UCLA Department of Electrical Engineering
EE101 – Engineering Electromagnetics
Winter 2012
Midterm, February 14 2012, (1:45 minutes)

Name [redacted] Student number [redacted]

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

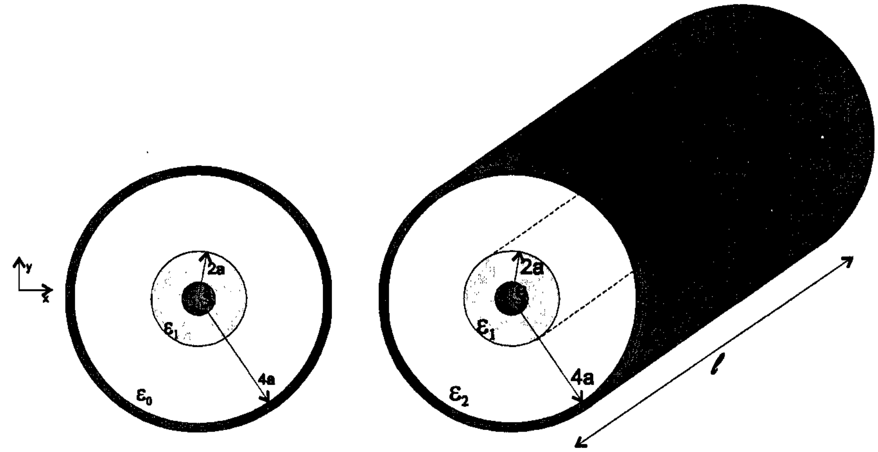
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

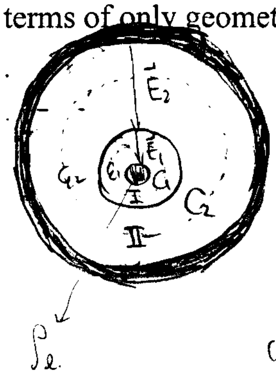
	Topic	Max Points	Your points
Problem 1	Coaxial Capacitor	50	50
Problem 2	Inductor	50	48
Problem 3			
Total		100	98

1. Coaxial capacitor (50 points)

Consider a piece of coaxial cable of length ℓ with two dielectric layers with permittivities ε_1 and ε_2. You may consider the inner conductor (radius a) and the outer conductor shell (radius $4a$) to be perfect conductors.



(a) What is the capacitance C between the inner and outer conductors? (Your answer should be in terms of only geometric and material parameters.)



(1) In region I: $\oint \vec{E}_1 \cdot d\vec{s} = \frac{\rho_\ell \ell}{\varepsilon_1} \Rightarrow E(2\pi r \ell) = \frac{\rho_\ell \ell}{\varepsilon_1} \Rightarrow E_1 = \frac{\rho_\ell}{2\pi r \varepsilon_1}$

$V_1 = -\int_{2a}^{a} \vec{E}_1 \cdot d\vec{l} = -\int_{2a}^{a} \frac{\rho_\ell}{2\pi r \varepsilon_1} dr = -\frac{\rho_\ell}{2\pi\varepsilon_1} \ln r \Big|_{2a}^{a}$

$= -\frac{\rho_\ell}{2\pi\varepsilon_1}\left(\ln\frac{1}{2}\right) = \frac{\rho_\ell}{2\pi\varepsilon_1}\ln(2)$

(2) In region II: $\oint \vec{E}_2 \cdot d\vec{s} = \frac{\rho_\ell \ell}{\varepsilon_2} \Rightarrow E_2(2\pi r \ell) = \frac{\rho_\ell \ell}{\varepsilon_2} \Rightarrow E_2 = \frac{\rho_\ell}{2\pi r \varepsilon_2}$

$V_2 = -\int_{4a}^{2a} \vec{E}_2 \cdot d\vec{l} = -\int_{4a}^{2a} \frac{\rho_\ell}{2\pi r \varepsilon_2} dr = -\frac{\rho_\ell}{2\pi\varepsilon_2} \ln r \Big|_{4a}^{2a} = -\frac{\rho_\ell}{2\pi\varepsilon_2}\ln\left(\frac{1}{2}\right)$

$= \frac{\rho_\ell}{2\pi\varepsilon_2}\ln(2)$

(3) $C_1 = \frac{Q}{V_1} = \frac{\rho_\ell \ell}{\frac{\rho_\ell}{2\pi\varepsilon_1}\ln(2)} = \frac{2\pi\varepsilon_1 \ell}{\ln(2)}$

