = \boxed{\frac{3d}{4}E_1 = V_0}$

$Q = A\rho_s = AE_1\varepsilon_0$

$$\boxed{C = \frac{Q}{V_0} = \frac{AE_1\varepsilon_0 4}{3dE_1} = \frac{4\varepsilon_0 ab}{3d}}$$

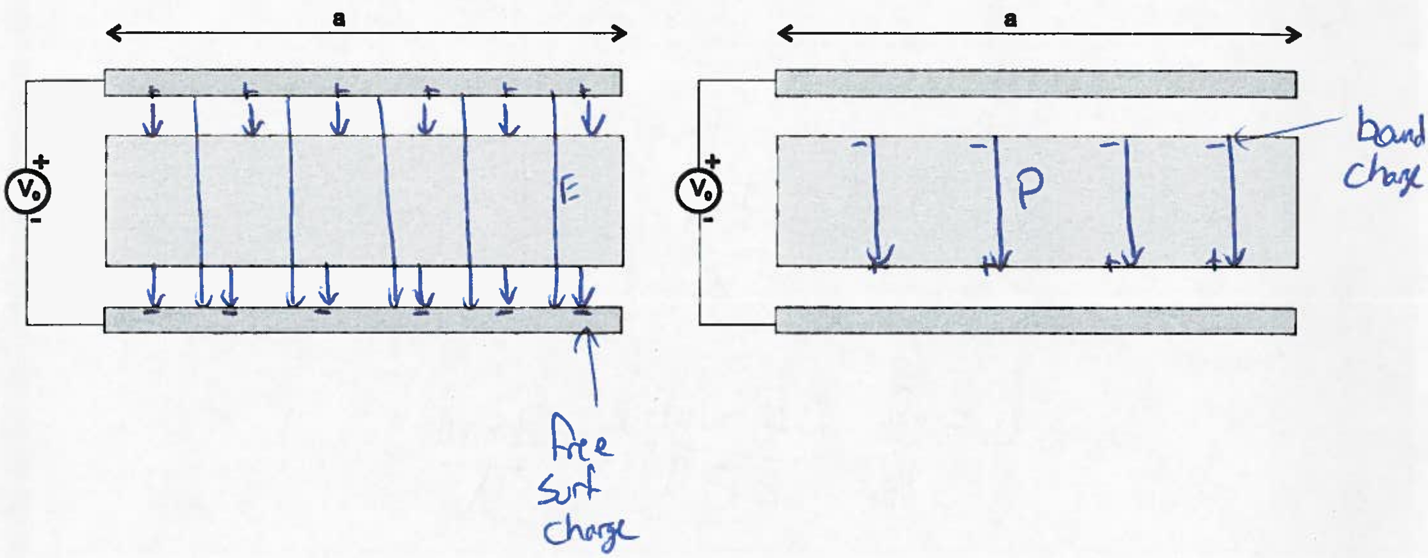
(b) (10 points) If a potential difference V_0 is applied to the capacitor, what is the E-field magnitude in the upper vacuum region, the dielectric, and the lower vacuum region.

Upper vacuum region: $E_1 = \frac{4V_0}{3d}$

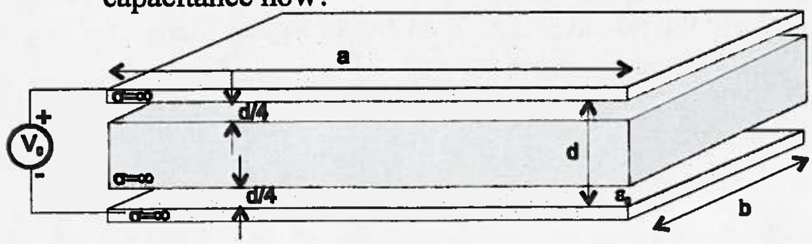
Lower vacuum region: $E_1 = \frac{4V_0}{3d}$

Dielectric : $E_2 = \frac{2V_0}{3d}$

(c) (10 points) On the diagrams below, on the left side of the picture, sketch the E-field vectors, and the location and sign of the free charge. On the right side of the picture, sketch the polarization field P vectors and the location and sign of the bound charge. Remember to pay attention to the relative strengths of the fields (stronger field means larger density of field vectors).



(d) (10 points) Now consider that the dielectric is replaced by a perfect conductor σ=∞. What is the capacitance now?



Now $V_0 = \frac{2E_1 d}{4} = \frac{E_1 d}{2}$

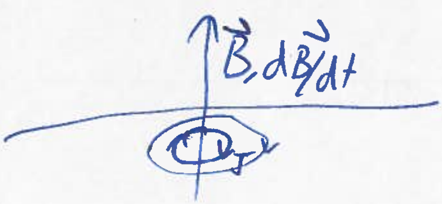
$C = \frac{Q}{V_0} = \frac{E_1 \varepsilon_0 ab\, 2}{E_1 d} = \frac{2\varepsilon_0 ab}{d}$

2. Conductors and fields (30 points)
   (a) (10 points) Explain qualitatively physically why the E-field must go to zero inside a perfect conductor.

