UCLA Department of Electrical Engineering
EE101 – Engineering Electromagnetics
Winter 2013
Midterm, February 12 2013, (1:45 minutes)

Name ______________________________ Student number ______________________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

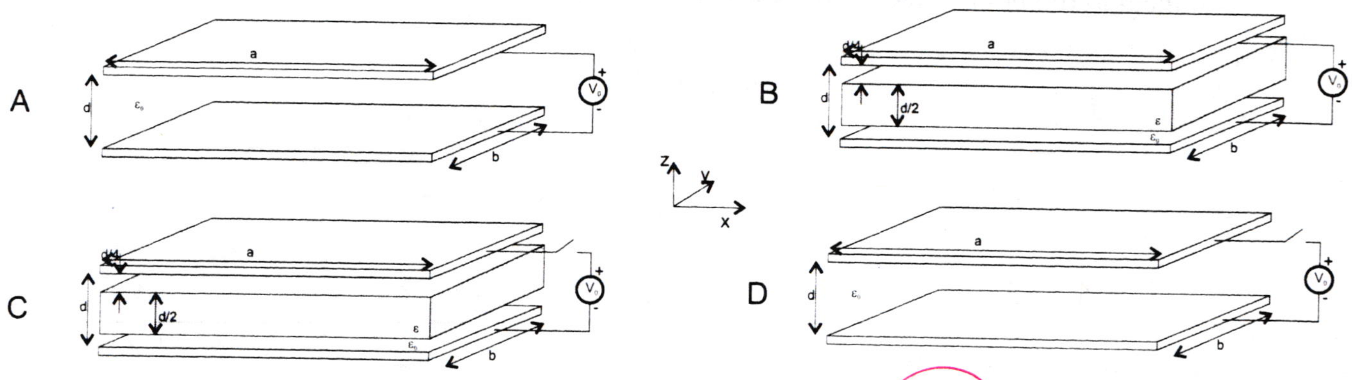
Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Capacitor	40	25
Problem 2	Transmission Line	30	30
Problem 3	Boundary condition	30	-8-
Total		100	63

1. Capacitor (40 points)



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(a) (10 points) Consider the following parallel plate capacitor with perfectly conducting metal plates, and only vacuum (ε_0) in between as shown below in figure (A). Assume the plates are held at a constant potential difference V_0 using a voltage source. Give an expression for the electric field in the gap between the plates (don't forget vector direction) in terms of V_0, and the dimensional and material quantities (i.e. a, b, d, ε, - (NOT C!)).

$\nabla^2 V = 0$ $\qquad C_{\text{parallel plate}} = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 ab}{d}$

$\frac{d^2 V}{dz^2} = 0$

$V = Az + B$

$V(0) = 0, \quad V(d) = V_0$

$0 = 0 + B \qquad V_0 = Ad$

$\rightarrow B = 0 \qquad A = V_0/d$

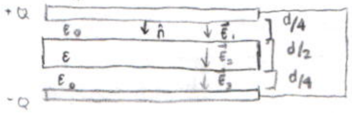
$V = \frac{V_0}{d} z$

$$\boxed{\vec{E} = -\nabla V = -\frac{V_0}{d}\hat{z}}$$

15

(b) (15 points) Now, we insert a piece of dielectric of thickness $d/2$ and with permittivity ε halfway in between the plates as shown in figure (B). Give an expression for electric field E (direction and magnitude) both inside the dielectric and in the vacuum regions.

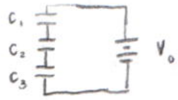
Area = ab



$C = \frac{Q}{V} = \frac{\int \vec{D} \cdot d\vec{S}}{-\int \vec{E} \cdot d\vec{l}}$

$\vec{E}_1 = \vec{E}_3$

Capacitance of parallel plate capacitor $= \frac{\varepsilon A}{d}$



$C_1 = C_3$

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\frac{1}{C_T} = \frac{d/4}{\varepsilon_0 ab} + \frac{d/2}{\varepsilon ab} + \frac{d/4}{\varepsilon_0 ab}$