$C_2 = \frac{Q}{V_2} = \frac{\rho_\ell \ell}{\frac{\rho_\ell}{2\pi\varepsilon_2}\ln(2)} = \frac{2\pi\varepsilon_2 \ell}{\ln(2)}$

(4) C_1 and C_2 are in series, so

$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\ln(2)}{2\pi\varepsilon_1\ell} + \frac{\ln(2)}{2\pi\varepsilon_2\ell} = \frac{\ln(2)(\varepsilon_2+\varepsilon_1)}{2\pi\varepsilon_1\varepsilon_2\ell}$

$\Rightarrow \boxed{C_{total} = \frac{2\pi\varepsilon_1\varepsilon_2\ell}{\ln(2)(\varepsilon_1+\varepsilon_2)} \text{ F}}$

25

(b) Consider that the dielectric layers are lossy with conductivities σ_1 and σ_2 respectively. What is the resistance R between the inner and outer conductors?

(1) $$R_1 = \frac{\varepsilon_1}{C_1\sigma_1} = \frac{\cancel{\varepsilon_1}}{\frac{2\pi\cancel{\varepsilon_1}l}{\ln(2)}\sigma_1} = \frac{\ln(2)}{2\pi l\sigma_1}$$

$$R_2 = \frac{\varepsilon_2}{C_2\sigma_2} = \frac{\cancel{\varepsilon_2}}{\frac{2\pi\varepsilon_2 l}{\ln(2)}\sigma_2} = \frac{\ln(2)}{2\pi l\sigma_2}$$

(2) R_1 and R_2 are in series, so:

$$R_{total} = R_1 + R_2 = \frac{\ln(2)}{2\pi l\sigma_1} + \frac{\ln(2)}{2\pi l\sigma_2}$$

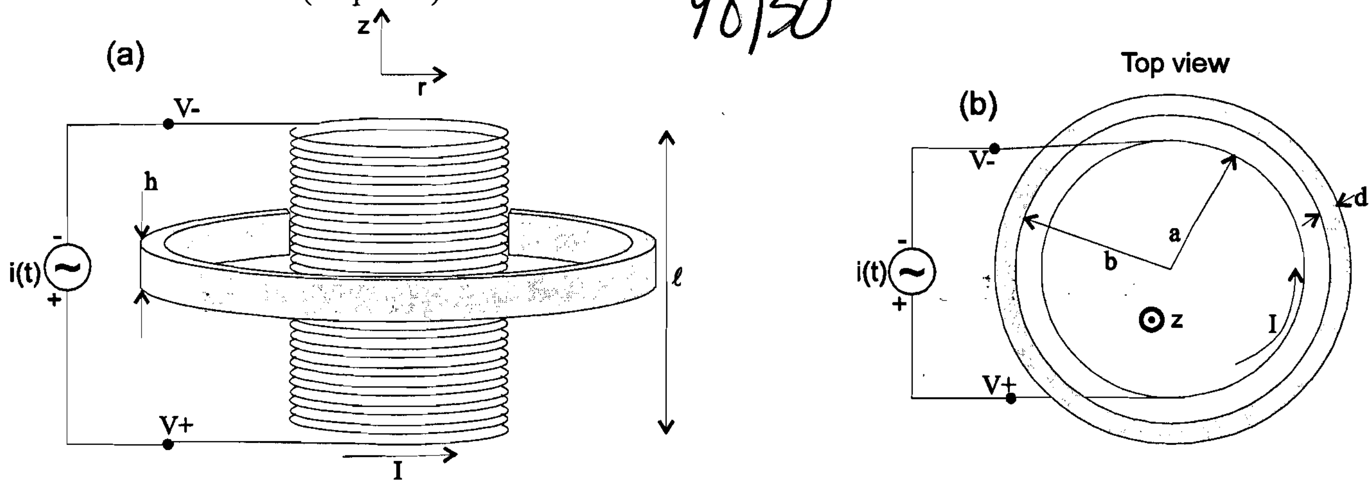
$$= \frac{\ln(2)}{2\pi l}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)$$

$$\boxed{R_{total} = \frac{\ln(2)}{2\pi l}\left(\frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}\right) \ (\Omega)}$$

25

2. Inductor (50 points)

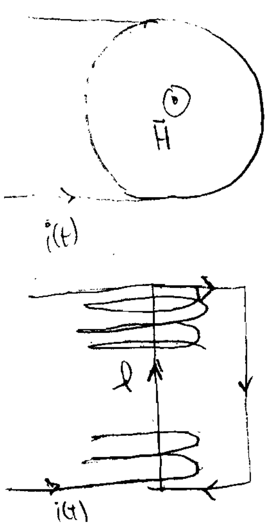
48/50



(a) (20 points) For part (a), ignore the metal ring in the figure.