Any applied field will cause current to flow. This will continue until the charge has arranged itself in such a way to cancel any applied fields. This takes approx time $\tau = \varepsilon/\sigma$ to occur. Any net charge must reside on surface so that $\rho = 0$ inside and $\nabla \cdot E = 0$ inside.
This occurs through Coulomb law.

   (b) (10 points) Explain qualitatively physically why the B-field must go to zero inside a perfect conductor.

Any time changing B-field flux extending through a conductor will cause solenoidal current to flow in such a way to produce its own B-field & cancel the field.



$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday's Law
Causes solenoidal E-field

$\vec{J} = \sigma \vec{E}$ Ohm's Law
E-field results in current flow

$\nabla \times \vec{B} = \mu \vec{J}$ Ampere's Law
Solenoidal current flow causes opposing B-field.

(c) (10 points) In a real conductor where σ is finite, we can only make approximations. Does a finite conductor act more like a perfect conductor at high or low frequencies for E-fields? How about for B-fields? Explain why

A finite conductor acts perfect at long times / low frequencies for $\vec{E}$. It takes longer than the dieletric relaxation time $\tau = \epsilon/\sigma$ for the charges to respond more to cancel the fields.

A finite conductor acts perfect at short times / high freq for $\vec{B}$. Any currents created by a changing B-field have an impulse response with time constant τ_m (magnetic diffusion time). For $t \ll \tau_m$ or $\omega \gg \frac{1}{\tau_m}$, induced currents can effectively screen applied B-fields, because the currents haven't had time to die out before the driving field switches.

Alternatively, the amount of driving current is

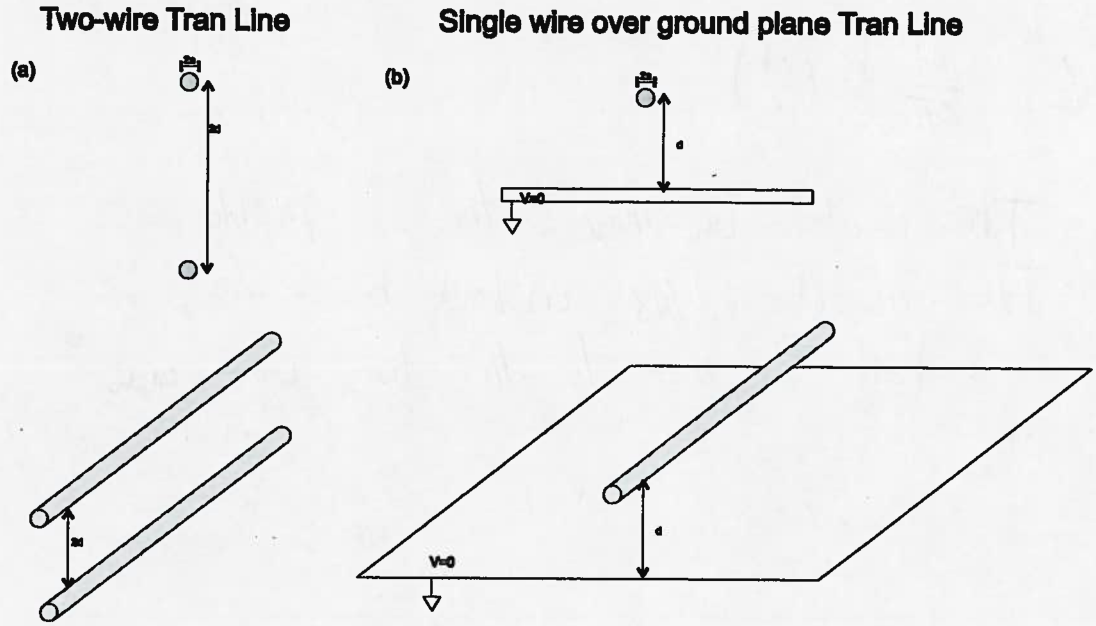
$$J \propto \nabla \times E \propto j\omega B$$

proportional to

which is larger at higher frequencies.

3. Transmission line (30 points)

Consider the lossless two-wire transmission line in case (a) and the single wire over a perfectly conducting semi-infinite ground plane shown in case (b).



(a) (15 points) The capacitance per unit length of the two-wire line with dimensions shown above (wire radius = a, separation = $2d$) is approximately: $C' = \dfrac{C}{\ell} = \dfrac{\pi\varepsilon}{\ln(2d/a)}$ (valid when $d \gg a$).

What is the capacitance per unit length of the single wire over ground plane (wire radius = a, distance from ground plane = d)?

This is easily solved by image theory.

$$C' = \frac{2\pi\varepsilon}{\ln(2d/a)}$$

The voltage difference between the wire & the plane is one half of the difference for the two wire case (for identical charge density).

(b) (15 points) The inductance per unit length of the same two-wire line is approximately: $L' = \frac{L}{\ell} = \frac{\mu}{\pi}\ln(2d/a)$. What is the inductance per unit length of the single wire over ground ?

$$L' = \frac{\mu}{2\pi} \ln\left(\frac{2d}{a}\right)$$

This is also an image theory problem.
The magnetic flux enclosed by a loop is one half of that for the two wire case.