$\frac{1}{C_T} = \frac{d/2}{\varepsilon_0 ab}\left(\frac{\varepsilon}{\varepsilon}\right) + \frac{d/2}{\varepsilon ab}\left(\frac{\varepsilon_0}{\varepsilon_0}\right)$

$\frac{1}{C_T} = \frac{\frac{\varepsilon d}{2} + \frac{\varepsilon_0 d}{2}}{\varepsilon \varepsilon_0 ab}$

$\frac{1}{C_T} = \frac{d(\varepsilon + \varepsilon_0)}{2\varepsilon\varepsilon_0 ab}$

$C_T = \frac{2\varepsilon\varepsilon_0 ab}{d(\varepsilon + \varepsilon_0)}$

$\frac{Q}{V_0} = \frac{2\varepsilon\varepsilon_0 ab}{d(\varepsilon + \varepsilon_0)}$

$Q = \frac{2\varepsilon\varepsilon_0 ab V_0}{d(\varepsilon + \varepsilon_0)}$

Without the dielectric, $\vec{E} = -\frac{V_0}{d}\hat{z}$

At the interface between the top plate and the vacuum region,

$\vec{D}_1 \cdot \hat{n} = \rho_s$

$-\varepsilon_0 E_{1z} = \frac{Q}{ab}$

$\vec{E}_1 = -\frac{Q}{\varepsilon_0 ab}\hat{z} = -\frac{1}{\cancel{\varepsilon_0 ab}} \cdot \frac{2\varepsilon \cancel{\varepsilon_0 ab} V_0}{d(\varepsilon + \varepsilon_0)}\hat{z}$

$\vec{E}_1 = -\frac{2\varepsilon V_0}{d(\varepsilon + \varepsilon_0)}\hat{z}$

At the interface between vacuum and dielectric,

$\varepsilon_0 E_{1n} - \varepsilon E_{2n} = \rho_s$

In a dielectric, $\rho_s = 0$

$\varepsilon_0 E_{1n} = \varepsilon E_{2n}$

$E_{2n} = \frac{\varepsilon_0}{\varepsilon} E_{1n}$

$\vec{E}_2 = -\frac{Q}{\varepsilon ab}\hat{z} = -\frac{1}{\cancel{\varepsilon ab}} \cdot \frac{2\cancel{\varepsilon}\varepsilon_0 \cancel{ab} V_0}{d(\varepsilon + \varepsilon_0)}\hat{z}$

$\vec{E}_2 = -\frac{2\varepsilon_0 V_0}{d(\varepsilon + \varepsilon_0)}\hat{z}$

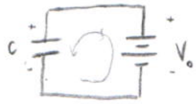
$$\boxed{\begin{array}{l} \text{In vacuum regions: } \vec{E} = -\frac{2\varepsilon V_0}{d(\varepsilon + \varepsilon_0)}\hat{z} \\ \text{In dielectric: } \vec{E} = -\frac{2\varepsilon_0 V_0}{d(\varepsilon + \varepsilon_0)}\hat{z} \end{array}}$$

0

Compare A & D

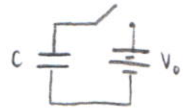
(c) (15 points) Now imagine that we open a switch connecting the voltage source (as shown in (C)) and remove the dielectric (as shown in (D)). Is the electrostatic energy in the system the same, larger, or smaller than the original configuration shown in (A)? If your answer is "same", explain why. If your answer is "larger" or "smaller", explain why, and where the energy came from or went to. (Explanations required for full credit)

Energy stored in a capacitor: $\frac{1}{2}CV^2$



$V_0 = V_c$

vs.



Open switch → no current flow

In A, the energy stored in the capacitor is $\frac{1}{2}\frac{\varepsilon_0 ab}{d}(V_0)^2$.