Consider a solenoid with N turns and length ℓ, and radius a, is driven by a current source $i(t)=i_0\cos(\omega t)$. As shown in the figure, we use the convention that a positive current is associated with current in the ϕ direction. You may assume that the solenoid is long ($\ell \gg a$). What is the voltage difference $v(t)=V_+ - V_-$ that appears across the terminals?

Your answer should be in terms of geometric, material, and fundamental parameters only (i.e. N, ℓ, a, b, ϵ, μ, σ, etc.).



(1) $\oint \vec{H}\cdot d\vec{l} = I \Rightarrow H\ell = NI \Rightarrow H = \frac{NI}{\ell} \Rightarrow \vec{H}(t) = \frac{N i_0 \cos(\omega t)}{\ell}\hat{z}$

(2) $\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 N i_0 \cos(\omega t)}{\ell}\hat{z}$ ✓

(3) $\Phi = \int_S \vec{B}\cdot d\vec{S} = \int_S \frac{\mu_0 N i_0 \cos(\omega t)}{\ell}\hat{z}\cdot(\hat{z}\,dS) = \frac{\mu_0 N i_0 \cos(\omega t)\pi a^2}{\ell}$ ✓ +10

(4) $V_{emf} = -N\frac{d\Phi}{dt} = -\frac{\mu_0 N^2 i_0(-\omega\sin(\omega t))\pi a^2}{\ell} = \frac{\mu_0 N^2 i_0 \omega \sin(\omega t)\pi a^2}{\ell}$

(5) $V(t) = V_+ - V_- = V_{emf} = -\boxed{\frac{\mu_0 N^2 i_0 \omega \sin(\omega t)\pi a^2}{\ell}\ (V)}$ +8

18/20

(b) (20 points) A metal ring with height h, radius b, and thickness d, and conductivity σ is placed around the center of the solenoid as shown in the figure. You may assume $d \ll b$. What is the current density $\mathbf{J}(t)$ that flows in the metal ring (give vector direction)? When answering this, you may neglect any contribution due the currents $\mathbf{J}$(t) in the metal ring.

(1) Flux Φ is flowing inside the metal ring, and it's changing

so,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} \Rightarrow 2\cancel{\pi} b E_\phi = -\frac{\mu_0 N i_0 (-\omega \sin(\omega t)) \cancel{\pi} a^2}{l} \Rightarrow E_\phi = \frac{\mu_0 N i_0 \omega \sin(\omega t) a^2}{2bl}$$

$$\vec{E}_\phi = E_\phi \hat{\phi}$$ ✓

(2) $\vec{J}(t) = \sigma \vec{E}$

$$\boxed{\vec{J}(t) = \frac{\sigma \mu_0 N i_0 \omega \sin(\omega t) a^2}{2bl} \hat{\phi} \ \left(\frac{A}{m^2}\right)}$$ ✓ ✓

+20

20/20

(c) (~~20~~ 10 points) Does the approximation used in part (b) (neglecting any contribution due the currents **J**(t) in the metal ring) correspond to the low frequency limit (ω is small compared to τ_m^{-1}), or the high frequency limit (ω is large compared to τ_m^{-1})? Give a qualitative explanation why.

10

Neglecting any contribution due to $\vec{J}(t)$ in the metal ring means neglecting the induced H-field (call it H_1) caused by $\vec{J}(t)$. In other words, $H_1 \approx 0$. Insulators show this kind of behavior since insulators do not have a lot of free electrons, so their $\vec{J}(t)$ is negligible, too. So, lets treat this metal as an insulator. Insulator has very low conductivity ($\sigma \approx 0$), so its $\tau_m \approx 0$ since $\tau_m \propto \sigma$. Because $\tau_m \approx 0$, ω will always be small compared to τ_m^{-1} since $\tau_m^{-1} = \frac{1}{\tau_m} \to \infty$. Therefore, the approximation used in part (b) correspond to the low frequency limit.

✓