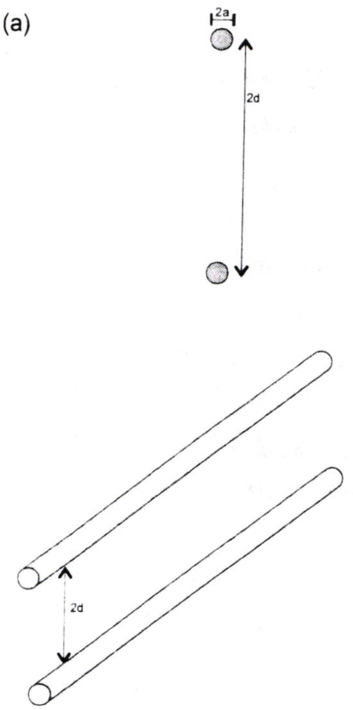
When the switch is opened, there is no longer a current flow through the system and charge begins to build up on the plates of the capacitor. The capacitance remains unchanged because it is purely a geometric quantity. Thus, the electrostatic energy remains the same.

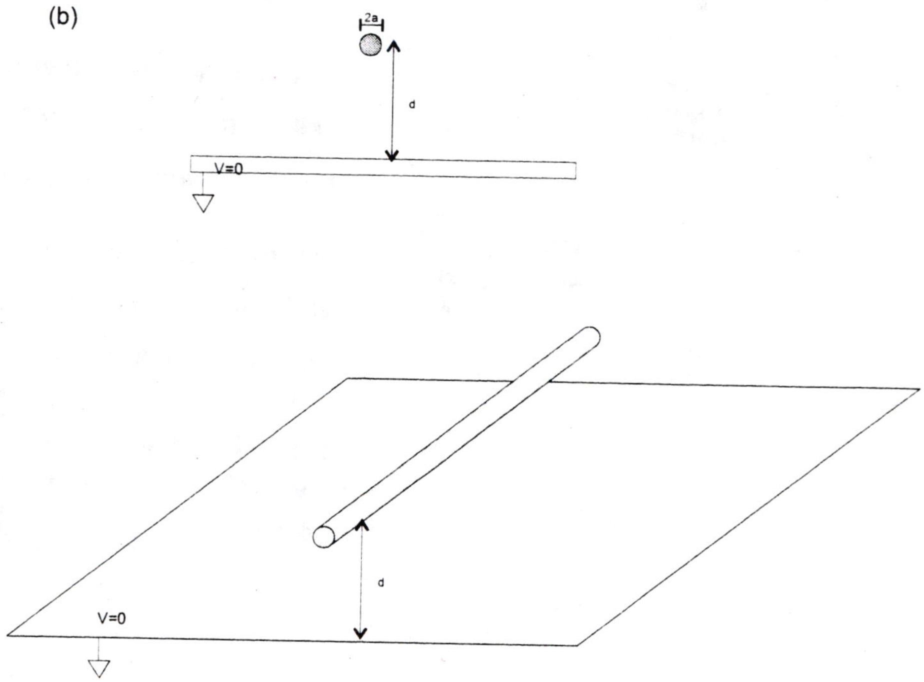
2. Transmission line (30 points)

Consider the lossless two-wire transmission line in case (a) and the single wire over a perfectly conducting semi-infinite ground plane shown in case (b)?

Two-wire Tran Line

Single wire over ground plane Tran Line





(a) (15 points) The capacitance per unit length of the two-wire line is approximately: $C' = \frac{C}{\ell} = \frac{\pi\varepsilon}{\ln(d/2a)}$. What is the capacitance per unit length of the single wire over ground plane?

The capacitance per unit length of the single wire over the ground plane is 2 times more because the potential is halved (the distance is halved).

$$\boxed{\frac{C}{\ell} = \frac{2\pi\varepsilon}{\ln(d/2a)}}$$

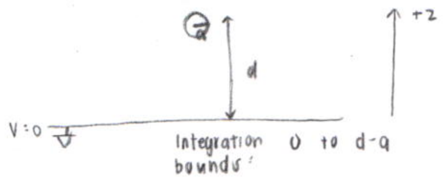
+15

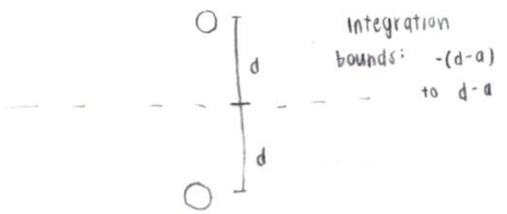
$C = \frac{Q}{V} = \frac{\oint \vec{D}\cdot d\vec{s}}{-\int \vec{E}\cdot d\vec{\ell}} \rightarrow$ For a 2-wire case, we will get $E(2d)$ whereas for a single wire, we will get $E(d)$

V for a 2-wire line will be twice as large as that of a single wire over a plane because in the two-wire case, our integral would be $-\int_{-(d-a)}^{d-a} E\,dz$ and for the single wire case, it would be $\int_{0}^{d-a} E\,dz$.

(b) (15 points) The inductance per unit length of the two-wire line is approximately:

$L' = \frac{L}{\ell} = \frac{\mu}{\pi}\ln(d/2a)$. What is the inductance per unit length of the single wire over ground plane?





$$L = \frac{N\Phi}{I} = \frac{N\int_s \vec{B}\cdot d\vec{s}}{I} = \frac{\ell\int B\, dz}{I}$$

Depending on the situation, the integration bounds for B in the z component will differ. For the 2-wire case, we would do $\int_{-(d-a)}^{d-a} B\, dz$, whereas for the single wire case, we would do $\int_0^{d-a} B\, dz$. Thus, the former integral would be twice as much as the latter. Since this is the only difference between the two equations, the inductance per unit length of the single wire would be half that of the 2-wire case: $\boxed{\frac{L}{\ell} = \frac{1}{2}\frac{\mu}{\pi}\ln\left(\frac{d}{2a}\right)}$

+15

3. Boundary conditions (30 points)

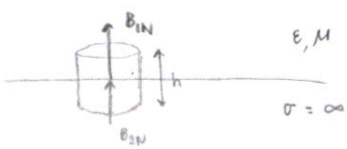
8/30

Consider the case of time-harmonic fields described by phasors $\tilde{H}$, $\tilde{B}$ for (i.e. fields varying with a single angular frequency ω).

(a) (15 points) At the interface between a perfect conductor (region 2) and a material with permittivity ε and permeability μ (region 1), we can write the boundary conditions for magnetic fields:

$$\begin{cases} \tilde{H}_{1t} = \tilde{J}_s, \tilde{H}_{2t} = 0 & \text{(transverse fields)} \\ \tilde{B}_{1n} = \tilde{B}_{2n} = 0 & \text{(normal fields)} \end{cases}$$

Explain qualitatively why the normal B-field must go to zero on both sides of the interface at a perfect conductor (σ = ∞). You should identify in your explanation which of Maxwell's laws is responsible. If you wish, you may supplement your explanation with diagrams or equations.



$\nabla \cdot \vec{B} = 0$

when finding boundary conditions for the magnetostatic case, ✓ to find the normal components, we use a Gaussian cylinder at the interface. $\oint_S \vec{B} \cdot d\vec{S} = 0$ by Gauss's Law for Magnetism $(\nabla \cdot \vec{B} = 0) \rightarrow B_{1n} = B_{2n}$. However, in a perfect conductor, the electric field must be zero. Because this is the case, by Ohm's Law, $\vec{J} = \sigma \vec{E}$, with no electric field, there can be no current not true since $\sigma = \infty$ in the conductor, so B_{2n} must be zero, which means B_{1n} must also be zero.

why? How? Maxwell's Eq?

-10

5/15

(b) (15 points) Now consider the case where the conductor is not perfect ($\sigma \neq \infty$). For which case is the boundary condition listed in (a) a better approximation: the low frequency case ($\omega \to 0$), or the high frequency case ($\omega \to \infty$). Explain why.

$t \gg \tau_m$

$t \ll \tau_m$ Free currents in conductor will cancel applied magnetic fields.

In the high frequency case, free currents in the conductor will cancel or screen the applied magnetic fields. However, in the low frequency case, the opposite occurs. $\vec{J}$ is very small and the applied magnetic fields are not very affected by $\vec{J}$ and the induced magnetic field tends to zero.

This low frequency case seems to be a better approximation, because the condition in $\vec{B}$ is unaffected by $\vec{J}$.

~~0/15~~ 3/15